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UNIVERSITY ARITHMETIC

EMBRACING THE

SCIENCE OF NUMBERS,

AND

GENERAL RULES FOR THEIR APPLICATION.

By CHARLES DAVIES, LL. D.,

AUTHOR OF PRIMARY, INTELLECTUAL, AND SCHOOL ARITHMETICS; ELEMENTARY
ALGEBRA; ELEMENTARY GEOMETRY; PRACTICAL MATHEMATICS; ELEMENTS
OF SURVEYING; ELEMENTS OF ANALYTICAL GEOMETRY; DESCRIPTIVE
GEOMETRY; SHADES, SHADOWS, AND PERSPECTIVE; DIFFERENTIAL
AND INTEGRAL CALCULUS; AND LOGIC AND
UTILITY OF MATHEMATICS.

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TO

THE TEACHERS OF THE UNITED STATES,

THIS

TREATISE ON ARITHMETIC,

THE LAST OF A SERIES OF WORKS DESIGNED TO LESSEN THE LABOR
AND IMPROVE THE SYSTEMS OF TEACHING,

IS

RESPECTFULLY DEDICATED,

BY

THE AUTHOR.

IT IS OFFERED AS A TOKEN OF HIS GRATEFUL APPRECIATION OF THE INDULGENCE
WITH WHICH HIS OTHER WORKS HAVE BEEN RECEIVED, AND AS A TESTIMONY
OF HIS REGARD FOR THOSE WITH WHOM HE HAS LONG BEEN A CO-LABORER
IN THE WORK OF PUBLIC INSTRUCTION

PREFACE.

SCIENCE, in its popular signification, means knowledge reduced to order; that is, knowledge so classified and arranged as to be easily remembered, readily referred to, and advantageously applied. More strictly, it is a *knowledge of laws, relations, and principles.*

ARITHMETIC is the science of numbers, and the art of applying numbers to all practical purposes. It is the foundation of the exact and mixed sciences, and an accurate knowledge of it is an important element either of a liberal or practical education.

It is the first subject, in a well-arranged course of instruction, to which the reasoning faculties of the mind are applied, and is the guide-book of the mechanic and man of business. It is the first fountain at which the young votary of knowledge drinks the pure waters of intellectual truth.

It has seemed, to the author, of the first importance that this subject should be carefully treated in our Elementary Text-books. In the hope of contributing something to so desirable an end, he has prepared a series of arithmetical works, embracing four books, entitled, Primary Arithmetic; Intellectual Arithmetic; Practical Arithmetic; and University Arithmetic—the latter of which is the present volume.

PRIMARY ARITHMETIC. This first-book is adapted to the capacities and wants of young children. Sensible objects are employed to illustrate and make familiar the simple combinations and relations of numbers. Each lesson embraces one combination of numbers, or one set of combinations.

Intellectual Arithmetic. This work is designed to present a thorough analysis of the science of numbers, and to form a complete course of mental arithmetic. I have aimed to make it accessible to young pupils by the simplicity and gradation of its methods, and to adapt it to the wants of advanced students by a scientific arrangement and logical connection, in all the higher processes of arithmetical analysis.

Practical Arithmetic. Great pains have been taken, in the preparation of this book, to combine *theory* and *practice;* to explain and illustrate principles, and to apply them to the common business transactions of life—to make it *emphatically a practical work.* The student is required to demonstrate every principle laid down, by a course of mental reasoning, before deducing a proposition or making a practical application of a rule to examples. He is required to fix and apprehend the *unit* or *base* of all numbers, whether integral or fractional—to reason with constant reference to this base, and thus make it the *key* to the solution of all arithmetical questions. It is hoped, that the language used in the statement of principles, in the definition of terms, and in the explanation of methods, will be found to be clear, exact, brief, and comprehensive.

University Arithmetic. This work is designed to answer another object. Here, the entire subject is treated as a *science.* The pupil is supposed to be familiar with the simple operations in the four ground rules, and with the first principles of fractions, these being now taught to small children, either orally or from elementary treatises. This being premised, the language of figures, which are the representatives of numbers, is carefully taught, and the different significations of which the figures themselves are susceptible, depending on the manner in which they are written, are fully explained. It is shown, for example, that the simple numbers in which the value of the unit increases from right to left according to the scale of tens, and the Denominate or Compound numbers in which it increases according to a varying scale, belong to the same class of numbers, and that both may be treated under the same rules. Hence, the rules for Notation, Addition, Subtraction, Multiplication, and Division, have been so constructed as to apply equally to all numbers. This arrangement, which the author has not seen elsewhere, is deemed an essential improvement in the science of Arithmetic.

In developing the properties of numbers, from their elementary to their highest combinations, great labor has been bestowed on classification and arrangement. It has been a leading object to present the entire subject of arithmetic as forming *a series of dependent and connected propositions:* so that the pupil, while acquiring useful and practical knowledge, may at the same time be introduced to those beautiful methods of reasoning which science alone teaches.

Great care has been taken to demonstrate every proposition —to give a complete analysis of all the methods employed, from the simplest to the most difficult, and to explain fully the reason of every rule. A full analysis of the science of Numbers has developed but *one law;* viz, *the law which connects all the numbers of arithmetic with the unit one,* and *which points out the relations of these numbers to each other.*

In the Appendix, which treats of Units, Weights, and Measures, &c., the methods of determining the Arbitrary Unit, as well as the general law which prevails in the formation of numbers, are fully explained. I cannot too earnestly recommend this part of the work to the special attention of Teachers and pupils.

In fine, the attention of Teachers is especially invited to this work, because *general methods* and *general rules* are employed to abridge the common arithmetical processes, and to give to them a more *scientific* and *practical* character. In the present edition, the matter is presented in a new form; the arrangement of the subjects is more natural and scientific; the methods have been carefully considered; the illustrations abridged and simplified; *the definitions* and rules thoroughly revised and cor rected; and a very large number and variety of practical examples have been added. The subjects of Fractions, Proportion, Interest, Percentage, Alligation, Analysis, and Weights and Measures, present many new and valuable features, which are not found in other works.

A Key to the present work has also been published for th use of such Teachers as may desire it,—prepared with great care, containing not only the answers and solutions of all the examples, but a full and comprehensive *analysis* of the more difficult ones.

The author has great pleasure in acknowledging the interest which Teachers have manifested in the success of his labors: they have suggested many improvements, both in rules and methods, not only in his elementary, but also in his advanced works. The recitation-room is the final tribunal, and the intelligent teacher the final judge, before which all text-books must stand or fall.

A Key to this volume has been prepared for the use of *Teachers only.*

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UNIVERSITY ARITHMETIC.

Definitions.

1. A **Unit** is a single thing, or one.

2. A **Number** is a unit, or a collection of units.

3. **Science** treats of the properties and relations of things **Art** is the practical application of the principles of Science.

4. **Arithmetic** is the Science of Numbers, and also the Art of applying numbers to practical purposes.

5. A **Proposition** is something to be done, or demonstrated.

6. An **Analysis** is an examination of the separate parts of a proposition.

7. An **Operation** is the doing of something with numbers.

8. A **Rule** is the direction for performing an operation.

9. An **Answer** is the result of a correct operation.

Operations of Arithmetic.

10. There are, in Arithmetic, five fundamental operations: Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

---

1 What is a Unit?—2. What is a Number?—3. Of what does Science treat? What is Art?—4. What is Arithmetic?—5. What is a Proposition?—6. What is an Analysis?—7. What is an Operation?—8. What is a Rule?—9. What *is an Answer*?—10. How many fundamental oper*ations are there in Arithmetic?* What are they?

Expressing Numbers.

11. There are three methods of expressing numbers:

1. By words, or common language;
2. By letters, called the Roman method;
3. By figures, called the Arabic method.

Expressing Numbers by Words.

12. A single thing is called	*One.*
One and one more	*Two.*
Two and one more	*Three*
Three and one more	*Four.*
Four and one more	*Five.*
Five and one more	*Six.*
Six and one more	*Seven.*
Seven and one more	*Eight.*
Eight and one more	*Nine.*
Nine and one more	*Ten.*

Each of the words, *one, two, three, four,* &c., expresses a number, and denotes how many units are taken. These words are generally called numbers; though, in fact, they are but the *names* of numbers.

NOTATION AND NUMERATION.

13. Notation is the method of expressing numbers, either by letters or figures.

Numeration is the art of reading, correctly, any number expressed by letters or figures.

There are two methods of Notation: the one by letters, the other by figures. The method by letters is called the *Roman Notation;* the method by figures is called the *Arabic Notation.*

Roman Notation.

14. In the Roman Notation, seven capital letters are used They express the following values :

I	V	X	L	C	D	M
one,	five,	ten	fifty,	one hundred,	five hundred,	one thousand.

All other numbers are expressed by combining these letters according to the following principles :

1. Every time a letter is repeated, the number which it denotes is repeated.

2. If a letter denoting a *less* number be written on the *right* of one denoting a *greater*, the number expressed will be the *sum* of the numbers.

3. If a letter denoting a *less* number be written on the *left* of one denoting a *greater*, the number expressed will be the *difference* of the numbers.

4. A dash (—), placed over a letter, increases the number for which it stands, a thousand times.

Roman Table.

I	One.
II	Two.
III	Three.
IV	Four.
V	Five.
VI	Six.
VII	Seven.
VIII	Eight.
IX	Nine.
X	Ten.
XX	Twenty.
XXX	Thirty.
XL	Forty.
L	Fifty.
LX	Sixty.
LXX	Seventy.
LXXX	Eighty.
XC	Ninety.
C	One hundred
CC	Two hundred.
CCC	Three hundred.
CCCC	Four hundred.
D	Five hundred.
DC	Six hundred.
DCC	Seven hundred.
DCCC	Eight hundred.
DCCCC	Nine hundred.
M	One thousand.
MD	Fifteen hundred.
MM	Two thousand.
$\overline{V}$	Five thousand.
$\overline{X}$	Ten thousand.

NOTE.—This Notation was used by the Romans hence its name It is still *used for dates, numbering* chapters, pages, &c.

Examples

Express the following numbers in Roman Notation:

1. Eleven.
2. Fourteen.
3. Sixteen.
4. Seventeen.
5. Nineteen.
6. Twenty-two.
7. Twenty-eight
8. Twenty-nine.
9. Thirty-three.
10. Thirty-seven.
11. Thirty-eight.
12. Forty-three.
13. Forty-seven.
14. Forty-nine.
15. Fifty-six.
16. Fifty-eight.
17. Fifty-nine.
18. Sixty-five.
19. Sixty-nine.
20. Sixty-seven.
21. Seventy-five.
22. Seventy-six.
23. Eighty-one.
24. Eighty-seven.
25. Eighty-nine.
26. Ninety-four.
27. Ninety-five.
28. Ninety-seven.
29. Ninety-nine.
30. One hundred and fifteen
31. Seven hundred and fifty
32. One thousand and sixty
33. Two thousand and forty
34. Five hundred and sixty.
35. Nine hundred and sixty
36. Six hundred and ninety.
37. One thousand and fifty.
38. Four thousand and four.
39. Six thousand and nine.
40. Nine thousand and nine.
41. Eight hundred and six.
42. Six hundred and eight.
43. Eight thousand and six.
44. Two thousand and one.

---

11. How many methods are there of expressing numbers? What are they?

12. What does each of the words, *one*, *two*, *three*, &c., denote? What are these words generally called? What are they, in fact?

13. What is Notation? What is Numeration? How many methods of Notation are there? What are they?

14. How many letters does the Roman notation employ? Which are they? What value does each represent? What is the effect of repeating a letter? What is the number, when a letter denoting a less number is placed on the right of one denoting a greater? What is the number, when a letter denoting a less number is placed on the left of one denoting a greater? What is the effect of placing a dash over a letter?

Arabic Notation.

15. Arabic Notation is the method of expressing numbers by figures. Ten figures are used, and they form the *Alphabet* of the Arabic Notation. They are,

0	1	2	3	4	5	6	7	8	9
naught,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

The *naught*, 0, is also called *cipher*. It denotes no number but the absence of a thing. Thus, if there are no apples in a basket, we write, the number of apples in the basket is 0 The other nine figures are called *Significant Figures*, or *Digits*

Orders of Units.

16. We have no single figure for the number ten. We therefore *combine* the figures already known. This we do by writing 0 on the right hand of 1:

Thus, ... 10

which is read, *ten*.

This 10 is equal to *ten* of the units expressed by 1. It is, however, but a *single ten*, and may be regarded as a *unit*, *ten times* as great as the unit 1. It is called, a unit of the *second order*.

17. When two figures are written by the side of each other, the one on the right is in the *place of units*, and the other in the *place of tens*, or of *units of the second order*. *Each unit of the second order is equal to ten units of the first order.*

When units simply are named, *units of the first order are always meant.*

Units of the second order are written thus:

One ten, or	10	Six tens, or sixty, . . .	60
Two tens, or twenty, . .	20	Seven tens, or seventy,. .	70
Three tens, or thirty, .	30	Eight tens, or eighty, . .	80
Four tens, or forty, . . .	40	Nine tens, or ninety, . .	90
Five tens, or fifty, . .	50	One hundred, . .	100

18. To express *ten units of the second order*, or *one hundred*, we form a new combination:

Thus, ... 100

by writing two ciphers on the right of 1. This number is read, *one hundred*, and is a unit of the *third order*.

We an now express any number less than one thousand.

In the number two hundred and fifty-five, there are 5 units, 5 tens, and 2 hundreds. Write, therefore, 5 units of the first order, 5 units of the second order, and 2 of the third; and read from the right, *units, tens, hundreds;* and from the left, *two hundred and fifty-five.*

huns.	tens.	units
2	5	5

In the number five hundred and ninety-five, there are 5 units of the first order, 9 of the second, and five of the third; and it is read from the right, *units, tens, hundreds.*

huns.	tens.	units
5	9	5

In the number six hundred and four, there are 4 units of the first order, 0 of the second, and 6 of the third.

huns.	tens.	units.
6	0	4

The right-hand figure always expresses units of the first order; the second, units of the second order; and the third, units of the third order.

19. To express *ten units of the third order*, or *one thousand*, we form a new combination:

Thus, ... 1000

by writing three ciphers on the right of 1. This number is read, *one thousand*, and is a unit of the *fourth order*.

We may now form as many orders of units as we please:

A single unit of the first order is expressed by	1
A unit of the second order by 1 and 0; thus, ...	10
A unit of the third order by 1 and two 0's; . .	100
A unit of the fourth order by 1 and three 0's; . .	1000
A unit of the fifth order by 1 and four 0's;	10000

And so on, for units of higher orders.

Hence, the following principles:

1st. *The same figure expresses different units according to the place which it occupies:*

2d. *Units of the first order occupy the place at the right, units of the second order, the second place; units of the third order, the third place; and the unit of any figure is determined by the number of its place:*

3d. *Ten units of the first order make one of the second, ten of the second, one of the third; ten of the third, one of the fourth; and so on for the higher orders:*

4th. *When figures are written by the side of each other, ten units in any one place make one unit of the place next at the left.*

Examples in Writing the Orders of Units.

1. Write 7 units of the 1st order.
2. Write 8 units of the 2d order.
3. Write 9 units of the 4th order.
4. Write 3 units of the 1st order, with 9 of the 2d.

---

15. What is the Arabic Notation? How many figures are used? What do they form? Name the figures. What does 0 express? What are the other figures called?

16. Have we a separate character for ten? How do we express ten? To how many units 1 is 1 ten equal? May ten be regarded as a single unit? Of what order?

17. When two figures are written by the side of each other, what place does the right-hand figure occupy? The figure on the left? When units simply are named, what units are meant?

18. How do you write one hundred? To how many units of the second order is it equal? To how many of the first order? How may it be regarded? Of what order? How many units of the third order in 200? In 600? In 900?

19. To what are ten units of the third order equal? How do you write it? How do you write a single unit of the first order? How do you write a unit of the second order? Of the third? Of the fourth? Ten units of the first order, make what? Ten of any order, make what? When figures are written by the side of each other, how many units of any place *make one unit of* the place next to the left?

5. Write 9 units of the 3d order, with 6 of the 2d, and 1 of the 1st.

6. Write 0 units of the 2d order, 8 of the 1st, with 4 of the 3d, and 7 of the 4th.

7. Write 8 units of the 6th order, 7 of the 4th, 9 of the 5th, 0 of the 3d, 2 of the 2d, and 1 of the 1st.

8. Write 8 units of the 8th order, 6 of the 7th, 0 of the 1st, 3 of the 2d, 4 of the 3d, 9 of the 4th, 0 of the 6th, and 2 of the 5th.

9. Write 4 units of the 10th order, 8 of the 7th, 3 of the 9th, 2 of the 8th, 0 of the 6th, 3 of the 1st, 6 of the 2d, 0 of the 3d, 1 of the 4th, and 2 of the 5th.

10. Write 3 units of the 2d order, 2 of the 1st, 9 of the 3d, 0 of the 4th, 9 of the 9th, 6 of the 8th, 7 of the 7th, 0 of the 6th, and 4 of the 5th.

11. Write 3 units of the 11th order, 0 of the 10th, 8 of the 4th, 0 of the 5th, 2 of the 6th, 0 of the 7th, 3 of the 8th, 4 of the 9th, 1 of the 3d, 2 of the 2d, and 3 of the 1st.

12. Write 3 units of the 12th order, 6 of the 11th, 3 of the 8th, 7 of the 6th, 2 of the 4th, and 1 of the 2d.

13. Write 5 units of the 13th order, 8 of the 12th, 0 of the 9th, 6 of the 7th, 8 of the 3d, and 12 of the 1st.

14. Write 7 units of the 14th order, 5 of the 13th, 6 of the 12th, 5 of the 10th, 7 of the 8th, 9 of the 6th, 5 of the 4th, and 8 of the 1st.

15. Write 9 units of the 15th order, 4 of the 13th, 8 of the 9th, 2 of the 6th, 7 of the 3d, and 2 of the 2d.

16. Write 6 units of the 16th order, 9 of the 12th, 7 of the 9th, 4 of the 7th, 0 of the 6th, 8 of the 4th, 9 of the 5th, and 2 of the 2d.

17. Write 8 units of the 20th order, 5 of the 18th, 6 of the 13th, 4 of the 11th, 9 of the 9th, 1 of the 17th, 4 of the 5th, and 9 of the 3d.

18. Write 6 units of the 10th order, 5 of the 8th, 9 of the 9th, 0 of the 4th, and 1 of the 1st

19. Write 9 units of the 18th order, and then diminish the figure of each order by 1 till you come to and include 0; then increase the figure of each order by 1, till you reach the first order; and then read each order.

Numeration Table

7th Period. Quintillions.			6th Period. Quadrillions.			5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		
Hundreds of Quintillions.	Tens of Quintillions.	Quintillions.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
3	7	0,	8	9	4,	2	1	6,	6	3	6,	8	0	6,	3	0	4,	6	2	5

NOTES.—1. Numbers expressed by more than three figures are written and read by periods, as shown in the above table.

2. Each period always contains three figures, except the left-hand period, which may contain one, two, or three figures.

3. The unit of the first, or right-hand period, is 1; of the second period, 1 thousand; of the third, 1 million; of the fourth, 1 billion; and so on, for periods, still to the left.

4. To Quintillions succeed Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c.

5. The pupils should be required to commit, thoroughly, the names of the periods, so as to repeat them in their regular order from left to right, as well as from right to left.

6. Formerly, in the English Notation, *six* places were given to Millions. They were read, Millions, Tens of Millions. Hundreds of Millions, *Thousands* of Millions, *Tens of Thousands* of Millions, *Hundred of Thousands* of Millions. This method produced great irregularity in the Notation, as it gave *three* places to the units of the first two periods (viz.: units and thousands), and six places to the next denomination. The French method, which gives *three places to the unit* of each period, is *fully adopted in this* country, and must soon become universal.

Notation and Numeration.

Rule for Notation.

I. *Begin at the left hand and write each period in order, as if it were a period of units:*

II *When the number, in any period except the left-hand period, can be expressed by less than three figures, prefix one or two ciphers; and when a vacant period occurs, fill it with ciphers.*

Rule for Numeration.

I. *Separate the number into periods of three figures each, beginning at the right hand:*

II. *Name the unit of each figure, beginning at the right:*

III. *Then, beginning at the left hand, read each period as if it stood alone, naming its unit.*

Examples for Practice.

Express the following numbers in figures.

1. Six hundred and twenty-one.
2. Five thousand seven hundred and two.
3. Eight thousand and one.
4. Ten thousand four hundred and six.
5. Sixty-five thousand and twenty-nine.
6. Forty millions two hundred and forty-one.
7. Fifty-nine millions three hundred and ten.
8. Eleven thousand eleven hundred and eleven.
9. Three hundred millions one thousand and six.
10. Sixty-nine billions three millions and two hundred.

Let the pupil point off and read the following numbers; then write them in words:

11. 97
12. 326
13. 3302
14. 65042
15. *742604*
16. 32045607
17. 90464213
18. 47364291
19. 4037902169
20. 91046302
21. 784236704
22. 7403026054
23. 21704080495
24. 21896720421
25. 8140290308091

26. 8504680467023
27. 90403040720156
28. 172304736893210
29. 30467214302704
30. 167320410341204
31. 2164032189765421

Let each of the above examples, after being written on the blackboard, be analyzed as a class exercise; thus—

1. In how many ways may the number 97 be read?
   1st. The common way, ninety-seven.
   2d. We may read, 9 tens, and 7 units.
2. In how many ways may 326 be read?
   1st. By the common way, three hundred and twenty-six.
   2d. Three hundred, 2 tens, and 6 units.
   3d. Thirty-two tens, and six units.
3. In how many ways may the number 5302 be read?
   1st. Five thousand three hundred and two.
   2d. Five thousand, three hundred, 0 tens, and 2 units.
   3d. Fifty-three hundred, 0 tens, and 2 units.
   4th. Five hundred and thirty tens, and 2 units.
4. In 65042, how many ten thousands? How many thousands? How many hundreds? How many tens? How many units?
5. In 742604, how many hundred thousands? How many ten thousands? How many thousands? How many hundreds? How many tens? How many units?

Let the pupil express the following in figures:

32. Forty-seven quadrillions, sixty-nine billions, four hundred and sixty-five thousand, two hundred and seven.

33. Eight hundred quintillions, four hundred and twenty-nine millions, six thousand and nine.

34. Ninety-five sextillions, eighty-nine millions, eighty-nine thousand, three hundred and six.

35. Six quintillions, four hundred and fifty-one billions, sixty five millions, forty-seven thousand, one hundred and four.

36. Nine hundred and ninety-nine billions, sixty-five millions, eight hundred and forty-one thousand, four hundred and eleven.

Formation of Numbers.

20. ONE refers to a *single thing*, and has no reference to *kind* or *quality*. It is called an *Abstract Unit:* hence, AN ABSTRACT NUMBER is one whose unit is abstract.

21. A DENOMINATE NUMBER is one whose unit is named, or denominated: thus, three feet, four dollars, five pounds are denominate numbers.

22. A CONCRETE NUMBER is a denominate number which carries with it the idea of matter; as six pounds, one ton.

23. A SIMPLE NUMBER is a single unit, or a single collection of units, either abstract or denominate.

Two numbers are of the *same* denomination when they have the same unit; and of *different* denominations when they have different units.

24. A COMPOUND DENOMINATE NUMBER is one expressed by two or more different units; as, 1 yard 2 feet 6 inches.

Laws of the Units and Scales.

25. We have seen that when figures are written by the side of each other, thus,

6 7 8 9 0 4,

the language implies that ten units, of any place, make one unit of the place next to the left.

When figures are written to express English Currency, thus,

£	s.	d.	far.
4	17	10	3,

the language implies, that four units of the lowest denomination

---

20. To what does one refer? What is it called? To what does one foot refer? What is it called? What is an Abstract Number?—21. What is a Denominate Number?—22. What is a Concrete Number?—23. What is a Simple Number? When are two numbers of the *same* denomination? When of different denominations?—24. What *is a Compound Denominate* Number?

make one unit of the next higher; twelve of the second, one of the third; and twenty of the third, one of the fourth.

When figures are written to express Avoirdupois weight, thus,

T.	cwt	qr.	lb.	oz.	dr.
27	17	2	24	11	10

the language implies, that 16 units of the lowest denomination make one unit of the next higher; 16 of the second, one of the third; 25 of the third, one of the fourth; 4 of the fourth, one of the fifth; and 20 of the fifth, one of the sixth. All the other compound denominate numbers are formed on the same principle: hence,

We pass from a lower to the next higher denomination by considering how many units of the lower make one unit of the next higher.

26. A Scale is a series of numbers expressing the law of relation between the different units of any number. There are two kinds of scales—*Uniform* and *Varying*.

A *Uniform Scale* is one in which the law of relation between the units, at any step of the scale, is the same.

A *Varying Scale* is one in which the law of relation between the units is different, at different steps of the scale.

The *Units of a Scale*, at any step, are denoted by the number of units of the lower denomination which make one unit of the next higher.

---

25. When several figures are written by the side of each other, what does the language imply?

In the English Currency, how many units of the lowest denomination make one of the next higher? How many of the second make one of the third? How many of the third, one of the fourth?

In Avoirdupois weight, how many units of the lowest denomination make one of the next higher? How many of the second, one of the third?

26. What is a *Scale?* *How many* kinds of scales are there? Name *them.* *What is a Uniform Scale?* What is a Varying Scale?

Uniform Scale of Tens.

27. If we write a row of 1's thus:

1 Hundred Billion.	1 Ten Billion.	1 Billion.	1 Hundred Million.	1 Ten Million.	1 Million.	1 Hundred Thousand.	1 Ten Thousand.	1 Thousand.	1 Hundred.	1 Ten.	1 Unit.
1	1	1,	1	1	1,	1	1	1,	1	1	1,

the language of figures expresses that the unit of each place increases from right to left, according to the scale of *tens*. This is called the decimal system of numbers, and the scale is uniform.

United States Currency.

28. United States Currency affords an example of a system of denominate units, increasing according to the scale of tens: thus,

Eagle.	Dollar.	Dime.	Cent.	Mill.
1	1 .	1	1	1

in which ten units of any denomination make one unit of the next higher.

The dollars are denoted by $, and separated from the dimes, cents, and mills by a period (.), called the decimal point.

Varying Scales.

29. If we write the well-known signs of the English Currency, and place 1 under each denomination, we shall have

£	s.	d.	far.
1	1	1	1

---

27. If several figures are written by the side of each other, what does the language express? What name is given to this system of numbers? What is the scale?—28. How do the different units compare with each other in United States Currency?

The signs, £ *s.* *d.* and *far.*, denote the value of the unit 1 in each denomination; and they also determine the relations between the different units. For example, this simple language expresses the following ideas:

1st. That the unit of the right-hand place is 1 farthing; of the place next at the left, 1 penny; of the next place, 1 shilling; of the next place, 1 pound: and

2d. That 4 units of the lowest denomination make one unit of the next higher; 12 of the second, one of the third; and 20 of the third, one of the fourth. Hence, 4, 12, and 20 are the numbers which make up the scale.

30. If we take the denominate numbers of Avoirdupois weight, we have

T.	cwt.	qr.	lb.	oz.	dr.
1	1	1	1	1	1

in which the units increase in the following manner: viz., counting from the right, 16 units of the lowest denomination make 1 unit of the next higher; 16 of the second, 1 of the third; 25 of the third, 1 of the fourth; 4 of the fourth, 1 of the fifth; 20 of the fifth, 1 of the sixth. The scale, therefore, for this class of denominate numbers, varies according to the above law

If we take any other class of denominate numbers, as the Troy weight, we shall have a different scale, and the scale will continue to vary as we pass from one class of numbers to another. But in all the formations, we shall recognize the application of the same general principles.

31. There are, therefore, two general methods of forming the different systems of integral numbers, from the unit one. The first consists in preserving a uniform law of relation between the different units. If that law of relation is expressed by 10 we have the system of decimal or common numbers.

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29. Is the scale uniform or varying in the English Currency? Name the units of the scale at each change of denomination.—30. Name the units of the *scale, at each step,* in the Avoirdupois weight. Name them *also in the Apothecaries* weight?

The second method consists in the application of known, though varying laws of change in the units. These changes in the units, produce different systems of denominate numbers, each of which has its appropriate scale.

Integral Units of Arithmetic.

32. The Integral Units of Arithmetic are divided into eight classes:

1. Units of Abstract Numbers;
2. Units of Currency;
3. Units of Length, or Linear Units;
4. Units of Surface;
5. Units of Volume, or Cubic Units;
6. Units of Weight;
7. Units of Time;
8. Units of Angular Measure.

First among the units of arithmetic is the abstract unit 1. This is the primary base of all abstract numbers, and becomes the base, also, of any denominate number, by merely naming the particular thing to which it is applied.

Of the Signs.

33. The sign =, is called the sign of *equality*. When placed between two numbers, it denotes that they are *equal;* that is, that each contains the same number of units.

The sign +, is called *plus*, which signifies *more*. When placed between two numbers, it denotes that they are to be added together. Thus, $3 + 2 = 5$.

The sign −, is called *minus*, a term signifying *less*. When placed between two numbers, it denotes that the one on the right is to be taken from the one on the left. Thus, $6 - 2 = 4$.

---

31. How many general methods are there of forming numbers from the unit one? What is the first? What is the second?—32. Into how many classes are the Units of Arithmetic divided? Name them.

The sign $\times$, is called the sign of *multiplication.* When placed between two numbers, it denotes that they are to be multiplied together. Thus, 12×3, denotes that 12 is to be multiplied by 3.

The parenthesis is used to indicate that the sum or difference of two or more numbers is to be regarded as a single number, Thus,

$$(2 + 3 + 5) \times 6,$$

shows, that the sum of 2, 3, and 5, is to be multiplied by 6. And

$$(5 - 3) \times 6,$$

denotes that the difference between 5 and 3, is to be multiplied by 6.

The sign $\div$, is called the sign of *division.* When placed between two numbers, it denotes that the one on the left is to be divided by the one on the right. Thus, $4 \div 5$, denotes that 4 is to be divided by 5.

Properties of the 9's.

34. In any number, written with a single significant figure, as, 4, 40, 400, 4000, &c., the excess over exact 9's is equal to the number of units in the significant figure. For, any such number may be written thus,

$4 = 4.$

Also, ... $40 = (9 + 1) \times 4,$

" . . . . $400 = (99 + 1) \times 4,$

" . . . . $4000 = (999 + 1) \times 4,$

&c., &c., &c.

Each of the numbers 9, 99, 999, &c., contains an exact number of 9's; hence, when multiplied by 4, the several products will contain an exact number of 9's: therefore,

---

33. What is the sign of Equality? What is the sign of Addition? What of Subtraction? What of Multiplication? For what is the parenthesis used? What is the sign of Division?

34. What will be the excess over exact 9's in any number expressed by a single significant *figure?* How may the excess over exact 9's be *found in any number whatever?*

The excess over exact 9'*s, in each number, is* 4; *and the same may be shown for each of the other significant figures.*

If we write any other number, as

6 2 5 3,

we may read it, 6 thousands, 2 hundreds, 5 tens, and 3. Now, the excess of 9's in the 6 thousands, is 6; in 2 hundreds, it is 2; in 5 tens, it is 5; and in 3, it is 3: hence, in them all, it is 16, which is one 9, and 7 over: therefore, 7 is the excess over exact 9's in the number 6253. In like manner,

The excess over exact 9'*s, in any number whatever, is found by adding together the significant figures, and rejecting the exact* 9'*s from the sum.*

NOTE.—It is best to reject or drop the 9, as soon as it occurs: thus, we say, 3 and 5 are 8 and 2 are 10; then, dropping the 9, we say, 1 to 6 is 7, which is the excess; and the same for all similar operations.

1. What is the excess of 9's in 48701? In 67498?
2. What is the excess of 9's in 9472021? In 2704962?
3. What is the excess of 9's in 87049612? In 4987051?

REDUCTION.

35. REDUCTION is the operation of changing a number from one unit to another, without altering its value.

36. REDUCTION DESCENDING is the operation of changing a number from a greater unit to a less.

37. REDUCTION ASCENDING is the operation of changing a number from a less unit to a greater.

38. If we have 4 yards, in which the unit is 1 yard, and wish to change to feet, the units of the scale will be 3, since 3 feet make 1 yard; therefore, the number of feet will be

$$4 \times 3 = 12 \text{ feet.}$$

---

35. What is Reduction?—36. What is Reduction Descending?—37 *What is Reduction* Ascending?

If it were required to reduce 12 feet to inches, the units of the scale would be 12, since 12 inches make 1 foot: hence,

4 yards = 4 × 3 = 12 feet = 12 × 12 = 144 inches.

If on the contrary, we wish to change 144 inches to feet, and then to yards, we would first divide by 12, the units of the scale in passing from inches to feet; and then by 3, the units of the scale in passing from feet to yards. Hence,

1st. To reduce a number from a higher unit to a lower

Multiply the highest denomination by the number of units in the scale at that place, and then add to the product the units of the next lower denomination. Proceed in the same manner through all the denominations till the number is brought to the required denomination.

2d. To reduce a number from a lower unit to a higher:

Divide the given number by the number of units in the scale, and set down the remainder, if there be one. Divide the quotient thus obtained, and each succeeding quotient in the same manner, till the number is reduced to the required denomination: the last quotient, with the several remainders annexed, will be the answer.

Examples.

1. Reduce £3 14s. 4d. to pence. We first multiply the £3 by 20, which gives 60 shillings. We then add 14, making 74 shillings: we next multiply by 12, and the product is 888 pence: to this we add 4d. and we have 892 pence, which are of the same value as £3 14s. 4d.

If, on the contrary, we wish to change 892 pence to pounds shillings, and pence, we should first divide by 12: the quotien is 74 shillings, and 4d. over. We next divide by 20, and the quotient is £3, and 14s. over: hence, the result is £3 14s. 4d., which is equal to 892 pence.

The reductions, in all the denominate numbers, are made in the same *manner.*

2. In £5 5s., how many shillings, pence, and farthings?

```
 £5   5s.
  20
 ----
 105   5 shillings added
  12
 ----
1260
   4
 ----
5040
```

Here the reduction is from a greater to a less unit.

3. In 5040 farthings, how many pence, shillings, and pounds?

```
   4) 5040  farthings.
     ------
  12) 1260  pence.
     ------
 2|0) 10|5  shillings.
     ------
     £5 5s.
```

In this example, the reduction is from a less to a greater unit.

4. In 34 T. 16 cwt. 3 qr. 19 lb., how many pounds?

```
   34
   20
 -----
  696   16 cwt. added.
    4
 -----
 2787   3 qr. added.
   25
 -----
13954   19 lb. added.
5574
 -----
69694 lb.
```

5. In 69694 lb., how many tons, cwt., qr., and lb.?

```
25) 69694
   -------
    4)2787 qr. .    19 lb.
     -----
  2|0)69|6 cwt. .    3 qr.
      -----
         34 T. . .  16 cwt.
```

Ans. 34 T. 16 cwt. 3 qr. 19 lb

6. In $426, how many cents? How many mills?

7. In 36 eagles 8 dollars and 6 dimes, how many cents?

8. In 8750 mills, how many dollars and cents?

9. In 43 eagles 3 dollars and 5 mills, how many mills?

10. In £37 9s. 8d., how many pence?

11. In 1569 farthings, how many pounds, shillings, pence and farthings?

12. In 7 T. 14 cwt. 1 qr. 20 lb. Avoirdupois, how many pounds?

13. In 15445 lb. Avoirdupois, how many tons, cwts., qrs., *and lbs.?*

14. How many grains of silver in 4 lb. 6 oz. 12 dwt. and 7 gr.?

15. How many pounds, ounces, pennyweights, and grains of gold in 704121 grains?

16 In 5 ℔ 1 ℥ 1 ʒ 1 ℈ 2 gr. Apothecaries' weight, how many grains?

17. In 174947 grains, how many pounds, ounces, drams, scruples, and grains?

18. In 6 yards 2 feet 9 inches, how many inches?

19. In 5 miles, how many rods, yards, feet, and inches?

20. In 2730 inches, how many yards, feet, and inches?

21. In 56 square feet, how many square yards?

22. In 355 perches, or square rods, how many acres, roods, and perches?

23. In 456 square chains, how many acres?

24. In 3 A. 2 R. 8 P., how many perches?

25. In 14 tons of round timber, how many cubic inches?

26. In 31 cords of wood, how many cubic feet?

27. In 56320 cubic feet, how many cords?

28. In 157 yards of cloth, how many nails?

29. In 192 ells Flemish, how many yards?

30. In 97 yd. 3 qr., how many ells English?

31. In 4 hhd. wine measure, how many quarts?

32. In 7560 pints, wine measure, how many hogsheads?

33. In 7 hogsheads of ale, how many pints?

34. In 74304 half-pints of ale, how many barrels?

35. In 31 bushels, dry measure, how many pints?

36. In 2110 pints, dry measure, how many bushels?

37 In 2 solar years of 365 d. 5 h. 48 m. 48 sec., each, how many seconds?

38. How many months, weeks, and days in 254 days, reckoning the month at 30 days?

ADDITION.

39. Addition is the operation of finding the sum of two or more numbers.

The Sum of two or more numbers, is a number containing as many units as all the numbers taken together.

Operations of Addition.

The operations of Addition depend on four principles, viz.:

1. A single number expresses a collection of like units.

2. Like units alone can be added together; that is, units must be added to units, tens to tens, dollars to dollars, &c.

3. Every number expressed by two or more figures, is the sum of its various units.

4. The sum of several numbers is equal to the sum of all their parts.

1. What is the sum of 769 and 487?

Analysis.—Write the numbers, so that the like units may fall in the same column, thus:

OPERATION.

$$\begin{array}{lrrrr} & & 7 & 6 & 9 \\ & & 4 & 8 & 7 \\ \hline \text{Sum of the units} \;.\;.\;.\;.\;.\;. & & & 1 & 6 \\ \text{Sum of the tens} \;.\;.\;.\;.\;.\;. & & 1 & 4 & \\ \text{Sum of the hundreds} \;.\;.\;.\;. & 1 & 1 & & \\ \hline \text{Entire sum} \;.\;.\;.\;. & 1 & 2 & 5 & 6 \end{array}$$

The example may be done in another way, thus:

Set down the numbers as before: then say, 7 and 9 are 16: set down 6 in the units' place, and the 1 ten under the 8 in the column of tens. Then say, 1 to 8 are 9, and 6 are 15. Set down the 5 in the column of tens, and the 1 hundred in the column of hundreds. We then add the hundreds, and find their sum to be 12: hence, the entire sum of 1256.

OPERATION.

$$\begin{array}{rrrr} & 7 & 6 & 9 \\ & 4 & 8 & 7 \\ & {\scriptstyle 1} & {\scriptstyle 1} & \\ \hline 1 & 2 & 5 & 6 \end{array}$$

Note.—1. Observe, that units of the *same value* are always written in the same column.

2. When the sum in any column equals or exceeds the units of the scale 10, it produces *one or more units* of a higher order, which belong to the next column at the left. In that case, write down the excess, and add the higher units to the next column. This is called *carrying to the next column*. The number to be carried, should not, in *practice* be *written* under the column at the left, but added mentally.

(2;	(3)	(4)
85468	672143	4783614
9104	79161	504126
379	8721	872804
94951	760025	6160544

5. What is the sum of 35 dollars 4 dimes 6 cents 5 mills 4 dollars 7 mills, and 97 cents 3 mills?

OPERATION.

$35.465
4.007
.973
$40.445

ANALYSIS.—Write the figures expressing units of the same value in the same column, separating the dollars from the cents and mills by a period: then add the columns as in simple numbers.

6. Let it be required to find the sum of £14 7s. 8d. 3far., and £6 18s. 9d. 2far.

OPERATION.

£	s.	d.	far.
14	7	8	3
6	18	9	2
21	6	6	1

ANALYSIS.—Write the numbers, as before, so that units of the same order shall fall in the same column. Beginning with the lowest denomination, we find the sum to be 5 farthings. But since 4 farthings make a penny, we set down the excess, 1 farthing, and carry one penny to the column of pence. The sum of the pence then becomes 18, which is 1 shilling, and 6 pence over. Set down the 6 pence, and carry the 1 shilling to the column of shillings, the sum of which becomes 26; that is, 1 pound and 6 shillings. Setting down the 6 shillings, and carrying 1 to the column of pounds, we find the entire sum to be £21 6s. 6d. 1far.

Rule.

I. *Write the numbers so that units of the same value shall fall in the same column:*

II. *Add the units of the lowest denomination, and divide their sum by so many as make one unit of the denomination next higher: set down the remainder, and carry the quotient to the next higher denomination. Proceed in the same manner through all the denominations, and set down the entire sum of the last column*

Proof.

40. The proof of an operation, in Addition, consists in showing that the answer contains as many units as there are in all the numbers added. There are three methods of proof.

I. *Begin with the units' column and add, in succession, all the columns in an opposite direction. If the work is right, the results will agree:*

II. *Divide the given numbers into parts, and add the parts separately: then add together the partial sums: if the work is right, the results will agree:*

III. *Find the excess of* 9's *in each number, and place it at the right* (Art. 34). *Add these numbers, and note the excess of 9's in their sum. This excess should be equal to the excess of 9's in the sum of the numbers.*

NOTE.—The third method of proof applies only to simple numbers.

1. What is the sum of 182796, 143274, 32160, and 47047? and what the proof?

1st Method.	2d Method.	
182796	182796	326070
143274	143274	
32160	32160	79207
47047	47047	
405277		405277

---

39. What is Addition? What is the sum of two or more numbers? on how many principles do the operations of Addition depend? What is the first principle? What the second? What the third? What the fourth? What is the Rule for Addition?

40. How many methods of Proof are there for Addition? What is the process in the first method? What in the second? What in the third?

41. What is the process of reading? How does it differ from *spelling?*

3d Method of Proof.

	182796	. . .	6 excess of 9's,
	143274	. . .	3 " "
	32160	. . .	3 " "
	47047	. .	4 " "
Sum	405277 . . . 7		16 . . 7 excess of 9's

Reading

41 The pupil should be early taught to omit the *intermediate words* in the addition of columns of figures. Thus, in the above example, instead of saying, 7 and 0 are 7; 7 and 4 are eleven; 11 and 6 are seventeen; he should simply say, seven, eleven, seventeen. Then, in the column of tens, he should say, five, eleven, eighteen, twenty-seven; and similarly, for the other columns at the left. This is called *reading* the columns. Let the pupils be often practised in the readings, both separately and in concert in the class.

Examples.

(1)	(2)	(3)	(4)
94201	80032	98800	10304
46390	4291	10926	67491
37467	2376	321	1324
4572	840	479	46

5. What is the sum of 1376, 38940, 8471, 23607, 891?

6. What is the sum of 3480902, 3271, 567321, 91243, 6001, 169?

7. What is the sum of 42300, 6000, 347001, 525, 47?

(8)	(9)	(10)	(11)	(12)
days.	bushels.	rods.	minutes.	gallons.
1276	47917	9003	67321	760321
3718	12031	1881	4702	18720
9024	5672	6035	1067	5762
1028	728	3176	377	1082
9131	47	2004	99	47269

(13)	(14)	(15)	(16)	(17)
miles.	furlongs.	pounds.	dollars.	casks.
1600	47468	76389	1602	40506
2588	59012	1036	9614	37219
9101	23419	2671	4732	50170
6793	15760	5132	5675	32614
8267	27900	6784	8211	73462
4572	12317	1672	4455	10001

(18)	(19)	(20)	(21)	(22)
$175.365	$30.365	$180.000	$300.40	$4802.279
278.056	28.779	489.007	167.275	1642.107
420.96	10.101	76.119	18.197	3026.267
76.125	9.08	16.423	29.94	125.092
41.04	7.14	9.011	10.08	42.75

(23)				(24)			(25)			(26)		
£	s.	d.	far.	lb.	oz.	dwt.	℔	℥	ʒ	lb.	oz.	dr.
14	11	3	1	174	11	19	17	11	7	17	15	12
17	18	10	2	75	10	13	94	10	6	29	32	10
29	7	6	0	642	3	10	60	9	2	84	10	9
42	14	11	3	125	7	5	42	3	9	14	3	7
17	10	0	1	62	0	16	12	0	6	40	9	9
84	0	1	0	39	1	4	98	7	5	76	4	7
16	19	8	2	176	10	15	127	1	0	18	11	15

(27)			(28)			(29)			(30)		
cwt.	qr.	lb.	yd.	qr.	na.	E. E.	qr.	na.	L.	mi.	fur.
174	2	20	74	3	3	14	4	3	17	2	7
320	1	14	60	1	2	75	1	2	10	1	4
136	3	23	14	0	1	84	3	1	7	0	6
47	0	12	45	2	3	17	2	0	5	2	3
84	1	24	69	1	0	10	0	2	25	1	0
90	2	9	11	0	0	19	1	1	36	2	2
7	3	5	36	3	1	29	3	2	40	1	0

(31)

yd.	ft.	in.
174	1	11
260	0	2
150	2	10
126	1	9
96	0	7
72	1	4
8	2	6

(32)

A.	R.	P.
77	3	39
64	2	37
16	1	29
72	0	18
36	2	20
42	2	14
11	3	7

(33)

Tun.	hhd.	gal.
714	3	56
626	1	48
320	0	29
156	2	31
225	1	42
84	0	17
96	1	34

(34)

gal.	qt.	pt.
14	3	1
74	2	1
96	1	0
47	2	1
22	0	1
65	1	0
19	0	0

(35)

chal.	bu.	qt.
14	31	6
25	14	2
36	29	7
42	24	3
39	32	1
56	19	5
14	20	4

(36)

yr.	wk.	da.
127	9	2
320	10	3
146	8	1
75	6	0
70	11	2
54	7	1
27	4	3

(37)

da.	hr.	min.
140	12	27
340	16	40
227	20	56
102	13	25
67	21	37
14	9	10
10	19	46

(38)

qr.	lb.	oz.
44	21	14
14	16	12
22	10	11
36	19	7
51	13	9
30	22	11
16	15	15

39. The population of the United States and Territories, in 1850, was as follows: White population, 19553068; Free Colored population, 434495; Slave population, 3204313; Indians, 400674: what was the entire population?

40. In the year 1850, the expenditures of the United States amounted to 43002168 dollars; in 1851, to 48905879 dollars; in 1852, to 46007893 dollars: what were the expenditures of the United States for these three years?

41. A man of fortune bequeathed to each of his three sons, 10492 dollars; to each of his two daughters, 5976 dollars; to his wife, the remainder of his property, which exceeded the amount bequeathed to his children by twelve hundred dollars: find the amount of his property.

42. A stage goes in one day 27 miles 3 furlongs 36 rods;

the next, 32 miles 10 rods; the next, 36 miles 2 furlongs; the next, 25 miles 6 furlongs 38 feet: how far did it go in 4 days?

43. Bought a barrel of flour for eight dollars and seventy-five cents; a ton of plaster for five dollars sixty-two and a half cents; a hat for three dollars twelve cents and five mills; fifty pounds of sugar for four dollars fifty cents and nine mills: what was the amount of my bill?

44. A lady bought a bonnet for $5.375; some silk for $12.03; some ribbon for $0.875; a shawl for $9.46: what did the whole amount to?

45. A wine-merchant taking an invoice of his liquors, finds that he has 5 hhd. 36 gal. 2 qt. of wine; 3 hhd. 15 gal. 1 qt. 1 pt. of rum; 1 hhd. 2 qt. of gin; 40 gal. 1 pt. of whiskey: how much liquor in all?

46. Tea was imported into the United States, in the year 1851, to the value of $4798005; in 1852, $7285817; in 1853, $8224853: what was the value of the tea imported during these three years?

47. The United States exported tobacco, in the year 1851, to the amount of $9219251; in 1852, $10031283; in 1853, $11319319: what was the entire value of tobacco exported in these three years?

48. A man sold his house and lot for $25840, which was $3186 less than he gave for them; how much did they cost him?

49. A speculator bought three city lots: for the first he paid $2870.43; for the second, $2346.75; for the third, $1563.82 He sold the same at an average profit upon each of $476.25; what amount did he receive for the lots?

50. What is the fortune of a merchant who has $79650 in real estate, $25640 in merchandise, $9654 in furniture and library $16835 in stocks, $12642 in debts due him, and $5685 in cash?

51. The churches of the United States and Territories, in *1850, were: Baptists,* 9375; Congregationalists, 1706; Presby-

terians, 4824; Methodists, 13280; Universalists, 529: what was the whole number of churches belonging to these five denominations?

52. In the same year, the value of the church property owned by the Baptists in the United States and Territories was $11020855: by the Congregationalists, $7970195; by the Presbyterians, $14543789; by the Methodists, $14822870; by the Universalists, $1752316: what was the entire amount?

53. During the year 1853, there was coined in the United States, $51888882 of gold; $7852571 of silver; and $67059 of copper; what was the amount of money coined in the United States in 1853?

54. A farmer sends to market the following quantities of butter: 18 cwt. 2 qr. 16 lb.; 1 ton 5 cwt. 21 lb.; 2 qr. 14 lb.: how much did he send in all?

55. A man having 84 acres 3 roods 26 perches of land, buys 120 acres 14 perches more: how much did he then have?

56. Suppose a father divides his estate equally among his three sons, giving each twenty-five thousand dollars seven dimes six cents and five mills: what was the value of the estate?

57. A farmer has three fields of grain: The first yields 1375 bushels; the second, 1810 bushels; the third, 1265 bushels; he values his entire farm at $2975 more than the number of bushels of grain raised from these three fields: what was the value of his farm?

58. Bought a silver teapot weighing 1 lb. 6 oz. 12 dwt.; a cream-cup, weighing 10 oz. 18 dwt. 20 gr.; a porringer, weighing 11 oz. 16 gr.; a dozen large spoons, weighing 1 lb. 14 dwt 12 gr.: what was the weight of the whole?

59. The whole number of adults in the United States and Territories, over twenty years of age, who could not read and write, in 1850, was as follows: Of whites, males, 389664; females, 573234; free colored, males, 40722; females, 49800: what was the *whole number*?

60. Cæsar was murdered B.C. 43, and Washington died A.D. 1799. How many years elapsed between the death of these great men?

61. A forwarding merchant had in his store-room, at one time, 7500 bushels of corn; 12865 bushels of wheat; 4680 bushels of oats; 3296 bushels of barley; and had room enough left to store 4000 bushels of oats: how many bushels of grain would the storehouse hold?

62. A man engaging in trade, had \$5164.50 in cash; \$11810.25 in goods; \$3004 in notes. His net profits averaged \$2384.16 a year, for 3 years: what was the total value of the property at the end of the three years?

63. A person paid two eagles for a coat; four dollars and six dimes for a hat; two dollars and sixty-three cents for a vest; eight dimes seven cents and five mills for a knife: what was the amount of his bill?

64. From a piece of cloth, 12 yd. 2 qr. were cut at one time; 16 yd. 1 qr. 3 na. at another, when there were 10 yd. 1 qr. 1 na. remaining: how much was there in the whole piece?

65. A farmer purchased a plough for \$$9\frac{1}{4}$; a wagon, for \$$45\frac{1}{2}$; a horse, for \$$110\frac{3}{4}$; a load of hay, for \$$12\frac{1}{8}$; a harrow, for \$$3\frac{1}{5}$: what was the cost of the whole?

66. If a certain warehouse be worth \$$12540.37\frac{1}{2}$, and one-fourth the contents is valued at \$5632.108: what is the value of the warehouse and the whole of its contents?

67. An English gentleman wishing to possess a certain horse, offers in exchange another horse, valued at £25 13s. 6d., a carriage valued at £15 8s. 9d. 2far., and £18 in cash. The offer was accepted: what did he pay for the horse?

68. In 1850, the State of New York produced 13121498 bushels of wheat; Pennsylvania, 15367691 bushels; Virginia, 11212616 bushels; Ohio, 14487351 bushels; Missouri, 2981652 bushels; Illinois, 9414575 bushels: what was the whole number of bushels produced by those States in that year?

69. A farmer sold his wheat for $825.87½; his barley for $67.12½; his pork for $80.10; his apples for $46: how much did he receive for the whole?

70. Three persons enter into copartnership: The first put in 7825 dollars capital; the second put in 1250 dollars more than the first; and the third put in as much as the other two: what was the whole amount of capital invested?

71. A farmer raised in one field 240 bush. 3 pk. 2 qt. of wheat; in another, 97 bush. 6 qt.; in another, 42 bush. 1 pk.: how much did he raise in the three fields?

72. Add together three hundred dollars, ten eagles, forty dimes, ninety-six cents, seven mills, nine dollars, forty-seven cents, five mills, four eagles, three dollars, and nine dimes.

73. What is the sum of £17 10s. 6d.; £25 4s. 10½d.; 18s. 6d. 3far.; £11 9¼d.; £1 18s.; 21s. 6¼d.?

74. A speculator bought a house and lot for $4750; he paid $695 for its thorough repair, and $165 for the introduction of gas; he then sold the house at an advance of $625 above all costs: what did he receive for it?

75. One town is in latitude 37° 34′ N., and another town in latitude 29° 16′ S.: how far apart are they in latitude?

76. A merchant bought 4 hogsheads of sugar weighing respectively, 19 cwt. 3 qr., 22 cwt. 1 qr. 18 lb., 16 cwt. 2 qr. 12 lb., 24 cwt. 1 qr. 19 lb.; he paid $582.68 for the sugar, and $83.24 for freight and other charges; he sold the whole, and gained $166.48: at what price per lb. did he sell?

77. The Deluge, according to Chronology, occurred 1656 years after the Creation; the call of Abraham, 427 after the Deluge; the departure of the Israelites, 430 after the call of Abraham the foundation of the Temple, 479 after the departure of the Israelites; the end of the Captivity, 476 after the foundation of the Temple; and the birth of Christ, 536 years after the end of the captivity: how many years from the Creation to the present time, *it being the year* 1864?

SUBTRACTION.

42. SUBTRACTION is the operation of finding the difference between two numbers.

43. The DIFFERENCE between two numbers is such a number as, added to the less, will give the greater.

44. THE MINUEND is the greater of the two numbers

45. THE SUBTRAHEND is the less of the two numbers.

46. THE REMAINDER, or DIFFERENCE between two numbers, is the result of the operation.

47. When the two numbers are equal, either may be the minuend, and the remainder is 0.

48. Principles which control the operations.

1. The difference of two numbers added to the less number, gives the greater;
2. Like units alone can be taken from each other;
3. The difference is the same, if both numbers be equally increased

49. Operations and Rule.

1. From 869 take 327; that is, from 8 hundreds 6 tens and 9 units, take 3 hundreds 2 tens and 7 units.

ANALYSIS.—Place the numbers so that units of the same order may fall in the same column Beginning with the lowest order, we take units from units; then tens from tens; then, hundreds from hundreds; and find the remainder to be 542.

OPERATION.

8 6 9	min.
3 2 7	sub.
5 4 2	rem.

---

42. What is the difference between two numbers?—43. What is subtraction?—44. What is the minuend?—45. What is the subtrahend?—46. What is the remainder, or difference?—47. When is the remainder 0?—48. What are the three principles that control the operations of Subtraction?—49 Give the rule for finding the difference of two numbers.

2. From the number 624 take 393.

Analysis.—Having written down the numbers, we subtract 3 from 4, and find a remainder 1. At the next step we meet a difficulty, for we cannot subtract 9 tens from 2 tens.

OPERATION.

		Hun.	Tens.	
10				
624	=	5	12	4
393	=	3	9	3
1				
231	=	2	3	1

Take 1 hundred = 10 tens, from the 6 hundreds, and add it to the 2 tens. Then, 9 tens from 12 tens, leaves 3 tens, and 3 hundreds from 5, leaves 2 hundreds, and the remainder is 231.

The remainder can be found by adding, mentally, 10 to 2 tens, and then saying, 9 from 12, leaves 3 tens; then adding 1 to 3 hundreds, and say, 4 from 6, leaves 2 hundreds.

The process of adding 10 to a figure of the *minuend*, and returning 1 to the next figure of the *subtrahend*, at the left, is called *borrowing.*

3. From 6 T. 14 cwt. 2 qr. 20 lb. 12 oz., take 4 T. 17 cwt. 1 qr. 21 lb. 10 oz.

Analysis.—Taking 10 oz. from 12 oz., 2 oz. remain. At the next step we find a difficulty, for 21 lb. cannot be taken from 20 lb. We then take 1 qr. = 25 lb. from the 2 qr. and add it to the 20 lb., making 45 lb.; then say, 21 lb. from 45 lb. leaves 24 lb.; we then add 1 to the next left-hand figure of the subtrahend, and say, 2 qr. from 2 qr. leaves 0; then 17 cwt. from 34 cwt. leaves 17 cwt., and 5 from 6 leaves 1 ton.

OPERATION.

T.	cwt.	qr.	lb.	oz.
	20		25	
6	14	2	20	12
4	17	1	21	10
1		1		
1	17	0	24	2
5	34	1	45	12
4	17	1	21	10
1	17	0	24	2

Rule.

I. *Set down the less number under the greater, so that units of the same value shall fall in the same column:*

II. *Begin with the units of the lowest denomination, and subtract each number from the one above it:*

III. *When the number of units in any denomination of the minuend is less than in the same denomination of the subtrahend, suppose so many units to be added as make one unit of the next higher denomination; after which, add 1 to the next denomination of the subtrahend and subtract as before.*

Proof.

50. There are three methods of proving Subtraction:

I. *Add the remainder to the subtrahend. If the work is right, the sum will be equal to the minuend.*

II. *Subtract the remainder from the minuend. If the work is right, the remainder will be equal to the subtrahend.*

III. *Find the excess of 9's in the minuend, in the subtrahend, and in the remainder. If the work is right, the excess of 9's in the two last numbers will be equal to the excess of 9's in the first.*

Note.—The third method is only applicable to simple numbers.

What is the difference between 874136 and 45302?

1st Method.		2d Method.
874136	828834	874136
45302	45302	828834
828834	874136	45302

3d Method.

874136 . . . 2 excess of 9's in the first.
45302 . . . 5 " " second.
828834 . . . 6 " "

5 + 6 = 11: hence, the excess of 9's in the last two numbers is 2.

Reading.

51. What is the difference between 426 and 295?

By the common method, which is spelling, we say, 5 from 6, leaves 1; 9 from 12, leaves 3; 1 to carry to 2, are 3; 3 from 4, leaves 1.

By reading the words which express the final result, we make the operations mentally, and say, *one, three, one.*

OPERATION
4 2 6
2 9 5
1 3 1

---

50. How many methods of proof are there? What is the first? What the second? What the third?

51. What is spelling of numbers? What is reading?

52. How do you find the difference between two dates?

Time between Dates.

52. What time elapsed between the inauguration of Mr. Jefferson March 4th, 12 o'clock M., 1801, and July 4th, 3 P. M., 1855?

OPERATION.			
yr.	mo.	da.	hr
1855	7	4	15
1801	3	4	12
54	4	0	3

ANALYSIS.—Place the earlier date under the later, writing the number of the year, reckoned from the beginning of the Christian Era, on the left. Then, write in the same line the number of the month, reckoned from the first of January, the number of the day, reckoned from the first of the month, the number of the hour, reckoned from 12 at night, and write the number of minutes and seconds, if there are any, still at the right. Hence, to find the time between two dates,

Rule.—*Write the earlier date under the later, and subtract as in compound numbers* (Art. 49).

NOTE.—1. In finding the difference between dates, as in casting interest, the month is regarded as the twelfth part of the year, and as containing 30 days.

2. The civil day begins and ends at 12 o'clock at night.

3. If the earlier date is before the Christian Era, the sum of the numbers will express the difference of time.

Examples.

	(1)	(2)	(3)	(4)
From	472567	103796	900372	1760134
Take	109271	47217	167301	48207

	(5)	(6)	(7)	(8)
	rods.	dollars.	mills.	barrels.
From	74623457	8600000	162347	8462
Take	32700169	761820	56321	4071

	(9)	(10)	(11)
	bushels	inches	minutes.
From	100000	200763194	3601789412
Take	37214	2142079	10031761

(12)

	cords.
From	4200000
Take	325

(13)

	gallons.
From	8888777
Take	9999

(14)

	pounds.
From	100000000
Take	23

(15)

From	$8475.656
Take	32.015

(16)

From	$1000.759
Take	194.375

(17)

From	$4871036.008
Take	17362.25

(18)

	£	s.	d.	far.
From	25	12	6	2
Take	10	14	3	1

(19)

	T.	cwt.	qr.	lb.
From	5	17	3	21
Take	2	9	1	14

(20)

	yd.	qr.	na.
From	137	1	3
Take	19	3	2

(21)

	L.	mi.	fur.	rd.
From	75	2	7	37
Take	16	1	4	9

(22)

	T.	hhd.	gal.	qt.	pt.
From	14	1	26	2	1
Take	5	3	35	3	1

(23)

	A.	R.	P.
From	100	2	27
Take	10	3	30

(24)

	bush.	pk.	qt.
From	1000	3	4
Take	25	1	6

(25)

	cord.	ft.	in.
From	225	42	1242
Take	100	112	720

(26)

	E. E.	qr.	na.
From	42	1	2
Take	16	4	3

(27)

℔	℥	ʒ
144	10	5
64	11	7

(28)

℥	ʒ	℈
27	4	1
14	7	2

(29)

E. E.	qr.	na.
174	3	1
49	4	2

(30)

E. F.	qr.	na.
171	1	3
74	3	2

(31)

T.	cwt.	qr.
14	12	2
1	14	3

(32)

cwt.	qr.	lb.
17	1	21
14	2	24

(33)

qr.	lb.	oz.
143	22	12
74	19	14

(34)

lb.	oz.	dr.
174	11	10
39	12	13

(35)			(36)			(37)			(38)		
A.	R.	P.	A.	R.	P.	da.	hr.	min.	hr.	min.	sec.
12	1	32	112	1	31	167	21	50	147	50	51
1	3	14	74	2	37	19	23	54	94	59	57

39. From $10000 take $1240.37½.

40. From 183701289 take 34627.

41. From 17 yr 9 mo. 1 wk. 6 da. take 10 yr. 11 mo. 2 wk. 5 da.

42. From 144 ℔ 7 ℥ 5 ʒ 1 ℈ take 56 ℔ 6 ℥ 7 ʒ 1 ℈.

43. From two eagles seven dimes, take twelve dollars and fifty cents.

44. From forty dollars twelve and a half cents, take twenty-five cents and seven mills.

45. From one eagle five dollars six dimes and ten cents, take five dollars seven cents and four mills.

46. What sum of money added to £11 14s. 9¼d. will make £133 11s. 9½d.?

47. An apprentice, who is 14 years 11 months 3 weeks 14 hours 58 minutes old, is to serve his master until he is 21 years of age. How long has he to serve?

48. The greater of two numbers is seven millions three hundred and four thousand and ten; the less is nine hundred and fifty thousand one hundred and forty. What is their difference?

49. Mont Blanc, the highest mountain in Europe, is 15680 feet high; Chimborazo, the highest in America, is 21427 feet. What is the difference in their heights?

50. A man sold his farm for seven thousand five hundre and thirty dollars, which was fifteen hundred and ten dollars more than he gave for it. How much did he give for it?

51. The revenue collected at the port of New York for the year *ending 30th June, 1853,* was $38289341.58; at Philadel-

phia, $4537046.16; at Boston, $7203048.52; at Baltimore, $836437.99. How much more was collected at the port of New York than at the other three?

52. A man engaging in trade, found, at the end of five years, that he had increased his capital ten thousand three hundred and ten dollars, and that his whole capital amounted to forty-six thousand five hundred dollars. How much did he commence with?

53. The minuend exceeds the remainder by 683021, and the remainder is 902563. What is the subtrahend?

54. The amount of tea consumed in the United States in the year 1846, was 16891020 pounds; the amount of coffee, 124336054 pounds. How many more pounds of coffee than of tea were consumed?

55. What number is that to which, if you add 3726, the sum will be ten thousand?

56. From a stack of hay containing 9 T. 3 qr. 20 lb., I sold 4 T. 17 cwt. 22 lb. How much was then left?

57. A owes B £25; after paying him £5 9¼d., how much will he still owe him?

58. If the distance from New York to Liverpool be 3100 miles, what distance remains after a ship has sailed 800 mi. 5 fur 36 rd.?

59. Mr. Jones bought a farm for three thousand five hundred dollars and fifty cents; he sold the same for three thousand three hundred dollars and eighty-seven and a half cents how much did he lose by the bargain?

60. If a lot of goods is bought for $750, and sold for $925.87½, what will be gained?

61. If I buy a bushel of wheat for $1.87½; ten gallons of molasses for $2.50; five yards of cloth for $12.37½: how much change must I receive back, if I give in payment two ten-dollar bills?

62. The population of the United States in the year 1850 was 23191876, of which 3204313 were slaves: what was the free population?

63. England contains 50922 square miles; Scotland, 31324 square miles; Wales, 7398 square miles; the United States contain 2988892 square miles. How many more square miles does the United States contain than the whole of Great Britain?

64. A gentleman of fortune owning an estate of two hundred thousand dollars, bequeathed thirty thousand dollars to objects of charity; twenty-five thousand two hundred and fifty dollars to each of his three sons; twenty thousand five hundred and seventy-five dollars to his daughter; and the remainder to his widow. How much did the widow receive?

65. The population of New Orleans, in 1850, was 116375; in 1854 it was 139190: what was the increase in four years?

66. Having deposited $1500 in a bank, I drew out at one time, $475.12½; at another time, $300; at another, $526,25: how much remained?

67. If the Declaration of Independence was made at precisely 12 o'clock, on the 4th day of July, 1776: how much time will have passed to the 4th day of March, 1857, at 30 minutes past 3 o'clock, P. M.?

68. If I borrow $1576 of a friend, and afterwards pay him $920.87½: how much will I still owe him?

69. The first settlement made in the United States was at Jamestown, in Virginia, May 23, 1607: how many years, months, and days, from that time to the 4th of July, 1856.

70. The sum of two numbers is 36804, and the greater is eighteen thousand nine hundred and twenty-seven: what is the less number?

71. The revenue of the United States in the year 1853 was $61337574; the expenditures, $54026818: how much did the revenue exceed the expenditures?

72. From a box of sugar, containing 19 cwt. 1 qr. 15 lb., there was 14 cwt. 3 qr. 22 lb. taken: how much was left?

73. A ship-builder sold a vessel for $50376, which cost him $42978: how much did he gain?

74. A farmer sold his farm for six thousand three hundred and seventy-five dollars; after paying his debts, he has four thousand and fifteen dollars left: what was the amount of his debts?

75. Gunpowder was invented in the year 1330: how many years from that time to the year 1856?

76. What number increased by five thousand eight hundred and twenty-nine, will become 12103?

77. A speculator bought a quantity of flour for $2084.50; of bacon, for $760.87½; of hops, for $1836.25. He sold the flour for $2375.60; the bacon, for $912.375; the hops, for $1750: what did he gain or lose on the whole?

78. A farmer has two pastures, one containing 9 A. 3 R. 32 P.; the other, 12 A. 29 P. He has also two meadows, one containing 10 A. 2 R.; the other, 15 A. 1 R. 20 P.: how much more meadow than pasture has he?

79. From a pile of wood containing 76 cords and 6 cord feet, was taken at one time 20 cords and 48 cubic feet; at another time, 14 cords 1 cord foot and 80 cubic feet: how much remained in the pile?

80. A gentleman purchased a house worth $9436; a carriage for $475.50; a span of horses for $840.40. He paid at one time, $5260; at another, $1275.37½; at another, $936.42: how much remained unpaid?

81. If a ship and cargo are valued at $47568.487, and the cargo alone at $3406.50: what is the value of the ship without the cargo?

82. A note on interest, dated July 1st, 1853, was to be paid *March 20th, 1856*: how long was it on interest?

83. A gentleman dying left an estate of $50000; after paying his debts, which amounted to $5647.50, he desired that each of his two sons should receive $15000, and his widow the remainder: how much did the widow receive?

84 Bought a hogshead of wine, from which was drawn 32 gal. 1 qt. 1 pt.: how much remained in the cask?

85. The population of Chicago, in 1850, was 29963; in 1855 it was 80025: what was the increase in five years?

86. A land speculator, owning twenty-five thousand acres of land, sells at one time fifteen hundred acres; at another, four thousand seven hundred; at another, twenty-five hundred acres; at another, seven hundred and fifty acres: what number of acres has he left?

87. The latitude of New Orleans is 29° 57′ 30″; that of Boston, 42° 21′ 23″: what is the difference in the latitude of these two places?

88. A person bought a span of horses for three hundred dollars; a carriage for $410.50; a harness for $50.675; he sold the whole for six hundred dollars: did he gain or lose, and how much?

89. The population of Great Britain and its adjacent islands, in the year 1841, was 18664761; in 1851 it was 20936468: what was the increase of population in ten years?

90. From a piece of cloth containing 47 yards, a tailor cut 14 yd. 3 qr. 2 na.: how much was left?

91. A tradesman failing in business, was indebted to A £105 19s. 11d.; to B, £127 10s. 9½d.; to C, £34 18s. 10d.; to D. £500 19s.; to E, £700 14s. 6½d. When this took place, he had in cash £50; in goods, £350 14s. 9d.; in household furniture, £24 11s.; and his book accounts amounted to £94 14s. 8d If all these were given up to the creditors, how much would they lose?

MULTIPLICATION.

53. Multiplication is the operation of taking one number as many times as there are units in another.

54. The Multiplicand is the number to be taken.

55. The Multiplier is the number denoting how many times the multiplicand is to be taken.

56. The Product is the result of the operation.

57. A Composite Number is one produced by multiplying two or more numbers together. Thus, 60 is a composite number, because 3 × 4 × 5 = 60.

58. A Factor is any one of the numbers which, multiplied together, produce a composite number. Thus, 3, 4, and 5 are factors of the composite number 60.

Note.—1. The product, after multiplication, is a composite number, and the multiplicand and multiplier are *factors* or *producers* of the product.

2. Multiplication is a *short method of addition*. For, if the multiplicand be written as many times as there are units in the multiplier, and the numbers added, the sum will be equal to the multiplicand taken as many times as there are units in the multiplier.

59. Product of two factors.

Multiply the number 6 by 4.

Analysis.—Write, in a horizontal line, as many 1's as there are units in the multiplicand, and write as many such lines as there are units in the multiplier. It is then evident that the product will contain as many units as there are in one line, taken as many times as there are lines.

$$4\left\{\overbrace{\begin{matrix}1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\end{matrix}}^{6}\right.$$

Change now the multiplier into the multiplicand: that is, multiply *4 by 6.*

Write, in a vertical line, as many 1's as there are units in the new multiplicand (4), and as many vertical lines as there are units in the new multiplier (6), when it is again evident that all the 1's will represent the number of units in the product. Hence,

The product of two factors is the same, whichever factor is used as the multiplier.

Thus, $3 \times 7 = 7 \times 3 = 21$: also, $6 \times 3 = 3 \times 6 = 18$
$9 \times 5 = 5 \times 9 = 45$: also, $8 \times 6 = 6 \times 8 = 48$

60. Product of Several Factors.

Multiply the number 7 by the composite number $6 = 2 \times 3$.

$$
\begin{array}{r}7\\ \underline{3}\\ 21\\ \underline{2}\\ 42\end{array}
\qquad
2\left\{\begin{array}{l}
3\left\{\begin{array}{ccccccc}1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\end{array}\right.\\
3\left\{\begin{array}{ccccccc}1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\end{array}\right.
\end{array}\right.
\qquad
\begin{array}{l}
\left.\begin{array}{l}\\ \\ \end{array}\right\} 2 \times 7 = 14\\
\left.\begin{array}{l}\\ \\ \end{array}\right\} 2\\
\left.\begin{array}{l}\\ \\ \end{array}\right\} 2
\end{array}
\qquad
\begin{array}{r}14\\ \underline{3}\\ 42\end{array}
$$

Analysis.—Write 6 horizontal lines with 7 units in each, and it is evident that the product of $7 \times 6 = 42$, will express the number of units in all the lines.

Let us first connect the lines in sets of two each, as at the right, the number of units in each set will then be expressed by $7 \times 2 = 14$. But there are three sets; hence, the number of units in all the sets is $14 \times 3 = 42$.

Again, if we divide the lines into sets of 3 each, as at the left, the number of units in each set will be equal to $7 \times 3 = 21$; and since there are two sets, the whole number of units will be expressed by $21 \times 2 = 42$.

---

53. What is Multiplication?—54. What is the number to be taken called?—55. What is the multiplier?—56. What is the product?—57. What is a composite number?—58. What is a factor? Is the product, after multiplication, a composite number? What are its factors? Why is multiplication a short method of addition?

59. In how many ways may 6 and 4 be multiplied together? How do the two products compare with each other? What principle does this prove?—60. If *several factors* be multiplied together, is the product *changed by changing* their order?

Since the product of either two of the three factors, 7, 3, and 4, will be the same, whichever be taken for the multiplier (Art. 59), and since the same principle will apply to that product and to the other factor, as well as to any additional factor, if introduced, it follows that,

The product of any number of factors will be the same in whatever order they are multiplied.

61. When the multiplier is a composite number.

1. Multiply 215 by $36 = 3 \times 3 \times 2 \times 2$.

Now, $215 \times 36 = 215 \times 3 \times 3 \times 2 \times 2$:

hence, from the last principle,

I. *Separate the composite number into its factors:*

II. *Multiply the multiplicand and the partial products by the factors, in succession, and the last product will be the entire product sought.*

NOTE.—Any number whatever, as 440, ending with 0, is a composite number of which 10 is a factor: for, $440 = 44 \times 10$. If there are two 0's on the right of the significant figures, then 100 is a factor; and so on for a greater number of ciphers. Hence, when there are ciphers on the right of significant figures, either in the multiplicand or multiplier, or both

Multiply the significant figures together, and then annex the ciphers to the product.

62. General Case and Rule.

1. Multiply the number 627 by 214.

ANALYSIS.—The multiplicand 627 is to be taken 214 times; that is, 4 units times, 1 ten times, and 2 hundred times. Taking it 4 units times, gives 2508; taking it 1 ten times, gives 627, of which the *lowest unit* is 1 *ten;* hence, 7 is written in the *tens place;* taking it 2 hundred times, gives 1254, the *lowest unit* of which is 1 hundred. Adding, we have 134178 for the product.

OPERATION

$$\begin{array}{r} 627 \\ \underline{214} \\ 2508 \\ 627 \\ 1254 \\ \hline 134178 \end{array}$$

NOTE.—When the multiplier contains more than one figure, the product obtained by multiplying the multiplicand by a single figure is called a *partial product.* In the example, there are three partial products, 2508, 627, and 1254. The *sum* of the partial products is equal to the product sought.

Principles from the Analysis.

1. If units be multiplied by units, the unit of the product will be 1.

2. If tens be multiplied by units, the unit of the product will be 1 ten.

3. If hundreds be multiplied by units, the unit of the product will be 1 hundred; and so on.

4. If units of the first order be multiplied by units of a higher order, the units of the product will be the same as that of the higher order.

5. If units of any order be multiplied by units of any other order, the unit of the product will be of an order one less than the sum of the units denoting the two orders.

2. Multiply the compound number £3 8s. 6d. 3far. by 6

ANALYSIS.—Multiplying 3 farthings by 6, we have 18 farthings, equal to 4d. and 2far.; set down the 2far.: then, 6 times 6d. are 36d., and 4 pence to carry, are 40d., equal to 3 shillings and 4d.: then, 6 times 8s. are 48s., and 3s. to carry, are 51 shillings, equal to £2 and 11 shillings; then, 6 times £3 are £18, and £2 to carry, are £20, which set down.

OPERATION.

£	s.	d.	far
3	8	6	3
			6
20	11	4	2

NOTE.—The unit of each product will be the same as the unit of the multiplicand. Hence, for the multiplication of all numbers, we have the following

Rule.

Multiply each order of units in the multiplicand, in succession, beginning with the lowest, by each figure in the multiplier and divide each product by so many units as make one unit of the next higher denomination: write down each remainder under the units of its own order, and carry the quotient to the next product.

NOTE.—In multiplying United States money, care must be taken to point off as many places for cents and mills as there are in the multiplicand.

63 Principles governing Multiplication.

The principles governing the operations of multiplication, are mainly the following:

1. There are three parts in every operation of Multiplication: First, the *multiplicand;* second, the *multiplier;* third, the *product.*

2. The multiplier is always an abstract number, and shows how many times the multiplicand is to be taken.

3. The unit of the product is always the same as the unit of the multiplicand.

4. The product is equal to the sum of the partial products which arise from multiplying the multiplicand, in succession, by each figure of the multiplier.

5 If the multiplier is 1, the product will be equal to the multiplicand.

6. If the multiplier is greater than 1, the product will be as many times greater than the multiplicand as the multiplier is greater than 1.

7. If the multiplier is less than 1, the product will be such a part of the multiplicand as the multiplier is of 1.

---

61. How do you multiply when the multiplier is a composite number? What is one factor of a number ending in 0? What is one factor when the number ends in two 0's? In three 0's? &c. How do you multiply such numbers together?

62. Explain the operation of multiplying 627 by 214. What is a partial product? Explain the five principles which come from this analysis. Give the general rule for multiplication.

63. What is the first principle governing multiplication? What the second? What the third? What the fourth? What the fifth? What the sixth? What the seventh?

64. How many methods are there of proving Multiplication? What *are they?*

Proof.

64. There are three methods of proving Multiplication:

I. *Write the multiplier in the place of the multiplicand, and find the product, as before: if the work is right, the two products will be the same.*

II *By casting out the 9's*

III. *Divide the product by the multiplier, and the result will be the multiplicand.*

First Method.

Multiply	80432	506	506
By	506	80432	80432
	482592	1012	4048
	402160	1518	2024
	40698592	2024	1518
		4048	1012
		40698592	40698592

Note.—Although we generally begin the multiplication by the figure of the lowest unit, yet we may multiply in any order, if we only preserve the *places of the different orders of units.* In the example at the right, we began with the order of tens of thousands, or 5th order

Second Method.

Let it be required to multiply any two numbers together, as 641 and 232.

OPERATION.

641 = 639 + 2
232 = 225 + 7

4473	+ 14
450	
3195	
1278	
1278	
148698	+ 14

Analysis.—We first find the excess over exact 9's in both factors, and then separate each factor into two parts, one of which shall contain exact 9's, and the other the excess, and unite the two by the sign plus. It is now required to take 639 + 2 = 641, as many times as there are units in 225 + 7 = 232.

Every partial product, in this multiplication, contains exact 9's, except 14, which contains one 9, *and 5 over; and* as the same may *be shown for any two numbers* we see that, *If we find the excess of 9's in each of two factors, and then multiply them together, the excess of 9's in their product will be equal to the excess of 9's in the product of the factors.*

Examples.

	(1)	Ex.	(2)	Ex.
Multiply	87603	. . 6	818327	. . 2
By	9865	. . 1	9874	. . 1
Prod.	864203595	. . 6	8080160798	. . 2

3. By multiplication, we have,

$$\overset{\text{Ex. 4.}}{7285} \times \overset{\text{Ex. 8.}}{143} \times \overset{\text{Ex. 4.}}{976} = \overset{\text{Ex. of product, 2.}}{1016752880.}$$

Analysis.—The excess of 9's in the product is found by multiplying together the excess of 9's in the factors, and casting out the 9's from the product. The excess thus found is equal to the excess of 9's in the final product of the numbers.

4. We have, also, $\overset{\text{Ex. 5.}}{869} \times \overset{\text{Ex. 4.}}{49} \times \overset{\text{Ex. 0.}}{36} = \overset{\text{Ex. 0.}}{1532916.}$

Note.—When the excess of 9's in any factor is 0, the excess of 9's in the product is always 0.

Examples.—Simple Numbers.

(1)	(2)	(3)	(4)
847046	9807602	570409	216987
8	7	6	9

(5)	(6)	(7)
103672	8163021	90031746
42	126	274

(8)	(9)	(10)
14168	894126	20034645
235	4514	6481

When the multiplier is a Composite Number (Art. 61).

(11)	(12)	(13)	(14)
67537	87456	390462	75046
12	27	81	72

(15)	(16)	(17)	(18)
270456	315900	390762	910000
460	6300	8100	640000

When the multiplicand is United States Currency.

(19)	(20)	(21)	(22)
$8704.04	$69.476	$481.694	$749.972
12	36	48	96

(23)	(24)	(25)	(26)
$67.492	$219.864	$67.492	$890.46
104	140	320	436

(27)	(28)	(29)
$87.041	$95.004	$946.274
3204	3992	9809

When the multiplicand is a Compound Number.

(30)	(31)	(32)
£ s. d.	T. qr. lb. oz.	yd. ft. in.
20 6 8	3 3 21 14	16 2 9
4	8	7

(33)	(34)	(35)
deg. ′ ″	hhd. gal. qt. pt.	E. F. qr. na.
12 42 55	4 42 2 1	24 2 3
9	12	24

General Examples.

1. Multiply 18 T. 2 qr. 16 lb. 9 oz. by 48.

2. Multiply 5 yr. 8 mo. 2 wk. 3 da. 42 m. by 56.

3. Multiply 68 by the factors 9 and 8 of the composite number 72.

4. Multiply 67046 by 10: also by 100.

5. Multiply 57049 by 100: also by 1000.

6. Multiply 4980496 by 1000: also by 10000

7. Multiply 90720400 by 100: also by 10000.

8. Multiply 74040900 by 1: also by 10.

9. Multiply 674936 by 100: also by 100000.

10. Multiply 478400 by 270400.

11. Multiply 367000 by 37409000.

12. Multiply 7849000 by 84694000.

13. Multiply 89999000 by 97770400.

14. Multiply 9187416300 by 274987650000.

15. Multiply 86543291213456 by 12637482965.

16. Multiply 76729835645873 by 217834569.

17. If it costs 2479 dollars to build one mile of plank-road, how much will it cost to build 25 miles?

18. How far would a vessel sail in 9 days, of 24 hours each, at the rate of 15 miles an hour?

19. A man bought two farms, one of 125 acres, at 26 dollars an acre; another of 96 acres, at 32 dollars an acre; he paid at one time 2500 dollars; at another time, 1725 dollars: what remained to be paid?

20. In 9 pieces of kersey, each containing 14 yd. 3 qr. 2 na., how many yards?

21. What will 15 gallons of wine cost, at 5s. 3½d. per gallon?

22. What will be the value of 416 sheep, at $2.48 a head?

23. Bought 40 barrels of flour, at $8.75 a barrel, and sold them for $9.12½ a barrel: what was the whole gain?

24. What is the weight of 11 hogsheads of sugar, each weighing 7 cwt. 2 qr. 18 lb., and what is its value, at 6 cents a pound?

25. A merchant bought 36 pieces of broadcloth, each containing 44 yards, at 4 dollars a yard: what did the whole cost?

26. A gentleman, whose annual income is $3479, expends, for pleasure and travelling, $600; for books and clothing, $570; for board and other expenses, $1200: how much will he have saved in 5 years?

27. The number of milch cows in the State of New York, in 1850, was 931324 : what was their value, at $18 each?

28. If a man travel 20 mi. 5 fur. 16 rd. in one day, how far will he travel in 24 days?

29. If a man spends six cents a day for cigars, how much will he spend in thirty years, allowing three hundred and sixty-five days to the year?

30. A farmer sold 118 bushels of barley for 62½ cents a bushel, and received 5 barrels of flour at $9.87½ a barrel, and the remainder in cash: how much cash did he receive?

31. Two persons start at the same point and travel in opposite directions, one at the rate of 34 miles a day, the other at the rate of 28 miles a day: how far apart will they be at the end of 14 days?

32. An apothecary sold 8 bottles of laudanum, each containing 10 ℥ 6 ʒ 2 ℈ 14 gr.: what was the weight of the whole?

33. A farmer took 7 loads of oats to market, each load having 20 bags, and each bag containing 2 bush. 3 pk. 6 qt.: how many bushels of oats did he take to market?

34. The greatest number of whales ever captured in the northern seas, in one season, was 2018. Estimating the oil produced from each to have been 212 barrels, what was the amount of oil?

35. What is the value of an ox weighing 7 cwt. 2 qr. 16 lb., at 11 cents a pound?

36. What is the cost of 245 hogsheads of sugar, each weighing 984 pounds, at 7 cents a pound?

37. Bought 6 loads of hay, each weighing 18 cwt. 3 qr. 21 lb.: after letting a neighbor have 2 tons 15 cwt. 1 qr. 5 lb., how much was there left?

38. In an orchard there are 136 apple-trees, each tree yielding 17 bushels of apples: how many bushels did the whole orchard yield, and what would they be worth, at 42 cents a bushel?

39. A flour merchant bought 1845 barrels of flour, at 7 dollars per barrel. He sold at one time 528 barrels, at 9 dollars a barrel; at another time, 856 barrels, at 8 dollars a barrel: how many barrels had he left, and at what price could he sell them, without gain or loss on the flour?

40. What are 25 hogsheads of sugar worth, each weighing 872 pounds, at 6½ cents a pound?

41. It is estimated that the whole amount of land appropriated by the general government for educational purposes, to the 1st of January, 1854, was 52770231 acres. What was the value of this land, at the government price of one dollar and twenty-five cents an acre?

42. If 30 men can do a piece of work in 25 days, how long will it take one man to do it?

43. A man desired that his property should be equally divided among his 5 children, giving each twenty-seven hundred dollars: what was the amount of his property?

44. Bought 9 chests of tea, each containing 72 pounds, at 37½ cents a pound: what was the cost of the whole?

45. A merchant bought a box of goods containing 37 pieces, each piece containing 46 yards, worth 7 dollars a yard: what did the box of goods cost?

46. A farm, consisting of 127 acres, was sold at auction for $37.565 an acre: what sum of money did it bring?

47. A drover bought 127 head of beef cattle, at an average of 39 dollars per head; he sold 86 of them for 43 dollars per head: for how much per head must he sell the remainder, to clear on the first cost 1246 dollars?

48. What will 75 firkins of butter cost, each firkin weighing 56 pounds, at 16 cents a pound?

49. A bond was given April 20th, 1850, and was paid Sept. 4th, 1856: what will be the product, if the time which elapsed from the date of the bond to the time it was paid be multiplied by 45?

50. What distance will a wheel, 16 feet 8 inches in circumference, measure on the ground, if rolled over 84 times?

51. What is the difference between twice eight and fifty, and twice fifty-eight?

52. How much wood in 4 piles, each containing 5 cords, 6 cord feet, and 32 cubic feet?

53. A man bought 56 acres of land for $25 an acre, and 94 acres for $32 an acre; if he sells the whole at $30 an acre, will he gain or lose, and how much?

54. If 12 men can build a wall in 16 days, how many men will build a wall nine times as long in half the time?

55. A farmer sold 4 cows for $25.50 each; 12 sheep for $2.12½ each; and 3 calves for $7.25 each: what was the amount of the sale?

56. If it requires 116 tons of iron to construct one mile of railroad, how much would it require to construct a railroad from Albany to Buffalo, it being 326 miles?

57. A merchant bought 960 lb. of cheese, at 9 cts. a pound; 148 lb. of butter, at 12½ cts. a pound. He gave in payment, 12 yd. of cloth, at $4.75 a yard; 186 lb. of sugar, at 7 cts. a pound, *and the remainder* in cash: how much cash did he pay?

58. How much brandy will supply an army of 25,000 men for one month, if each man requires 1 gal. 2 qt. 1 pt. 2 gi.?

59. It is estimated that the French, during the years 1854 and 1855, transported to the Crimea 80000 horses, and that 70000 of them were lost in the same time. Supposing the first cost of each horse to be $100, and the cost of transportation $95 per head, what was the value of the horses lost?

60. A man purchased a piece of woodland containing 27 acres, at 39 dollars per acre; each acre produced on an average 70 cords of wood, which, being sold, yielded a net profit of 45 cents a cord: how much did the profit on the wood fall short of paying for the land?

Bills of Parcels.

CHICAGO, June 10, 1857.

61. *Mr. John C. Smith,* *Bought of David Toombs.*

14	pounds of	tea,	at	75 cents,	$
9	"	coffee,	"	14 "	
42	"	sugar,	"	11 "	
3	"	pepper,	"	12½ "	
5	"	chocolate,	"	56 "	
12	"	candles,	"	16 "	
					$

Received payment, DAVID TOOMBS.

NEW YORK, March 20, 1857.

62. *Mr. Jacob Johns,* *Bought of George Bliss & Co.*

48 pounds of sugar, at 9½ cents a pound, . . $
6 hhds. of molasses, each containing 63 gallons, at
27 cents a gallon, ...
8 casks of rice, 285 lb. each, at 5 cts. a pound,
9 chests of tea, 86 lb. each, at 87½ cts. a pound,
4 bags of coffee, each 67 lb., at 11 cts. a pound,
$

Received payment, GEO. BLISS & CO.

HARTFORD, November 21, 1856.

63 *Gideon Jones,* *Bought of Jacob Thrifty.*

78 chests of tea, at $55.65 per chest, ... $
251 bags of coffee, 100 lb. each, at 12½ cts. per lb.
317 boxes of raisins, at $2.75 per box, . .
1049 barrels of shad, at $7.50 per bbl. . . .
76 barrels of oil, 32 gal. each, at $1.08 per gal.

Amount, $

Received the above in full, JACOB THRIFTY.

BALTIMORE, January 1, 1855.

64. *Mr. Abel Wirt,* *Bought of Timothy Stout.*

10 yards of broadcloth, at $4.37½ . . . . $
75 " sheeting, " .09 . . . .
42 " plaid prints, " .45 . . . .
5 bbl. Genesee flour, " 7.87½ . . . .
7 pairs of boots, at $1.60 per pair, . . .
18 bushels of corn, at 72 cts. per bushel, . .

$

MONTREAL, October 16, 1855.

65. *Mr. Chas. Snow,* *Bought of Vose, Duncan & Co.*

		£	s.	d.
45 yards of broadcloth, at	9s. 6d. . . .			
56 " " "	12s. 9¼d. . . .			
16 " vestings, "	6s. 8½d. . . .			
24 lb. colored thread, "	5s. 4d. . . .			
72 pair silk hose, "	7s. 5¾d. . . .			
108 yards carpeting, "	14s. 10d . . .			
		£		

Received payment, VOSE, DUNCAN & Co.

DIVISION.

65. Division is the operation of finding how many times one number is contained in another; or, of dividing a number into equal parts.

66. The Dividend is the number to be divided.

67. The Divisor is the number by which we divide. It is the standard which measures the dividend; or, it shows into how many equal parts the dividend is to be divided.

68. The Quotient is the result of division. It shows how many times the divisor is contained in the dividend, or the value of one of the equal parts of the dividend.

69. The Remainder is what is left after the operation. When it is 0, the quotient is a whole number, and the division is *exact*.

Numbers in Division.

70. There are always three numbers in every division, and sometimes four: First, the dividend; second, the divisor; third, the quotient; fourth, the remainder.

There are three methods of denoting division; they are the following:

$12 \div 3$ expresses that 12 is to be divided by 3;
$\frac{12}{3}$ expresses that 12 is to be divided by 3,
$3\,)\,12$ expresses that 12 is to be divided by 3.

When the last method is used, if the divisor does not exceed 12, we draw a line beneath the dividend, and set the quotient under it. If the divisor exceeds 12, we draw a curved line on the right of the dividend, and set the quotient at the right.

Kinds of Division.

71. Short Division is the operation of dividing when the work is performed mentally, and the results only written down It is limited to the cases in which the divisors do not exceed 12.

72 Long Division is the operation of dividing when all the work is expressed. It is used when the divisor exceeds 12.

73. Operations and Rule.

1. Divide 456 by 4.

Analysis.—The number 456 is made up of 4 hundreds, 5 tens, and 6 units, each of which is to be divided by 4. Dividing 4 hundreds by 4, we have the quotient, 1 hundred: 5 tens divided by 4, gives 1 ten, and 1 ten over: reducing this to units, and adding in the 6, we have 16 units, which contains 4, 4 times: hence, the quotient is 114: that is, the dividend contains the divisor 114 times.

OPERATION.

4) 4 5

1 1

2. Divide £11 8s. 7d. 3far. by 5.

Analysis.—Dividing £11 by 5, the quotient is £2, and £1 remaining. Reducing this to shillings, and adding in the 8, we have 28s., which, divided by 5, gives 5s., and 3s. over. This being reduced to pence, and 7d. added, gives 43d. Dividing by 5, we have 8d., and 3d. remainder. Reducing 3d. to farthings, adding 3 farthings, and again dividing by 5, gives the last quotient figure, 3far. Hence, £2 5s. 8d. 3far., is one of the five equal parts of the dividend.

OPERATION.

	£	s.	d.	far.
5)	11	8	7	3
	2	5	8	3

3. Divide 11772 by 327.

Analysis.—Having set down the divisor on the left of the dividend, it is seen that 327 is not contained in the first three figures on the left, which are 117 hundreds. But by observing that 3 is contained in 11, 3 times, and something over, we conclude that the divisor is contained at least 3 times in the first *four* figures, 1177 tens, which is a *partial dividend.* Set down the quotient figure 3, and multiply the divisor by it: we thus get 981 tens, which being less than 1177, the quotient figure is not too great: we subtract the 981 tens from the first four figures of the dividend, and find a remainder 196 tens, which being less than the divisor, the quotient figure is not too small. Reduce this remainder to units, and add in the 2, and we have 1962.

OPERATION.

327) 11772 (36
981
1962
1962

As 3 is contained in 19, 6 times, we conclude that the divisor is contained in 1962 as many as 6 times. Setting down 6 in the quotient, and multiplying the divisor by it, we find the product to be 1962. Hence, the entire quotient is 36, or the divisor is contained 36 times in the dividend, and 36 is also one of the 327 equal parts of the dividend

Rule.

I. *Beginning with the highest order of units, take for a partial dividend the fewest figures that will contain the divisor: divide these figures by it, for the first figure of the quotient: the unit of this figure will be the same as that of the partial dividend:*

II. *Multiply the divisor by the quotient figure so found, and subtract the product from the partial dividend:*

III. *Reduce the remainder to units of the next lower order, and add in the units of that order found in the dividend: this gives a new partial dividend. Proceed in a similar manner until units of every order shall have been divided.*

74. Directions for the Operations.

1. There are five steps in the operation of Division: 1st, To write down the numbers; 2d, To divide, or find how many times; 3d, To multiply; 4th, To subtract; 5th, To bring down, to form the partial dividends.

2. The product of a quotient figure by the divisor must never be larger than the corresponding partial dividend: if it is, the quotient figure is too large, and must be diminished.

3. If any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased.

4. The unit of any quotient figure is the same as that of the partial dividend from which it is obtained. The pupil should always name the unit of every quotient figure.

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65. What is Division?—66. What is the dividend?—67. What is the divisor? What does it show?—68. What is the quotient? What does it show?—69. What is the remainder?

70. How many numbers are there in every division? What are they? How many signs of Division are there? Make and name them.

71. What is Short Division? When is it used?—72. What is Long Division? When is it used?—73. Explain each of the three examples. Give the rule for the division of numbers

5. If the dividend and divisor are both compound numbers, reduce them to the same unit before commencing the division

6. If any partial dividend is less than the divisor, the corresponding quotient figure is 0.

7. When there is a remainder, after division, write it at the right of the quotient, and place the divisor under it.

75. Principles resulting from Division.

1. When the divisor is equal to the dividend, the quotient will be 1

2. When the divisor is 1, the quotient will be equal to the dividend.

3. When the divisor is less than the dividend, the quotient will be greater than 1. The quotient will be as many times greater than 1, as the dividend is times greater than the divisor.

4. When the divisor is greater than the dividend, the quotient will be less than 1. The quotient will be such a part of 1, as the dividend is of the divisor.

74.—1. How many steps are there in division? Name them.

2. If a partial product is greater than the partial dividend, what does it indicate? What then do you do?

3. What do you do when any one of the remainders is greater than the divisor?

4. What is the unit of any figure of the quotient? When the divisor is contained in simple units, what will be the unit of the quotient figure? When it is contained in tens, what will be the unit of the quotient figure? When it is contained in hundreds? In thousands?

5. If the dividend and divisor are both compound numbers, what do you do?

6. If any partial dividend is less than the divisor, what is the corresponding figure of the quotient?

7. When there is a remainder after division, what do you do with it?

75.—1. When the divisor is equal to the dividend, what will the uotient be?

2. When the divisor is 1, what will the quotient be?

3. When the divisor is less than the dividend, how will the quotient compare with 1? How many times will it be greater than 1?

4. When the *divisor is greater* than the dividend, how will the *quotient compare with 1?* What part will the quotient be of 1?

Proofs of Division.

76. There are three methods of proving division:

I. *Multiply the divisor by the quotient, and add in the remainder, if any: the result should be the dividend.*

II. *Divide the dividend, diminished by the remainder, if any, by the quotient: the result should be the divisor.*

III. *Find the excess of 9's in the divisor and in the quotient; multiply them together, and note the excess of 9's in the product: this should be equal to the excess of 9's in the dividend, after being diminished by the remainder, if any.*

1st Method.

In the last example, we had 11772 ÷ 327 = 36.

Now, 327 × 36 = 11772.

2d Method.

Divisor,	327,	excess of 9's	. .	3	Product, 0.
Quotient,	36	" "	. .	0	
Dividend,	11772	" "	. .	0	

Examples.

(1)
8) 19737

(2)
4) 147368

(3)
5) 1346840

(4)
6) 1650930

(5)
6) 47689872

(6)
9) 10324683

(7)
7) 506321494

(8)
£ s. d.
3) 47 19 3

(9)
A. R. P.
9) 37 3 17

(10)
yd. qr. na.
5) 47 3 1

(11)
$ cts. m.
8) 634 75 6

(12)
$ cts. m.
7) 1468 09 6

(13)
$ cts.
12) 802346 16

Divide 734947644 by 48.
Divide 8536752 by 36.
Divide 3367598 by 19.
Divide 49300 by 725.
Divide 6477150 by 145.
Divide 770 by 28.
Divide $87.256 by 5.
Divide $495.704 by 129.

22. Divide $29.25 by 26.
23. Divide $10.125 by 27.
24. Divide $347.49 by 429.
25. Divide $751.50 by 150.
26. Divide $5711.04 by 108
27. Divide $315 by $35.
28. Divide $50065 by $527.
29. Divide $432 by 54.

30. Divide 334422198 by 438.
31. Divide 714394756 by 1754.
32. Divide 47159407184 by 3574.
33. Divide 5719487194715 by 45705.
34. Divide 4715714937149387 by 17493.
35. Divide 671493471549375 by 47143.
36. Divide 571943007145 by 37149.
37. Divide 1714347149347 by 57143.
38. Divide 49371547149375 by 374567.
39 Divide 171493715947143 by 571007.
40. Divide 6754371495671594 by 678957.
41. Divide 7149371478 by 121.
42. Divide 71900715708 by 57149.
43. Divide 147149371484 75 by 123456.
44. Divide 729 A. 2 R. 7 P. by 41.
45. Divide 365 da. 6 hr. by 240.
46. Divide 1298 mi. 2 fur. 33 rd. by 37.
47. Divide 95 hhd. 6 gal. by 120.
48. Divide 232 bush. 3 pk. 7 qt. by 105.

How many methods of proof are there for division? What are
What is the proof by *multiplication*? What is the proof by
's?

49. Divide $18306.25 into seven hundred and twenty-five equal parts.

50. Bought 7 yards of cloth for 16s. 4d.: what did it cost per yard?

51. A man travelled 265 mi. 6 fur. 16 rd. in 12 days: how far did he travel in one day?

52. If 569 A. 2 R. 23 P. be equally divided among 9 persons how much will 5 of them have?

53. The annual income of a gentleman is $10000: how much is that per day, counting 365 days to the year?

54. What number multiplied by 9999 will give the product 987551235?

55. A gentleman owning an estate of $75000, gave one-fourth of it to his wife, and the remainder was divided equally among his five children: how much did each receive?

56. The expenditure of the United States for 1853 was $54026818: how much would that be per day, allowing 365 days to the year?

57. If 28 yards of cloth cost one hundred and thirty-three dollars, what will one yard cost?

58. If I pay $637.50 for 51 yards of cloth, what is the price per yard?

59. The city of New York, in 1850, had 104 periodical publications, with an aggregate circulation of 78747600 copies: what was the average circulation of each?

60. Bought 19 bushels of wheat for $30.875: what was the cost of one bushel?

61. How long will 9125 loaves of bread last 5 families, if each family consumes 5 loaves a day?

62. The product of two numbers is 7207272072, and the multiplier 9009: what is the multiplicand?

63. How many rings, each weighing 4 dwt. 12 gr. can be made from 10 oz. 11 dwt. 12 gr. of gold?

64. If iron is worth 2 cents a pound, how much can be bought for $67.50?

65. If 14 sticks of hewn timber measure 12 T. 38 ft. 118 in., how much does each stick contain?

66. In 1850, Pennsylvania manufactured 285702 tons of pig iron, and employed 9285 hands: what was the average product of each hand?

67. The number of college libraries in the United States in 1850, was 213, containing 942321 volumes: what would be the average number of volumes in each?

68. Supposing the sun to make a complete revolution in 365 days, what is his daily velocity of rotation?

69. How many dozen spoons can be made from 3 lb. 11 oz. of silver, allowing 15 pwt. 16 gr. to each spoon?

70. A gentleman bought a piece of cloth, containing 48 yards, at $3.25 per yard. At what price per yard must he sell the cloth to gain $60?

71. If £75 18s. 9d. will pay 5 men for their weekly labor, what amount would pay 18 men at the same rate?

72. How many revolutions does the wheel of a railcar make in going a distance of 75 miles, supposing the wheel to be 9 ft. 6 in. in circumference?

73. There is a certain number, which being divided by 3, the quotient multiplied by 4, and 6 added to the product, will give 18: what is the number?

74. If a farm, containing 512 acres, costs $28672, how much oes it cost per acre?

75. If 288120 be divided into 432 equal parts, what will be the value of one of the parts?

76. Light moves with immense rapidity. It comes from the sun to the earth *in 8 minutes*, the distance being 96 millions *of miles: how far does it* move in 1 second?

CONTRACTIONS IN MULTIPLICATION.

77. Contractions in Multiplication are short methods of finding products when the multipliers are particular numbers.

78. To multiply by 25.

1. Multiply the number 356 by 25.

ANALYSIS.—If we annex two ciphers to the multiplicand, we multiply it by 100 (Art. 61): this product is four times too great: for the multiplier is but *one fourth* of 100; hence, to multiply by 25,

OPERATION.

$$\begin{array}{r} 4\,)\,\underline{3\,5\,6\,0\,0} \\ 8\,9\,0\,0 \end{array}$$

Annex two ciphers to the multiplicand, and divide the result by 4.

Examples.

1. Multiply 287 by 25.
2. Multiply 184 by 25.
3. Multiply 6741 by 25.
4. Multiply 3074 by 25.

79. When the multiplier contains a fractional unit.

1. What is the product of 15, multiplied by $3\frac{1}{5}$?

ANALYSIS.—The multiplicand is to be taken 3 and one-fifth times: taking it *one-fifth times*, which is done by dividing by 5, gives 3, which we write in the unit's place: then, taking it 3 times, gives 45, and the sum 48 is the product; hence,

OPERATION.

$$\begin{array}{r} 15 \\ \underline{3\frac{1}{5}} \\ 3 \\ \underline{45} \\ 48 \end{array} \textit{ Ans}$$

Take such a part of the multiplicand as the fraction is of 1; *then multiply by the integral number, and the sum of the products will be the required product.*

Examples

1 Multiply 327 by $8\frac{1}{3}$.
2 Multiply 23474 by $16\frac{1}{2}$.
3. Multiply 34700 by $127\frac{1}{5}$.
4. Multiply 1272 by $12\frac{1}{8}$.
5. Multiply 9824 by $272\frac{1}{4}$.
6. Multiply 3828 by $73\frac{1}{6}$.

---

77. What are contractions in Multiplication?—78. How do you multiply by 25?—79. How do you multiply when the multiplier contains a *fractional unit?*

80. To multiply by 12½.

1. Multiply the number 286 by 12½.

Analysis.—Since 12½ is *one-eighth* of 100,

Annex two ciphers to the multiplicand, and divide the result by 8.

OPERATION.

8)28600

3575

Examples.

1. Multiply 384 by 12½.
2. Multiply 476 by 12½.
3. Multiply 14800 by 12½.
4. Multiply 670418 by 12½.

81. To multiply by 33⅓.

1. Multiply the number 975 by 33⅓.

Analysis.—Annexing two ciphers to the multiplicand, multiplies it by 100: but the multiplier is *one-third* of 100: hence,

OPERATION.

3)97500

32500

Annex two ciphers, and divide the result by 3.

Examples.

1. Multiply 1679252 by 33⅓.
2. Multiply 1480724 by 33⅓.
3. Multiply 10675512 by 33⅓.
4. Multiply 4442172 by 333⅓.

82. To multiply by 125.

1. Multiply the number 1125 by 125.

Analysis.—Annexing three ciphers to the multiplicand, multiplies it by 1000: but 125 is but *one-eighth* of one thousand: hence,

OPERATION.

8)1125000

140625

Annex three ciphers, and divide the result by 8.

Examples.

1. Multiply 59264 by 125.
2. Multiply 17593408 by 125.
3. Multiply 1940812 by 125.
4. Multiply 140588 by 125.

---

80. How do *you multiply* by 12½?—81 How do you multiply by 33⅓?—82. *How do you multiply* by 125?

Applications.

83. The analysis of a practical question, in Multiplication, requires that the multiplier be an abstract number; and then the unit of the product will be *the same as the unit of the multiplicand.*

84. To find the cost of several things, when we know the price of one, and the number of things

1. What will six yards of cloth cost, at 8 dollars a yard?

Analysis.—Six yards of cloth will cost 6 times as much as 1 yard. Since 1 yard of cloth costs 8 dollars, 6 yards will cost 6 times 8 dollars, which are 48 dollars; therefore, 6 yards of cloth, at 8 dollars a yard, will cost 48 dollars: hence,

The cost of any number of things is equal to the price of a single thing multiplied by the number of things.

We have seen that the product of two numbers will be the same (that is, contain the same number of units), whichever be taken for the multiplicand (Art. 59). Hence, in practice, we may multiply the two factors together, taking either for the multiplier, and *then assign the proper unit to the product.* We generally take the *less* number for the multiplier.

85. To find the cost, when the price is an aliquot part of a dollar.

Note.—For definition of Aliquot part, see Art. 98.

1. Find the cost of 45 bushels of apples, at 25 cents a bushel.

Analysis.—If the price were 1 dollar a bushel, the cost would be as many dollars as there are bushels. But the price is 25 cents = $\frac{1}{4}$ of a dollar; hence, the cost will be one-fourth as many dollars as there are bushels; that is, as many dollars as 4 is contained times in 45, which is 11, and 1 dollar over. This is reduced to cents by adding two ciphers; then dividing again by 4, we have the entire cost: hence,

OPERATION.

$$\begin{array}{r} 4\,\underline{)\,45.00} \\ \$11.25 \end{array}$$

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83. What does the analysis of a practical question in Multiplication require? What then follows?—84. How do you find the cost of several things when you know the price of a single thing?—85. How do you find the cost when the price is an aliquot part of a dollar?

Take such a part of the number which denotes the amount of the commodity as the price is of 1 *dollar: the result will be the cost in dollars.*

86. To find the cost, when the price contains an aliquot part of a dollar

Find the cost of 45 bushels of wheat, at $2.25 a bushel.

ANALYSIS.—The cost, at 25 cents per bushel, would be $11.25; and at $2 per bushel, would be $90: hence, at $2.25, it is $101.25.

45	
$2¼	
11.25	at 25 cts.
90	at $2.00.
$101.25	cost.

Multiply the quantity by the aliquot part of the dollar: then multiply it by the integral part of the price: the sum of the products will be the entire cost.

Examples.

1. What would be the cost of 284 bushels of potatoes, at 50 cents a bushel?

2. At 33⅓ cents a gallon, what will 51 gallons of molasses cost?

3. What cost 112 yards of calico, at 12½ cents a yard?

4. If a pound of butter costs 20 cents, what will 175 pounds cost?

5. What will 576 bushels of wheat cost, at $1.50 a bushel?

6. What will it cost to dig a ditch 129 rods long, at $1.33⅓ a rod?

7. At $1.25 a barrel, what will 96 barrels of apples cost?

8. What will 3 pieces of cloth cost, each piece containing 25 yards, at $1 20 a yard?

---

86. How do you find the cost when the price contains an aliquot part *of a dollar?*

87 To find the cost of articles sold by the 100 or 1000.

1. What will 544 feet of lumber cost, at 2 dollars per 100?

ANALYSIS.—At 2 dollars a foot, the cost would be 544 × 2 = 1088 dollars; but as 2 dollars is the price of 100 feet, it follows that 1088 dollars is 100 times the cost of the lumber; therefore, if we divide 1088 dollars by 100 (which is done by cutting off two of the right hand figures, Art. 61), we obtain the cost

NOTE.—Had the price been so much per 1000, we should have divided by 1000: hence,

Multiply the quantity by the number denoting the price: if the price be by the 100, *cut off* two *figures on the right hand of the product; if by the* 1000, *cut off* three, *and the remaining figures will be the answer in the same denomination as the price, which, if cents or mills, may be reduced to dollars.*

Examples.

1. What will be the cost of 3742 feet of timber, at $3.25 per 100?

2. At $12.50 per 1000, what will 5400 feet of boards cost?

3. At $9.75 per hundred, what will be the cost of 7568 oranges?

4. What will be the cost of 19875 lemons, at the rate of $25 per thousand?

5. *Richard Ames,* *Bought of John Maple.*

1275	feet of	boards,	at	$9.00	per 1000,
3720	"	"	"	15.25	"
715	"	scantling,	"	8.75	"
1200	"	timber,	"	12.06	"
2550	"	lathing,	"	.75	100,
965	"	plank,	"	1.12½	"

Received payment, JOHN MAPLE.

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87. How do you find the cost of articles sold by the 100 or 1000?

88. To find the cost of articles sold by the ton.

What is the cost of 640 pounds of hay, at $11.50 per ton?

Analysis.—Since there are 2000 pounds in a ton, the cost of 1000 pounds will be half as much as of 1 ton: viz., $5.75. Multiply this by the number of pounds (640), and cut off three places from the right (Art. 87), in addition to the two places cut off for cents; hence,

OPERATION.

2) $11.50
5.75 price of 1000 lb.
640
23000
3450
$3.68000

Multiply one-half the price of a ton by the number of pounds, and cut off three figures from the right hand of the product The remaining figures will be the answer in the same denomination as the price of a ton.

Examples.

1. What will be the cost of 1575 pounds of plaster, at $3.84 per ton?

2. If one ton of coal costs $7.37½, what will be the cost of 3496 pounds?

3. What will 1260 pounds of hay cost, at $9.40 per ton? at $10.25? at $14.60?

4. What will be the cost of transportation of 5482 pounds of iron from Pittsburgh to New York, at $6.65 per ton?

5. What will be the cost of removing 785797 pounds of stone, at $1.87½ per ton?

6. What will 67418 pounds of hay cost, at 26 dollars a ton?

7 What will 497046 pounds of plaster cost, at $9.75 a ton?

8. What is the cost of 9047641 pounds of railroad iron, at $75 a ton?

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88. *How do you find the cost of articles sold by the ton?*

CONTRACTIONS IN DIVISION.

89. Contractions in Division are short methods of finding the quotient when the divisor is a particular number.

90. By reversing the processes of Arts. 78, 80, 81 and 82 we have the four following rules :

1. To divide any number by 25.

Multiply the number by 4, *and divide the product by* 100.

2. To divide any number by 12½:

Multiply the number by 8, *and divide the product by* 100.

3. To divide any number by 33⅓, or 333⅓, &c.:

Multiply the number by 3, *and divide the product by* 100, *or* 1000, *&c.*

4. To divide any number by 125:

Multiply by 8, *and divide the product by* 1000.

Examples.

1. Divide 6350 by 25.
2. Divide 656280 by 25.
3. Divide 7278675 by 25.
4. Divide 5287215 by 25.
5. Divide 12225 by 12½.
6. Divide 11925 by 12½.
7. Divide 1760600 by 12½.
8. Divide 67500 by 33⅓.
9. Divide 1308400 by 33⅓.
10. Divide 15851400 by 33⅓.
11. Divide 8072400 by 33⅓.
12. Divide 16144800 by 33⅓.
13. Divide 31702800 by 33⅓.
14. Divide 281250 by 125.
15. Divide 6015750 by 125.
16. Divide 2026875 by 125.
17. Divide 6080625 by 125.
18. Divide 18047250 by 125.

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89. What are Contractions in Division?

90. *What rules do we* get by reversing the four last processes?

91. When the divisor is a composite number.

1. How many feet and yards are there in 288 inches?

Analysis.—Since there are 12 inches in 1 foot, there will be as many feet in 288 inches as 12 is contained times in 288; viz., 24 feet, *in which the unit is* 1 *foot*. Since 3 feet make 1 yard, there will be as many yards in 24 feet as 3 is contained times in 24; viz., 8 yards, *in which the unit is* 1 *yard*. We have thus passed, by division, from the unit 1 inch to the unit 1 foot, and then to the unit 1 yard; that is, in each operation, we have *increased the unit as many times as there are units in the divisor.*

OPERATION.

$$\begin{array}{r} 12\underline{)\,288} \\ 3\underline{)\,24} \\ 8 \end{array}$$

Let us now use the same numbers, in a different question:

2. If 288 dollars be equally divided among 36 men, what will be the share of each?

Analysis.—Since 288 dollars is to be equally divided among 36 men, each will have as many dollars as 36 is contained times in 288. Dividing 288 into 12 equal parts, we find that each part is 24 dollars. If *each of these parts* be now divided into 3 equal parts, there will then be 36 parts in all, each equal to 8 dollars: here, *the unit of the result is the same as that of the dividend.* Hence, we may regard division under two points of view:

OPERATION.

$$\begin{array}{r} 12\underline{)\,288} \\ 3\underline{)\,24} \\ 8 \end{array}$$

1. *As a process of reduction, in which the unit of each succeeding dividend is increased as many times as there are units in the divisor:*

2. *As a process of separating a number into equal parts; in which case the unit of a part will be the same as that of the dividend.*

Hence, when the divisor is a composite number:

Divide the dividend by one of the factors of the divisor then divide the quotient, thus arising, by a second factor, and so on, till every factor has been used as a divisor: the last quotient will be the answer.

---

91. How do you divide when the divisor is a composite number?

Examples.

Divide the following numbers by the factors of the divisors:

1. 2322 by 6 = 2 × 3.
2. 37152 by 24 = 4 × 6.
3. 19152 by 36 = 6 × 6.
4. 38592 by 48 = 4 × 12.
5. 1145592 by 72 = 8 × 9.
6. 185760 by 96 = 8 × 12
7. 115776 by 64 = 8 × 8.
8. 463104 by 144 = 12 × 12

NOTE.—When there are remainders, after division, the operation is to be treated as one of reduction.

92. How to find the true remainder.

1. Divide the number 3671 by 30 = 2 × 3 × 5.

2) 3671			
3) 1835	. .	1 = 1st rem.	 1
5) 611	. .	2 = 2d rem.	2 × 2 = 4
122	. .	1 = 3d rem.	1 × 3 × 2 = 6
Ans. $122\frac{11}{30}$.			For remainder, 11

ANALYSIS.—Dividing 3671 by 2, we have a quotient 1835, and a remainder, 1. After the third division, the quotient is 122, and the remainder, 1. Now, it is plain, from the first analysis, that,

1. The unit of the first quotient is as many times greater than the unit of the dividend, as the divisor is times greater than 1; and similarly for all the following quotients.

2. The unit of the first remainder is the same as the unit of the dividend; and the unit of any remainder is the same as that of the corresponding dividend.

3. The unit of any dividend is reduced to that of the preceding dividend, by multiplying it by the preceding divisor.

Hence, to find the remainder in *units of the given dividend* is simply a case of reduction in which the divisors denote the units of the scale: therefore,

To the first remainder, add the products which arise by multiplying each of the following remainders by all the preceding divisors, except its own: the sum will be the true remainder.

Examples.

Divide the following numbers, and find the remainders:

1. 416705 by 315 = 7 × 9 × 5.
2. 804106 by 462 = 3 × 2 × 7 × 11.
3. 756807 by 3456 = 4 × 8 × 9 × 12.
4. 8741659 by 105 = 3 × 5 × 7.
5. 947043 by 385 = 5 × 7 × 11.
6. 4704967 by 1155 = 11 × 7 × 5 × 3.
7. 71874607 by 7560 = 8 × 7 × 9 × 5 × 3.

93. When the divisor is 10, 100, 1000, &c.

1. Divide 3278 by 1000 = 10 × 10 × 10

OPERATION.

$10\,)\,\underline{3\,2\,7|8}$

$10\,)\,\underline{3\,2|7}$. 8 rem.

$10\,)\,\underline{3|2}$ . . 7 rem.

3 . . 2 rem.

$3\frac{278}{1000}$, *Ans.*

ANALYSIS.—We divide 3278 by 10, by simply cutting off 8, giving 327 tens, and 8 units remainder. We again divide by 10, by cutting off the 7, giving 32 hundreds, and 7 tens remainder. We again divide by 10, by cutting off the 2, giving a quotient of 3 thousands, and 2 hundreds remainder. The quotient then is 3, and a remainder of 2 hundreds, 7 tens, and 8 units, or 278: hence,

Cut off from the right of the dividend as many figures as there are ciphers in the divisor, considering the figures at the left, the quotient, and those at the right, the remainder

94. When any divisor contains significant figures, with one or more ciphers at the right hand.

1 Divide 875896 by 32000.

OPERATION.

32|000) 875|896 (27

$\underline{64}$

235

$\underline{224}$

11896 rem

Ans. $27\frac{11896}{32000}$.

ANALYSIS.—The divisor 32000 = 32 × 1000. Dividing by 1000 gives a quotient 875, and 896 remainder. Then dividing by 32, gives a quotient 27, and 11 remainder, which *gives the result*, $27\frac{11896}{32000}$: *hence,* *Cut off, by a line, the ciphers from the right of the divisor, and an equal number of figures from the right of the dividend: divide the remaining figures of the dividend by the remaining figures of the divisor, and to the remainder, if any, annex the figures cut off from the dividend, and the result will form the true remainder.*

Examples.

Divide the following numbers:

1. 1972654 by 420000.
2. 1752000 by 12000.
3. 73199006 by 801400.
4. 11428729800 by 72000.
5. 36981400 by 146000.
6. 141614398 by 63000.

95. When the divisor contains a fraction.

1. Divide 856 by $4\frac{1}{5}$.

OPERATION.

$3\,)\,4280$

$7\,)\,\underline{1426}$. 2 rem.

203 . 5 rem.

Ans. $203\frac{17}{21}$.

ANALYSIS.—There are 5 fifths in 1; hence, in 4 there are 20 fifths; therefore, $4\frac{1}{5}$ = 21 fifths. In the dividend 856, there are 5 times as many fifths as units 1; that is, 4280 fifths; therefore, the quotient is 4280 divided by 21, equal $203\frac{17}{21}$. Hence, when the divisor contains a fraction,

Reduce the divisor and dividend to the fractional unit of the divisor, and then divide as in integral numbers.

Examples.

Find the quotients in the following examples:

1. $3245 \div 16\frac{1}{2}$.
2. $47804 \div 15\frac{1}{3}$.
3. $870631 \div 14\frac{1}{4}$.
4. $37214 \div 51\frac{1}{8}$.
5. $87317 \div 9\frac{3}{8}$.
6. $87906 \div 12\frac{4}{7}$.
7. $95675 \div 15\frac{5}{8}$.
8. $71096 \div 17\frac{3}{7}$.

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92. How do you find the true remainder?

93. How do you divide when the divisor is 10, 100, &c.

94 How do you divide when the divisor contains significant figures with ciphers at the right?

95. How do you divide when the divisor contains a fraction?

Applications.

96. Division has three applications.

1. Given the number of things and their cost, to find the price of one thing.
2. Given the cost of a number of things, and the price of one thing, to find the number of things.
3. To divide any number of things into a given number of equal parts.

Rules.

I. *Divide the number denoting the cost by the number of things: the quotient will be the price of one:*

II. *Divide the number denoting the cost by the price of one: the quotient will be the number of things:*

III. *Divide the number denoting the things by the number of parts into which they are to be divided: the quotient will be the number in each part.*

PRACTICE.

97. **Practice** is an easy and short method of applying the rules of arithmetic to questions which occur in trade and business.

98. **An Aliquot Part** of a number is any exact divisor of it, whether integral or fractional. Thus, 3 months is an aliquot part of a year, being one-fourth of it, and 12½ cents is an aliquot part of 1 dollar, being one-eighth of it.

Aliquot Parts of a Dollar.

$1	= 100 cents.	$\frac{1}{8}$ of a dollar	= 12½ cents.
$\frac{1}{2}$ of a dollar	= 50 cents.	$\frac{1}{10}$ of a dollar	= 10 cents.
$\frac{1}{3}$ of a dollar	= 33⅓ cents.	$\frac{1}{16}$ of a dollar	= 6¼ cents.
$\frac{1}{4}$ of a dollar	= 25 cents.	$\frac{1}{20}$ of a dollar	= 5 cents.
$\frac{1}{5}$ of a dollar	= 20 cents.	$\frac{1}{2}$ of a cent	= 5 mills.

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96. How many applications has division? What are they? Give the rules.

97. What is Practice?—98. What is an aliquot part of a number?

Aliquot Parts of a Pound.

£1 = 20 shillings.
½ of a pound = 10 shillings.
⅓ of a pound = 6s. 8d.
¼ of a pound = 5 shillings.

⅕ of a pound = 4 shillings.
⅙ of a pound = 3s. 4d.
⅛ of a pound = 2s. 6d.
$\frac{1}{12}$ of a pound = 1s. 8d.

Aliquot Parts of a Year.

1 year = 12 months.
½ of a year = 6 months.
⅓ of a year = 4 months.

¼ of a year = 3 months.
⅙ of a year = 2 months.
$\frac{1}{12}$ of a year = 1 month.

Aliquot Parts of a Month.

½ of a month = 15 days.
⅓ of a month = 10 days.
¼ of a month = 7¼ days.

⅕ of a month = 6 days.
⅙ of a month = 5 days.
⅒ of a month = 3 days.

1. What is the cost of 376 yards of cloth, at $1 75 a yard?

1	1	$376 = cost at $1 per yard
50	½	188 = cost at 50 cts. "
25	¼	94 = cost at 25 " "
		$658 = cost at $1.75 "

ANALYSIS.—At $1 per yard, 376 yards cost $376. Separating 75 cents into 50 cents, an aliquot part of one dollar, and 25 cents, an aliquot part of 50 cents, we take one-half of $376, and obtain $188, the cost of 376 yards at 50 cents per yard. Since 25 cents is one-half of 50 cents, we take ½ of $188, and thus obtain $94, the cost at 25 cents per yard. The sum $658 gives the cost at $1.75 per yard.

2. What is the cost of 196 yards of cotton, at 9d. per yard?

1	1	196s. = cost at 1s. per yard
6	½	98s. = cost at 6d. "
3	½	49s. = cost at 3d. "
		20) 147s. = cost at 9d. "
		£7 7s., entire cost.

ANALYSIS.—9d. = 6d. + 3d. The cost of 196 yards at 1s. = 196s. Since 6d. = ½s., the cost at 6d. = ½ of 196s. = 98s. The cost at 3d. = ½ as much as at 6d.; ½ of 98s. = 49s. The cost *at 9d = the sum* = 147s. = *£7 7s.*

Examples.

1. What is the cost of 425 yards of calico, at 1s. 6d. per yard?

2. What is the cost of 475 yd. of tape, at 1d. 1far. per yard?

3. What is the cost of 354 yards of cord, at 1¼d. per yard?

4. At 12½ cents = \$⅛ a yard, what will be the cost of 4756 yards of bleached shirting?

5. At 2s. 6d. = £⅛ per pair, what will be the cost of 3754 pairs of gloves?

6. If wheat is 3s. 6d. a bushel, what will be the cost of 5320 bushels?

7. If broadcloth costs £1 7s. a yard, what will be the cost of 435 yards?

8. If linen is 2s. 6d. = 2½s. a yard, what will be the cost of 660 yards?

9. What will be the cost of 40 lb. of soap, if 1 pound costs 6¾ cents?

10. What will be the cost of 148 yards of cloth, at \$3.75 a yard?

11. If one bushel of apples cost 62½ cents, what will be the cost of 876 bushels?

12. What will be the cost of 1000 quills, if every 5 quills cost 1½ cents?

13. If 1 yard of extra-superfine cloth costs \$9.50, what will be the cost of 85 yd. 2 qr.?

14. What will 6½ yards of cloth cost, at \$3.75 a yard?

15. What will 8¾ boxes of lemons cost, at \$7.25 a box?

16. What will 15⅕ pieces of calico cost, at \$20.75 a piece?

17. If one ton of iron costs \$124, what will be the cost of 3 T. 15 cwt. 2 qr. 15 lb.?

18. What will be the cost of 350 bushels of potatoes, at *3s. 6d. a bushel, English Currency?*

LONGITUDE AND TIME.

99. The equator of the earth is divided into 360 equal parts, which are called *degrees of longitude.*

100. The sun apparently goes round the earth once in 24 hours. This time is called a *day.*

Hence, in 24 hours, the sun apparently passes over 360° of longitude; and in 1 hour, over $360° \div 24 = 15°$.

Since the sun, in passing over 15° of longitude, requires 1 hour, or 60 minutes of time, in 1 minute he will pass over $15 \div 60 = 15'$ of longitude; and in 1 second of time, he will pass over $15' \div 60 = 15''$ of longitude: Therefore,

15° of longitude require 1 hour of time.
15′ " " 1 minute of time.
15″ " " 1 second of time.

Hence,

I. *If the longitude, expressed in degrees, minutes, and seconds, be divided by* $15 = 3 \times 5$, *the quotient will be hours, minutes, and seconds of time.*

II. *If time, expressed in hours, minutes, and seconds, be multiplied by* $15 = 3 \times 5$, *the product will be degrees, minutes, and seconds of longitude.*

Examples.

1. Reduce 45° 31′ 45″ of longitude to time.

ANALYSIS.—We divide by 15, as in compound numbers; giving us 3 hr. 2 m. 7 sec.

$$\begin{array}{r|lll} 5) & 45° & 31' & 45'' \\ \hline 3) & 9 & 06 & 21 \\ \hline & 3\text{ hr.} & 2\text{ m.} & 7\text{ sec.} \end{array}$$

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99 How is the equator of the earth divided?

100. How long is the sun in apparently going round the earth? What is this time called? How many degrees of longitude does the sun pass over in a day? How much in 1 hour? How much in 1 minute? How much in 1 second? How is longitude reduced to time? How is time reduced to longitude?

2. Reduce 8 hr. 16 m. 40 sec. of time, to longitude.

ANALYSIS.—We multiply the seconds, minutes, and hours, each by 15, carrying from one to the other as in the multiplication of compound numbers.

OPERATION.

8 hr. 16 m. 40 sec.
15
124° 10′ 00″

3. If the difference of time between two places be 42 m. 16 sec., what is the difference of longitude?

4. What is the difference of longitude between two places, if the difference of time is 2 hr. 20 m. 44 sec.?

5. When it is 12 M. at New York, it is 11 hr. 6 m. 28 sec. at Cincinnati: what is their difference of longitude?

101. Which place has the earlier time.

When the sun is on the meridian of any place, it is 12 o'clock, or noon, at that place. And since the sun apparently goes from east to west, it will be *past* noon for all places at the east, and *before* noon for all places at the west.

If, then, we find the difference of time between two places, and know the exact time at one of them, the corresponding time at the other will be found by *adding* the difference, if that other be *east*, or by subtracting it, if *west*.

102. Knowing the longitude of two places, and the time at one place, to find the corresponding time of the other.

1. The longitude of Albany is 73° 42′ west, and that of Buffalo 78° 55′ west: what is the time at Buffalo when it is 10 o'clock A. M. at Albany?

ANALYSIS.—The difference of longitude is found by subtraction, and is 5° 13′. This difference is changed into the time 20 m. 52 sec., by dividing by 15. Since Buffalo is west of Albany, this difference must be subtracted from 10 hr., the time at Albany, and the remainder shows the time at Buffalo to be 9 hr. 39 m. 8 sec.

OPERATION

78° 55′
73 42
15) 5° 13′ diff. long.
Diff. time, 20 m. 52 sec.

10 hr. 0 m. 0 sec.
20 52
9 hr 39 m. 8 sec.

Hence the following

Rule.—I. *Reduce the difference of longitude to time:*

II. *Add the result to the given time, when the place at which the time is required lies* east, *and subtract it, when* west.

NOTE.—If the longitudes are both east or both west, the difference of longitude is found by subtraction. If one is east and the other west, the difference of longitude is expressed by the sum.

Examples.

1. The longitude of New York is 74° 1′ west, and that of Springfield, Illinois, 89° 33′ west: what would be the time at New York when it is 12 M. at Springfield?

2. The longitude of Philadelphia is 75° 10′ west, and that of New York 74° 1′ west: what is the time at Philadelphia, when it is 3 o'clock P. M. at New York?

3. Washington is in longitude 77° 2′ west; New Orleans in 89° 2′ west. When it is 9 o'clock A. M. at Washington, what is the time at New Orleans?

4. The difference of longitude between St. Louis and New York is 15° 35′. In travelling from New York to St. Louis, will a watch, keeping accurate time, be fast or slow at St. Louis, and how much?

103. The time at each of two places, and the longitude of one, being known, to determine the longitude of the other.

1. New York is in longitude 74° 1′ west. In what longitude is that place whose time is 10 o'clock A. M., when it is 2 o'clock P. M. at New York?

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101. What is the time, at any place, when the sun is on the meridian? How will the time then be for any place at the east? How will it be for any place at the west? If you have the difference of time of two places, and know the time at one of them, how do you find the time at the other when it is east? When it is west?

102. Knowing the longitude of two places, and the time at one, how do you find the corresponding time at the other?

103. If the time at each of the places be known, and the longitude *of one, how do you* find the longitude of the other?

Analysis.—The difference of time is 4 hr., equal to 60°, which is the difference of longitude. Since the time at New York is later in the day than that of the required place, New York must be east of that place, and the longitude is found by adding 60° to 47° 1′ which gives 134° 1′ west,—the required longitude. Hence we have the following

OPERATION.

14 hr.	0 m.		time at N. Y.
10	0		" at place
4	0	=	diff. of time.
	15		
60°	0′	=	diff. of long
74	1		
134°	1′		

Rule.—I. *Reduce the difference of time to difference of longitude:*

II. *Add the result to the given longitude, when the place at which the longitude is required has the earlier time, and subtract when it has the later time.*

Examples.

1. Philadelphia is in longitude 75° 10′ west. In what longitude is a vessel, whose chronometer indicates 11 hr. 30 m., A. M., Philadelphia time, when it is 2 hr. 15 min., P. M., on board the vessel?

2. The longitude of St. Louis is 90° 15′ west. A person at that place observed an eclipse of the moon at 10 hr. 40 m., P. M.; another person, in a neighboring State, observed the same eclipse 22 m. 12 sec. earlier: what was the longitude of the latter place, and the time of observation?

3. Oxford, in England, is in longitude 1° 15′ 22″ west. What is the longitude of that place, whose local time is 9 o'clock P. M., when the time at Oxford, as shown by an accurate chronometer, is 10½ o'clock, P. M.?

4. In going from London, whose longitude is 0, to Oregon City, an accurate timekeeper was found to have gained 8 hours In what longitude is Oregon City?

5. A captain observed an eclipse of the moon at 11 hr. 18 m. 15 sec., P. M., which was seen at Greenwich, according to the Nautical Almanac, at 12 hr. 50 m. 19 sec., P. M. In what longitude was the vessel?

Applications in the Fundamental Rules

1. What will it cost to build a wall 96 rods long, at a rod?

2. A farmer wishes to put 1066 bush. 2 pk of potat 474 barrels: what quantity must he put into each ba

3. How many barrels of apples, each containing 2½ can I buy for $36, at 45 cents a bushel?

4. The quotient arising from a certain division is 12 divisor is 375, and the remainder 184: what is the di

5. The Croton Water Works of New York are ca discharging 60000000 gallons of water every 24 hour is the average amount per minute?

6. The population of the United States in 1850 was 2 It is estimated that one person in every 400, dies annu intemperance: how many deaths may be attributed an this cause in the United States?

7. If a quantity of provisions lasted 25 men 2 mo. 3 how long would it have lasted 10 men?

8. If a man's salary is $1200 a year, and his expe $640, how many years will be required to save $6720

9. How long will it take to count 20 millions, at th 80 per minute?

10. If 3160 barrels of pork cost $47400, how man can be bought for $11475?

11. What will be the cost of 6 firkins of butter, e taining 96 pounds, at 12½ cents a pound?

12. What will 1000 quills cost, at ½ cent apiece?

13. What will be the cost of 85½ yards of cloth, yard?

14. What will be the cost of 1 hhd. 2 gal. 3 qt. of *at 56¼ cents a quart?*

15. What will be the cost of 196 yards of cotton goods, at 1s. 6d. per yard?

16. At 2s. 8d. per bushel, what will 1246 bushels of oats cost?

17. If 112 lb. of cheese cost £2 16s., what is that per pound?

18. What will be the cost of 1426 pounds of hay, at $9.75 per ton?

19. How much must I pay for the transportation of 3840 pounds of iron, from Albany to Buffalo, at $4.50 per ton?

20. Bought 124 bbl. of potatoes, each containing 2¼ bush., at 33⅓ cents a bushel: what was the cost?

21. There are three numbers, whose continued product is 16200; one of the numbers is 25; another, 18: what is the third number?

22. If 1 pwt. of gold is worth 92 cents, what would be the weight of $10059.28 in gold?

23. A man sold his house and lot for $4200, and took his pay in railroad stock, at 84 dollars a share: how many shares did he receive?

24. A person bought 640 acres of land, at 15 dollars an acre. He afterwards sold 160 acres, at 20 dollars an acre; 240 acres, at 18 dollars an acre; and for the remainder he received $4560. What was his entire gain, and what did he receive per acre on the last sale?

25. A piece of ground, 65 feet long and 48 feet wide, is inclosed by a wall 12 feet high, and 2½ feet thick: how many cubic feet in the wall?

26. What will be the cost of transportation from Montreal to Boston of 325640 feet of lumber, at $2.37½ per thousand?

27. Bought 684 pounds of hay, at $12.40 a ton: what did it cost me?

28. *At $2.12½ a hundred,* what will 786 feet of lumber cost?

29. How many shingles will it require to cover the roof of a building 40 feet long and 26 feet wide, with rafters 16 feet long, allowing one shingle to cover 24 square inches?

30. If 14 lb. 8 oz. 12 pwt. 3 gr. of silver be made into 9 teapots of equal weight, what will be the weight of each?

31. A man bought 320 barrels of flour for $2688: at what rate must he sell it to gain $1.60 on each barrel?

32. A farmer has a granary containing 449 bush. 1 pk. 2 qt. of wheat; he wishes to put it into 182 bags: how much must he put into each bag?

33. A trader bought 750 barrels of flour, for which he paid $4875; he sold the same for $7.25 a barrel: what was his profit on each barrel?

34. How many sheep, at $1.62½ a head, can be bought for $169?

35. How many canisters, each holding 3 lb. 10 oz., can be filled from a chest of tea containing 58 lb.?

36. In 26 hogsheads the leakage has reduced the whole amount to 1358 gal. 2 qt.; if an equal quantity has leaked out of each hogshead, how much still remains in each?

37. A man bought a piece of land for $3475.25, and sold it for $3801.65, by which transaction he made $3.40 an acre: how many acres were there?

38. The whole amount of gold produced in California in the year 1855, was as follows: $43313281, sent to the Atlantic States; $6500000, sent directly to England; and $8500000 retained in the country. In 1854, the total product of gold in California was $57715000: how much more was produced in 1855 than in 1854?

39. If the forward wheels of a carriage, are 12 feet in circumference, and the hind wheels, 16 feet 6 inches, how many more times will the forward wheels turn round than the hind wheels in running a distance of 264 miles?

40. If a certain township is 9 miles long, and 4⅓ miles wide, how many farms of 192 acres each does it contain?

41. The total number of land warrants issued during the year ending September 30, 1855, was 34337, embracing 4093850 acres of land: what was the average number of acres to each warrant?

42. The longitude of Philadelphia is 75° 10′, and that of New Orleans 89° 2′, both west: when it is 12 M. at Philadelphia, what is the time at New Orleans?

43. The sun passes the meridian at 12 M., the moon at 8 hr. 30 m. P. M.: what is the difference in longitude between the sun and moon?

44. Two persons, A and B, observed an eclipse of the moon; A observed its commencement at 9 hr. 42 m. P. M.; B was in longitude 73° 20′, and observed its commencement 23 minutes earlier than A: what was A's longitude, and B's time of observation?

45. If in 11 piles of wood there are 120 cords 7 cord feet 5 cubic feet, how much is there in each pile?

46. If 16 cwt. 2 qr. 11 lb. 10 oz. of flour, be put into nine barrels, how much will each barrel contain?

47. A miller bought a quantity of wheat for $625.40, which he floured and put into barrels at an expense of $110.12½: what profit did he make by selling it for $900?

48. America was discovered October 11, 1492: how long to the commencement of the Revolution, April 19, 1775?

49. From a hogshead of wine, a merchant draws 18 bottles each containing 1 pt. 3 gi.; he then fills three 6-gallon demijohns, and 4 dozen bottles, each containing 2 qt. 1 pt. 3 gi.: how much remained in the cask?

50. In 753689 yards, how many degrees and statute miles?

51. In 189 *m.* 3 *fur.* 6 *rd.* 1 ft., how many feet?

52. If 24 men can build 768 rods of wall in 1 day, how many rods can 48 men build in 9 days?

53. A certain number increased by 1764, and the sum multiplied by 209, gives the product of 7913576: what is the number?

54. If a man travels 146 mi. 7 fur. 14 rd. 14 ft. in 5 days, how much is that for each one half-day?

55. If 325 acres of land costs $17712.50, how many acres can be bought for $545?

56. A merchant having $324, wishes to purchase an equal number of yards of two kinds of cloth; one kind was worth 4 dollars a yard, the other was worth 5 dollars a yard: how many yards of each can he buy?

57. From one-fourth of a piece of cloth, containing 68 yd. 3 qr., a tailor cut 5 suits of clothes: how much did each suit contain?

58. A manufacturer having £5 10s., distributed it among his laborers, giving every man 18d., every woman 12d., and every boy 10d.; the number of men, women, and boys was equal: what was the number of each?

59. It is estimated that 1 out of every 1585 persons in Great Britain is deaf and dumb. The population, according to the census of 1851, was 20936468: how many deaf and dumb persons were there in the entire population?

60. A grocer, in packing 6 dozen dozen eggs, broke half a dozen dozen, and sold the remainder for 1¼ cents a piece: how much did he receive for the eggs?

61. How much time will a man save in 50 years, beginning with a leap year, by rising 45 minutes earlier each day?

62. During the year 1855, there were shipped to Great Britain from the United States, 408434 barrels of flour; 2550092 bushels of wheat; 1048540 bushels of corn. Supposing the flour to have sold for $10.25 a barrel, the wheat for $2.12½ a bushel, and the corn for $0.94 a bushel, what was the value of the whole?

63. Richard Roe was born at 6 o'clock, A. M., June 24th. 1832: what was his age at 3 o'clock, P. M., on the 10th day of January, 1858?

64. A man dying without making a will, left a widow and 4 children. The law provides, in such cases, that the widow shall receive one-third of the personal property, and that the remainder shall be equally divided among the children. The estate was valued as follows: Stocks worth $5000; 5 horses, at $85 each; a yoke of oxen, at $110; 25 cows, at $22 each; 150 sheep, at $2 each; some lumber, at $45; farming utensils, at $174; household furniture, at $450; grain and hay, at $380: what was the share of the widow and each child?

65. How many shingles will it take to cover the two sides of the roof of a building, 55 feet long, with rafters 16½ feet in length, allowing each shingle to be 15 inches long and 4 inches wide, and to lay one-third to the weather?

66. The longitude of St. Petersburgh is 30° 45′ east, and that of Washington 77° 2′ west: what is the difference of longitude between the two places; and what is the time at St. Petersburgh when it is 6 o'clock A. M. at Washington?

67. A vessel sails from New York to Liverpool. After a number of days, the captain, by taking an observation of the sun, finds that his chronometer, which gives New York time, differs 1 hr. 44 m. from the time, at the place of observation. If his chronometer shows the time to be 3 hr. 12 m., P. M., what is the time at the place of observation, and how far is the vessel east of New York?

68. A cistern containing 960 gallons, has two pipes; 45 gallons run in every hour by one pipe, and 25 gallons run out by the other: how long a time will be required to fill the cistern?

69. The whole number of gallons of rum manufactured in the United States in 1850, was 6500500: if valued at 50 cents a gallon, how many *school*-houses could be built, worth $750 *each, with the proceeds?*

70. A speculator sold 840 bushels of wheat for $2180, which was $500 more than he gave for it: what did it cost him a bushel?

71. A farmer sold a grocer 30 bushels of potatoes, at 37½ cents a bushel, for which he received 6 gallons of molasses, at 45 cents a gallon; 60 pounds of mackerel, at 6¼ cents a pound; and the remainder in sugar, at 10 cents a pound: how many pounds of sugar did he receive?

72. If a man travels 12 mi. 3 fur. 20 rd. in one day, how long will it take him to travel 174 mi. 1 fur. at the same rate?

73. If a man sells 2 bar. 12 gal. 2 qt. of beer in one week, how much will he sell in 12 weeks?

74. A liquor merchant had 550 pint bottles, 400 quart bottles, 350 two-quart bottles, 375 three-quart bottles, and 150 jugs holding a gallon each: how many barrels of wine will fill them?

75. How many yards of carpeting, one yard wide, will it take to cover the floors of two parlors, each 18 feet long and 16 feet wide; and what will it cost, at $1.33⅓ a yard?

76. How many rolls of wall-paper, each 10 yards long and 2 feet wide, will it take to cover the sides of a room 22 feet long, 16 feet wide, and 9 feet high?

77. Two persons are 1 mi. 4 fur. 20 rd. apart, and are travelling the same way. The hindmost gains upon the foremost 5 rods in travelling 25 rods: how far must he travel to overtake him?

78. A man sold 500 bushels of wheat at $1.75 a bushel, and took his pay in sugar at 5 cents a pound. He afterwards sold one-half of the sugar: what quantity had he left?

79. A man bought 7 barrels of sugar, at $12.87½ a barrel; he kept two barrels for his own use, and sold the remainder for what the whole cost him: what did he receive per barrel?

80. A flour merchant bought a quantity of flour for $18750, and sold the same for $26250, by which he gained $3 a barrel: *how many barrels* were there?

81. Three men rented a farm, and raised 964 bush. 2 pk. 4 qt of grain, which was to be divided in proportion to the rent paid by each. The first was to have one-half the whole; the second, one-third the remainder; and the third what was left: how much did each have?

82. A vessel, in longitude 70° 25′ east, sails 105° 30′ 56″ west then 46° 50′ east, then 10° 5′ 40″ west, then 39° 11′ 36″ east: in what longitude is she then, and how many days will it take her to sail to longitude 77° west, if she sails 3° 20′ each day?

83. A privateer took a prize worth $25000, which was divided into 125 shares, of which the captain took 12 shares; 2 lieutenants, each 5 shares; 6 midshipmen, each 3 shares; and the remainder was divided equally among 85 seamen: how much did each receive?

84. If the longitude of Boston is 71° 4′, and a gentleman, in travelling from Boston to Chicago, finds that his watch is 1 hr. 5 m. 44 sec. too fast by the time of the latter place: what is the longitude of Chicago, provided his watch has kept accurate time?

85. What time would it be in Boston if it was 8 hr. 27 m. 30 sec., A. M., in Chicago?

86. What time would it be at Chicago if it was 12 M. at Boston?

87. Two places lie exactly east and west of each other, and by observation it is found that the sun comes to the meridian of the latter place 1 hour and 16 minutes after the former: how far apart are they in minutes and degrees of longitude?

88. In 12 bales of cloth, each bale containing 16 pieces, and each piece containing 20 ells English, how many yards?

89. A speculator gave $8968 for a certain number of barrel of flour, and sold a part of it for $2618, at $7 a barrel, and by so doing lost $2½ on each barrel: for how much must he sell the remainder, to *gain $1060* on the whole?

90. How many eagles can be made from 24 lb. 4 oz. 6 pwt. 18 gr. of gold, making no allowance for waste, if each eagle weighs 11 pwt. 9 gr.?

91. A man paid $3284.82 for some wheat. He sold 740 bushels at 2 dollars a bushel; the remainder stood him in $1.42 a bushel: how many bushels did he purchase?

92. A man sold 105 A. 2 R. 20 P. of land for as many dollars as there were perches of land, payable in instalments, at the rate of 1 dollar an hour. If the contract was closed at 12 o'clock, M., April 1st, 1856, what length of time will be allowed the purchaser to pay the debt, reckoning 365 days 6 hours to the year?

93. The sum of 2 numbers is 98, and their difference is 46: what are the numbers?

94. A farmer paid $76 dollars more for a horse than for a cow; he paid $190 for both: what was the value of each?

95. How many days intervene between March 5th and August 21st, both days inclusive?

96. A merchant buys 870 barrels of flour, at $9.50 a barrel. He finds one-half of it injured, and is willing to lose one-quarter on the value of that part: how much loss was that on each half barrel?

97. Three merchants, A, B, and C, are engaged together in business, and gain in one year $24612. This amount is to be equally divided among them, after paying A $675, and B $812, for extra services. How much did each receive?

98. Four merchants are in partnership. Their apparent profits during the year amount to $56895; but they have expended for clerk hire, $6750; for rent, $3500; for insurance, $156; and for incidental expenses, $364. The first is to have $250 for extra services; the second, $175 for travelling expenses; and the third, $95 for various articles furnished by him to the concern. What was the share of profit of each, after paying these expenses?

PROPERTIES OF NUMBERS.

Exact Divisors—Prime Numbers.

104. **An Exact Divisor** of a number, is any number, except 1 and the number itself, that will divide it without a remainder.

105 **One Number** is *divisible* by another, when the remainder is 0.

106. **An Odd Number** is one not divisible by 2.

107. **An Even Number** is one divisible by 2.

108 **A Prime Number** is one which has no exact divisor.

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, &c.,

are prime numbers.

109. What numbers are exact divisors.

1. **Any Factor** of a composite number is an exact divisor of the number.

2. **Three** is an exact divisor of any number, the sum of whose digits is divisible by 3.

3. **Four** is an exact divisor of a number when it will exactly divide the number expressed by the two right-hand digits.

4. **Five** is an exact divisor of every number whose right-hand figure is 0 or 5.

5. **Six** is an exact divisor of any even number of which 3 is an exact divisor.

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104. What is an exact divisor?—105. When is one number divisible by another?—106. What is an odd number?—107. What is an even number?—108. What is a prime number?

109.—1. Is a factor of a composite number an exact divisor?

2. What numbers will 3 exactly divide? 3. What numbers will 4 exactly divide? 4. What numbers will 5 exactly divide? 5. What numbers will 6 *exactly divide?* 6. What numbers will 9 exactly divide? *7. What numbers will 10* exactly divide?

6. **Nine** is an exact divisor of any number, the sum of whose digits is divisible by 9.

7. **Ten** is an exact divisor of every number whose right-hand figure is 0.

110. To find the prime factors of a composite number

1. What are the prime factors of 18?

Analysis.—Every factor of a composite number is an exact divisor. If any divisor is a composite number, resolve it again into factors, till each shall be prime: hence,

$$\begin{aligned}18 &= 0 \times 2\\ &= 3 \times 3 \times 2\end{aligned}$$

Every composite number is equal to the product of all its prime factors: hence, it is divisible by each of them.

Thus, $24 = 3 \times 8 = 3 \times 2 \times 4 = 3 \times 2 \times 2 \times 2$;
and $60 = 5 \times 12 = 5 \times 2 \times 6 = 5 \times 2 \times 3 \times 2$.

2. What are the prime factors of the composite number 105?

Analysis.—Three being an exact divisor, and a prime number, we divide by it, giving the quotient 35; then, 5 and 7 are prime divisors of the quotient: hence, 3, 5, and 7 are the prime factors of 105.

$$\begin{array}{r} 3)\underline{105} \\ 5)\underline{35} \\ 7 \end{array}$$

Hence, to find the prime factors of any composite number,

Rule.—*Divide the given number by any* PRIME *number that is an exact divisor: then divide the quotient by any other exact prime divisor, and so on, till a quotient is found which is a prime number: the several divisors and the last quotient will be the prime factors.*

Examples.

1. What are the prime factors of 9? 10? 12? 14? 16? 18? 24? 27? 28?

2. What are the prime factors of 30? 22? 32? 36? 38? 40? 45? 49?

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110. What is the rule for finding the prime factors of a composite *number?*

3. What are the prime factors of 50? 56? 58? 60? 64? 66? 68? 70? 72?

4. What are the prime factors of 76? 78? 80? 82? 84? 86? 88? 90?

5. What are the prime factors of 100? 102? 104? 275? 960? 472? 160? 836?

6. What are the prime factors of 105? 106? 108? 110? 115? 116? 120? 125? 1125? 360?

NOTE.—The prime factors, when the numbers are small, may generally be seen by inspection. The teacher can easily increase the number of examples.

111. **To find the prime factors common to two or more composite numbers.**

1. What are the common prime factors of 90, 120, and 150?

$$\begin{array}{r|rcrcr} 2 & 90 & .\,. & 120 & .\,. & 150 \\ \hline 3 & 45 & .\,. & 60 & .\,. & 75 \\ \hline 5 & 15 & .\,. & 20 & .\,. & 25 \\ \hline & 3 & .\,. & 4 & .\,. & 5 \end{array}$$

ANALYSIS.—It is plain that 2 is an exact divisor of all the numbers: hence, it is a prime factor of them. Since 3 is an exact divisor of the quotients, it is a prime factor of them; and since 5 will divide the second set of quotients, it is a prime factor. The quotients 3, 4, and 5 have no exact divisor; therefore, 2, 3, and 5 are all the common prime factors: hence,

The common prime factors of two or more numbers, are the exact divisors common to them all

Examples.

1. What are the prime factors common to 150, 210, and 270?
2. What are the prime factors common to 42, 126, and 168?
3. What are the prime factors common to 105, 315, and 525?
4. What are the prime factors common to 84, 126, and 210?
5. What are the prime factors common to 168, 256, 410, and 820?

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111. What is the rule for finding the prime factors common to two or more *composite numbers?*

CANCELLATION.

112. CANCELLATION is a method of shortening Arithmetical operations by omitting or *cancelling* common factors.

1. Divide 36 by 18. First, $36 = 9\times 4$; and $18 = 9\times 2$.

ANALYSIS.—Thirty-six divided by 18 is equal to 9×4 divided by 9×2: by *cancelling*, or striking out the 9's, we have 4 divided by 2, which is equal to 2.

OPERATION.

$$\frac{36}{18} = \frac{\cancel{9}\times 4}{\cancel{9}\times 2} = 2$$

NOTE.—The figures cancelled are slightly crossed.

113. Principles—Operations—and Rule.

The operations, in cancellation, depend on two principles:

1. The cancelling of a factor, in any number, is equivalent to dividing the number by that factor.

2. If the dividend and divisor be both divided by the same number the quotient will not be changed.

1. Divide 56 by 32.

ANALYSIS.—Resolve the dividend and divisor into factors, and then cancel those which are common.

OPERATION.

$$\frac{56}{32} = \frac{\cancel{8}\times 7}{\cancel{8}\times 4} = \frac{7}{4} = 1\frac{3}{4}$$

2. In 72 times 25, how many times 45?

ANALYSIS.—We see that 9 is a factor of 72 and 45. Divide by 9, and write the quotient 8 over 72, and the quotient 5 below 45. Again, 5 is a factor of 25 and 5. Divide 25 by 5, and write the quotient 5 over 25. Dividing 5 by 5, reduces the divisor to 1, hence, the true quotient is 40.

OPERATION.

$$\frac{\overset{8}{\cancel{72}}\times\overset{5}{\cancel{25}}}{\underset{\cancel{5}}{\cancel{45}}} = 40.$$

Rule.

I *Resolve the dividend and divisor into their prime factors or conceive them to be so resolved:*

II. *Cancel the common factors, and then divide the product*

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112. What is Cancellation?

113. On what principles does the operation of cancellation depend? *What is the rule for* the operation?

of the remaining factors of the dividend by the product of the remaining factors of the divisor.

NOTES.—1. Since every factor is cancelled by *division*, the quotient 1 always takes the place of the cancelled factor.

2. If one of the numbers contains a factor equal to the product of two or more factors of the other, all such factors may be cancelled.

3. If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, such factors may be cancelled.

Examples.

1. What is the quotient of 2×4×8×13×7×16, divided by 26×14×8?

2. What is the quotient of 42 × 3 × 25 × 12, divided by 28×4×15×6?

3. What is the quotient of 125×60×24×42, divided by 25×120×36×5?

4. How many times is 11×39×7×2 contained in 44×18×26×14?

5. What is the quotient of 8 times 240 multiplied by 5 times 114, divided by 24 times 57 multiplied by 6 times 15?

6. What is the value of (22+8+16) × (18+10+21) divided by (9+5+7) × (15+8)?

7. Divide (140 + 86 − 34) × (107 − 19) by (237 − 141) × (17 + 20 − 15)?

8. Divide [12×5−2×9] × (42+30) by (5×8) × (2×9) × (10+17)?

9. What is the quotient of 240×441×16 divided by 175×56×27?

10. What is the quotient of 64 times 840 multiplied by 9 times 124, divided by 32 times 560 multiplied by 4 times 31

11 How many dozen of eggs, worth 14 cents a dozen, must be given for 18 pounds of sugar, worth 7 cents a pound?

12. A dairyman sold 5 cheeses, each weighing 40 pounds, at 9 *cents a pound: how many* pounds of tea, worth 50 cents a *pound, must he receive for* the cheeses?

13. Bought 12 yards of cloth, at $1.84 a yard, and paid for it in potatoes at 48 cents a bushel: how many bushels of potatoes will pay for the cloth?

14. How many firkins of butter, each containing 56 pounds, at 25 cents a pound, will pay for 4 barrels of sugar, each weighing 175 pounds, at 8 cents a pound?

15. A man bought 10 cords of wood, at 20 shillings a cord and paid in labor at 12 shillings a day: how many days did he labor?

16. How many pieces of cloth, each containing 36 yards, at $3.50 a yard, must be given for 96 barrels of flour, at $10.50 a barrel?

17. A farmer exchanged 492 bushels of wheat, worth $1.84 a bushel, for an equal number of bushels of barley, at 87 cents a bushel; of corn, at 60 cents a bushel; and of oats, at 45 cents a bushel: how many bushels of each did he receive?

18. How many barrels of flour, worth $7 a barrel, must be given for 250 bushels of oats, at 42 cents a bushel?

19. If 48 acres of land produce 2484 bushels of corn, how many bushels will 120 acres produce?

20. A man worked 12 days, at 9 shillings a day, and received in payment wheat at 16 shillings a bushel: how many bushels did he receive?

21. A grocer sold 6 hams, each weighing 14 pounds, at 10 cents a pound, and received in payment apples, at 48 cents a bushel: how many bushels of apples did he receive?

22. How long will it take a man, travelling 36 miles a day, to go the same distance that another man travelled in 15 days, at the rate of 27 miles a day?

23. A man took four loads of apples to market, each load containing 12 barrels, and each barrel 3 bushels. He sold them at 45 cents a bushel, and received in payment a number of boxes of tea, each box containing 20 pounds, worth 72 cents a pound: how many boxes of tea did he receive?

LEAST COMMON MULTIPLE.

114. A **Multiple** of a number is any product of which the number is a factor; hence, any multiple of a number is exactly divisible by the number itself.

115. A **Common Multiple** of two or more numbers is any number which each will divide without a remainder.

116. **The Least Common Multiple** of two or more numbers is the *least* number which they will separately divide without a remainder.

117. Principles—Operations—and Rule.

1. Any divisible number, is divisible by any prime factor of the exact divisor.

2. If a number has several exact divisors, it will be divisible by all their prime factors.

3. Hence, the question of finding the least common multiple of several numbers is reduced to finding a number which shall contain all their prime factors, and *none others.*

1. What is the least common multiple of 6, 12, and 18?

OPERATION.

$$\begin{array}{r|rcrcr} 2 & 6 & .\,. & 12 & .\,. & 18 \\ \hline 3 & 3 & .\,. & 6 & .\,. & 9 \\ \hline & 1 & .\,. & 2 & .\,. & 3 \end{array}$$

$$2 \times 3 \times 2 \times 3 = 36$$

Analysis.—Having placed the given numbers in a line, if we divide by 2, we find the quotients 3, 6 and 9; hence, 2 is a prime factor of all the numbers. Dividing by 3, we find that 3 is a prime factor of the quotients 3, 6, and 9; and hence, the quotients 2 and 3 are prime factors of 12 and 18; therefore, the prime factors of all the numbers are 2, 3, 2 and 3; and their product, 36, is the *least common multiple.*

---

114. What is a multiple of a number?—115. What is a common multiple of two or more numbers?—116. What is the least common multiple of two or more numbers?

117. What is the first principle on which the operation for finding the least *common multiple* depends? What is the second? What is *the third? Give the rule* for finding the least common multiple.

Rule.

I. *Place the numbers on the same line, and divide by any* PRIME *number that will exactly divide two or more of them, and set down, in a line below, the quotients and the undivided numbers:*

II. *Then divide as before, until there is no prime number greater than* 1 *that will exactly divide any two of them:*

III. *Then multiply together the divisors and the numbers of the lower line, and their product will be the least common multiple.*

Note.—If the numbers have no common prime factor, their product will be their least common multiple.

Examples.

1. What is the least common multiple of 4, 9, 10, 15, 18, 20, 21?

2. What is the least common multiple of 8, 9, 10, 12, 25, 32, 75, 80?

3. What is the least common multiple of 1, 2, 3, 4, 5, 6, 7, 9?

4. What is the least common multiple of 9, 16, 42, 63, 21, 14, 72?

5. What is the least common multiple of 7, 15, 21, 28, 35, 100, 125?

6. What is the least common multiple of 15, 16, 18, 20, 24, 25, 27, 30?

7. What is the least common multiple of 9, 18, 27, 36, 45, 54?

8. What is the least common multiple of 4, 10, 14, 15, 21?

9. What is the least common multiple of 7, 14, 16, 21, 24?

10. What is the least common multiple of 49, 14, 84, 168, 98?

11. A can dig 9 rods of ditch in a day; B, 12 rods in a day; and C, 16 rods in a day: what is the smallest number of rods that would afford exact days of labor to each, working alone? In what time would each do the whole work?

12. A blacksmith employed 4 classes of workmen, at $15, 16, $21, and $24 per month, for each man respectively, paying o each class the same amount of wages. Required the least mount that will pay either class for 1 month; also, the number f men in each class?

13. A farmer has a number of bags containing 2 bushels each; of barrels, containing 3 bushels each; of boxes, containing 7 bushels each; and of hogsheads, containing 15 bushels each: what is the smallest quantity of wheat that would fill each an exact number of times, and *how many times* would that quantity fill each?

14. Four persons start from the same point to travel round a circuit of 300 miles in circumference. A goes 15 miles a day; B, 20 miles; C, 25 miles; and D, 30 miles a day. How many days must they travel before they will all come together again at the same point, and how many times will each have gone round?

NOTE.—First find the number of days that it will take each to travel round the circuit.

GREATEST COMMON DIVISOR.

118. A COMMON DIVISOR of two or more numbers, is any number that will divide each of them without a remainder; hence, it is always a *common factor* of the numbers.

119. THE GREATEST COMMON DIVISOR of two or more numbers, is the *greatest* number that will divide each of them without a remainder; hence, it is their greatest *common factor*.

120. Two numbers are said to be *prime to each other*, when they have no common divisor.

NOTE.—Since 1 will divide *every number*, it is not reckoned among the common divisors.

---

118. What is a common divisor?—119. What is the greatest common divisor?

120. *When are two numbers* said to be *prime* to each other?

121. To find the greatest common divisor of two or more numbers, when the numbers are small.

Since an exact divisor is a factor, the greatest common divisor of the given numbers will be their greatest common factor: hence,

I. *Resolve each number into its prime factors, and observe those which are common to all the numbers:*

II. *Multiply the common factors together, and their product will be the greatest common divisor.*

Examples.

1. What is the greatest common divisor of 12 and 20?

OPERATION.

$12 = 2 \times 2 \times 3$

$20 = 2 \times 2 \times 5$

Analysis.—There are three prime factors in 12; viz., 2, 2, and 3: there are three prime factors in 20; viz., 2, 2, and 5. The factors 2 and 2 are common; hence, 2 × 2 = 4 is the greatest common divisor.

2. What is the greatest common divisor of 18 and 36?

3. What is the greatest common divisor of 12, 24, and 60?

4. What is the greatest common divisor of 15, 50, and 40?

5. What is the greatest common divisor of 24, 18, and 144?

6. What is the greatest common divisor of 50, 100, and 80?

7. What is the greatest common divisor of 56, 84, and 140?

8. What is the greatest common divisor of 84, 154, and 210?

122. To find the greatest common divisor, when the numbers are large.

This method depends on the following principles:

ILLUSTRATION.

$30 - 8 = 22$

1. Any number which will exactly divide two numbers separately, will divide their difference; else, *we should have a whole number equal to a fraction, which is impossible.*

---

121. How do you find the greatest common divisor, when the numbers are small?—122. On what principle does finding the greatest common divisor depend? Give the rule.

2. Any number that will exactly divide the difference of two numbers, and one of them, will exactly divide the other: else, we should have a whole number equal to a fraction, which is impossible.

3. Any number which will exactly divide another, will divide any multiple of that other; because, the first dividend which is divisible is a factor of the multiple.

1. Let it be required to find the greatest common divisor of the numbers 216 and 408.

OPERATION.

```
216) 408 (1
     216
     ---
     192) 216 (1
          192
          ---
           24) 192 (8
               192
```

ANALYSIS.—The greatest common divisor cannot be greater than the least number, 216. Now, as 216 will divide itself, let us see if it will divide 408; for, if it will, it is the greatest common divisor. Making the division, we find a quotient 1, and a remainder, 192; hence, 216 is not a common divisor.

The greatest common divisor of 216 and 408 will divide the remainder 192; and if 192 will exactly divide 216, it will be the greatest common divisor. We find that 192 is contained in 216 once, and a remainder 24. The greatest common divisor of 192 and 216 will divide the remainder 24; and if 24 will exactly divide 192, it will also divide 216, and consequently 408; now, 24 exactly divides 192, and hence is the greatest common divisor sought.

Rule.

Divide the greater number by the less, and then divide the preceding divisor by the remainder, and so on, till nothing remains: the last divisor will be the greatest common divisor.

Principles from the Rule.

1. If the last remainder is 1, the numbers *are prime to each other.*

2. If, in the course of the operation, any one of the remainders is a *prime number,* and will not exactly divide the *preceding divisor,* it is certain that there is no common divisor.

3. To find the greatest common divisor of three or more numbers, find the greatest common divisor of two of them, and then the divisor of this *common divisor* and of the third number, and so on.

Examples.

1. What is the greatest common divisor of 3328 and 4592?

2. What is the greatest common divisor of 2205 and 4501?

3. What is the greatest number that will divide 16082 and 25740?

4. What is the greatest number that will divide 620, 1116 and 1488?

5. What is the greatest common divisor of 5270, 5952, 5394, and 3038?

6. What is the greatest common divisor of 4617, 7695, 6642, and 8424?

7. A farmer has 315 bushels of corn, and 810 bushels of wheat; he wishes to draw the corn and wheat to market separately in the fewest number of equal loads: how many bushels must he draw at a load?

8. The Illinois Central Railroad Company have 15750 acres of land in one location, and 21725 acres in another. They wish to divide the whole into lots of equal extent, containing the greatest number of acres that will give an exact division: how many acres will there be in each lot?

9. A man has a corner lot of land, 1044 feet long and 744 feet wide. The adjacent sides are bounded by the highway, and he wishes to build a board fence with the fewest panels of equal length: what must be the length of the panels?

10. A farmer has 231 bushels of barley, 369 bushels of oats, and 393 bushels of wheat, all of which he wishes to put into the smallest number of bags of equal size, without mixing: how many bushels must each bag contain?

11. Three persons, A, B, and C, agree to purchase a lot of 63 cows at the same price per head, provided each man can thus invest his whole money. A has $286; B, $462; and C, $638: *ow many cows could* each man purchase?

COMMON FRACTIONS

123. AN INTEGRAL, or *whole number*, is the unit 1, or a collection of such units.

NOTE.—All integral numbers are formed by the continual additions of 1: as, $1 + 1 = 2$, $2 + 1 = 3$, &c.

124. A UNIT is a single thing; as, an apple, a chair, a hat, &c.; and is denoted by 1.

If a unit be divided into two equal parts, each part is alled, *one-half.*

If a unit be divided into three equal parts, each part is alled, *one-third.*

If a unit be divided into four equal parts, each part is alled, *one-fourth.*

If a unit be divided into twelve equal parts, each part is alled, *one-twelfth;* and if it be divided into *any number* of qual parts, we have a like expression for each part.

The parts thus written:

$\frac{1}{2}$	is read,	one-half.	$\frac{1}{7}$	is read,	one-seventh.
$\frac{1}{3}$	. .	one-third.	$\frac{1}{8}$	. .	one-eighth.
$\frac{1}{4}$	. .	one-fourth.	$\frac{1}{10}$	. .	one-tenth.
$\frac{1}{5}$	. .	one-fifth.	$\frac{1}{15}$	. .	one-fifteenth.
$\frac{1}{6}$	. .	one-sixth.	$\frac{1}{50}$	. .	one-fiftieth.

The $\frac{1}{2}$, is an *entire half;* the $\frac{1}{3}$, an *entire third;* the $\frac{1}{4}$, an ntire *fourth;* and the same for each of the other equal parts; ence, *each equal part is an entire thing,* and is called a *fractional unit.*

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123. What is an integral, or whole number? How are integral umbers formed?

124. What is a unit? By what is it denoted? What is each part alled when the unit 1 is divided into two equal parts? When it is ivided into three? *Into four?* Into five? Into twelve?

125. THE UNIT OF A FRACTION is the single thing that is divided into equal parts.

126. THE FRACTIONAL UNIT is one of the equal parts of the unit that is divided.

127. A FRACTION is a fractional unit, or a collection of such units.

NOTE.—In every fraction, let the pupil distinguish carefully between the *unit of the fraction* and the *fractional unit*. The first is the *whole thing* from which the fraction is derived; the second, *one of the equal parts* into which that thing is divided.

128. Every whole number, except 1, has a fractional unit corresponding to it: thus the numbers,

2, 3, 4, 5, 6, 7, 8, 9, 10, &c.,

have, corresponding to them, the fractional units,

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, &c.

129. Expressing Fractions.

Each fractional unit may, like the unit 1, become the base of a collection: thus, suppose it were required to express 2 of each of the fractional units, we should then write

$\frac{2}{2}$ which is read 2 halves $= \frac{1}{2} \times 2$.
$\frac{2}{3}$ . . . 2 thirds $= \frac{1}{3} \times 2$.
$\frac{2}{4}$ . . . 2 fourths $= \frac{1}{4} \times 2$.
$\frac{2}{5}$ . . . 2 fifths $= \frac{1}{5} \times 2$.
&c., &c., &c.

If it were required to express 3 of each of the fractional units, we should write

$\frac{3}{2}$ which is read 3 halves $= \frac{1}{2} \times 3$.
$\frac{3}{3}$ . . . 3 thirds $= \frac{1}{3} \times 3$.
$\frac{3}{4}$ . . . 3 fourths $= \frac{1}{4} \times 3$.
$\frac{3}{5}$ . . . 3 fifths $= \frac{1}{5} \times 3$.
&c., &c., &c.

Hence, if we suppose a second unit to be divided into the same number of equal parts, such parts may be expressed in the same collection with the parts of the first: thus,

$\frac{3}{2}$ is read, 3 halves.
$\frac{7}{4}$ . . 7 fourths.
$\frac{16}{5}$ . . 16 fifths.
$\frac{18}{6}$ 18 sixths.
$\frac{25}{7}$ . . 25 sevenths.

A whole number may be expressed fractionally by writing 1 below it for a denominator. Thus,

3 may be written $\frac{3}{1}$ and is read, 3 ones.
5 . . . . . $\frac{5}{1}$ . . . . 5 ones.
6 . . . . . $\frac{6}{1}$ . . . . 6 ones.
8 . . . . . $\frac{8}{1}$ . . . . 8 ones.

But 3 ones are equal to 3, 5 ones to 5, 6 ones to 6, and 8 ones to 8; hence, *the value of a number is not changed* by placing 1 under it for a denominator: Hence, we see,

1. That Fractions are expressed by two numbers, one written above the other, with a line between them.

2. That every fraction may be divided into two factors, one of which is the fractional unit, and the other the number denoting how many times the fractional unit is taken.

130. The Denominator is the number written below the line. It shows into how many equal parts the unit of the fraction is divided.

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125. What is the unit of a fraction?—126. What is the fractiona unit?—127. What is a fraction?—128. What fractional unit correspond to the whole number 5? What to 6? What to 14?

129. May a fractional unit become the base of a collection? How are fractions expressed? Into how many factors may every fraction be divided? What are they?

130. What is the denominator of a fraction?

131. THE NUMERATOR is the number written above the line It shows how many fractional units are taken.

132. THE TERMS of a fraction are the numerator and denominator taken together: hence, every fraction has two terms.

133. THE VALUE of a fraction is the quotient of the numerator divided by the denominator.

134. THE ANALYSIS of a fraction is the naming of its unit—its fractional unit—and the number of fractional units taken.

135. Analysis of Fractions.

How is the fraction $\frac{7}{8}$ to be interpreted?

1. The unit of the fraction is 1.

2. The unit of the fraction is divided into 8 equal parts; hence, the fractional unit is one-eighth.

3. Seven fractional units are taken. In the fraction $\frac{7}{8}$, the base of the collection of fractional units is $\frac{1}{8}$, but it is not the *primary base*. For, $\frac{1}{8}$ is *one-eighth of the unit* 1; hence, the *primary base* of every fraction is the unit 1.

The expression may also be interpreted as the quotient of 7 divided by 8. In the latter case, the thing divided is the number 7; in the former, it was the number 1. The value in both cases is the same; for, seven times one-eighth of 1, is equal to one of the 8 equals of 7. Hence, *a fractional expression has the same form as an unexecuted division.*

136. Principles and Properties of Fractions.

1. A fraction is a fractional unit, or a collection of such units.

2. The denominator shows into how many equal parts the unit of the fraction is divided.

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131. What is the numerator of a fraction?—132. What are the terms of a fraction? How many terms has every fraction?—133. What is the value of a fraction?—134. What is the analysis of a fraction?

135. Analyze the fraction $\frac{7}{8}$. What is the base of the collection? What is the primary base? What else does $\frac{7}{8}$ express?

136. Explain the principles and properties of Fractions.

3. The numerator shows how many fractional units are taken.

4. The value of every fraction is equal to the numerator divided by the denominator.

5. When the numerator is less than the denominator, the value of the fraction is less than 1.

6. When the numerator is equal to the denominator, the value of the fraction is equal to 1.

7 When the numerator is greater than the denominator, the value of the fraction is greater than 1.

137. Writing and Reading Fractions.

1. Read and analyze the following fractions:

$$\frac{8}{9},\quad \frac{7}{12},\quad \frac{5}{3},\quad \frac{6}{15},\quad \frac{21}{9},\quad \frac{16}{7},\quad \frac{18}{104}.$$

2. Write 15 of the 19 equal parts of 1. Also, 37 of the 49 equal parts of 1. Write 24-thirtieths.

3. If the unit of the fraction is 1, and the fractional unit one-fortieth, express 27 fractional units. Also, 95. Also, 106 Also, 87. Also, 41.

4. If the unit of the fraction is 1, and the fractional unit one-68th, express 45 fractional units. Also, 56. Also, 85. Also, 95. Also, 37.

5. If the unit of the fraction is 1, and the fractional unit one-90th, express 9 fractional units. Also, 87. Also, 75. Also, 65

138. Six Kinds of Fractions.

1. A Proper Fraction is one whose numerator is less than the denominator.

The following are proper fractions:

$$\frac{1}{2},\quad \frac{1}{3},\quad \frac{1}{4},\quad \frac{3}{4},\quad \frac{3}{7},\quad \frac{5}{8},\quad \frac{9}{10},\quad \frac{8}{9},\quad \frac{5}{6}.$$

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137. Give an example in writing, reading, and analyzing fractions

138. How many kinds of fractions are there? Name and describe each.

2. An **Improper Fraction** is one whose numerator is equal to, or exceeds the denominator. The following are improper fractions:

$\frac{3}{2}$, $\frac{5}{3}$, $\frac{6}{5}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{12}{6}$, $\frac{14}{7}$, $\frac{19}{7}$.

Note.—Such a fraction is called *improper*, because its value equals or exceeds 1.

3. A **Simple Fraction** is one whose numerator and denominator are both whole numbers. The following are simple fractions:

$\frac{1}{4}$, $\frac{3}{2}$, $\frac{5}{6}$, $\frac{8}{7}$, $\frac{9}{2}$, $\frac{8}{3}$, $\frac{6}{5}$, $\frac{7}{5}$.

Note.—A simple fraction may be either proper or improper.

4. A **Compound Fraction** is a fraction of a fraction, or several fractions connected by the word *of*. The following are compound fractions:

$\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{6}$ of 3, $\frac{1}{7}$ of $\frac{1}{8}$ of 4.

5. A **Mixed Number** is made up of a whole number and a fraction. The following are mixed numbers:

$3\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{2}{8}$, $5\frac{3}{5}$, $6\frac{4}{8}$, $3\frac{1}{7}$.

6. A **Complex Fraction** is one which has a fraction in one or both of its terms. The following are complex fractions:

$$\frac{(\frac{1}{7})}{5} \qquad \frac{2}{19\frac{1}{3}} \qquad \frac{(\frac{2}{3})}{(\frac{4}{5})} \qquad \frac{45\frac{1}{8}}{69\frac{1}{7}}.$$

FUNDAMENTAL PRINCIPLES.

139. **Let it be required to multiply $\frac{3}{8}$ by 4.**

OPERATION.

$$\tfrac{3}{8} \times 4 = \tfrac{3 \times 4}{8} = \tfrac{12}{8}.$$

Analysis.—In $\frac{3}{8}$ there are 3 fractional units each of which is $\frac{1}{8}$, and these are to be taken 4 times. But three things taken 4 times, give 12 things of the *same kind;* that is, 12 eighths; hence the product is 4 times as great as the multiplicand; therefore,

Proposition I.—*If the numerator of a fraction be multiplied by any number, the value of the fraction will be multiplied as many times as there are units in the multiplier.*

Examples.

1. Multiply $\frac{3}{8}$ by 6, by 7.
2. Multiply $\frac{7}{8}$ by 4, by 9.
3. Multiply $\frac{5}{31}$ by 11, by 12.
4. Multiply $\frac{7}{36}$ by 12, by 14.
5. Multiply $\frac{47}{15}$ by 3, by 4.
6. Multiply $\frac{14}{19}$ by 7, by 9.
7. Multiply $\frac{47}{28}$ by 5, by 10.
8. Multiply $\frac{27}{29}$ by 3, by 11.

140. Let it be required to multiply $\frac{5}{12}$ by 4.

OPERATION.

$$\frac{5}{12} \times 4 = \frac{5}{12 \div 4} = \frac{5}{3}.$$

Analysis.—In $\frac{5}{12}$ there are 5 fractional units, each of which is $\frac{1}{12}$. If we divide the denominator by 4, the quotient is 3, and the fractional unit becomes $\frac{1}{3}$, which is 4 *times as great as* $\frac{1}{12}$; because, if $\frac{1}{3}$ be divided into 4 equal parts, each part will be $\frac{1}{12}$. If we take this fractional unit 5 times, the result, $\frac{5}{3}$, will be 4 *times as great as* $\frac{5}{12}$; therefore,

Proposition II.—*If the denominator of a fraction be divided by any number, the value of the fraction will be multiplied as many times as there are units in the divisor.*

Hence, to multiply a fraction by any number, *divide its denominator.*

Examples.

1. Multiply $\frac{17}{16}$ by 8, by 4, by 2.
2. Multiply $\frac{9}{24}$ by 2, 3, 4, 6, 8.
3. Multiply $\frac{7}{30}$ by 6, 5, 10, 15.
4. Multiply $\frac{17}{48}$ by 2, 3, 4, 6, 8.
5. Multiply $\frac{6}{40}$ by 4, 5, 10, 20.
6. Multiply $\frac{7}{35}$ by 7, by 5.
7. Multiply $\frac{6}{42}$ by 21, 6, 7, 3, 2.
8. Multiply $\frac{19}{36}$ by 3, 4, 6, 9, 12.

141. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9 \div 3}{11} = \frac{3}{11}.$$

Analysis.—In $\frac{9}{11}$ there are 9 fractional units, each of which is $\frac{1}{11}$, and these are to be divided by 3. But 9 things, divided

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139. What is proved in Proposition I.?—140. What is proved in Proposition II.?

by 3, gives 3 *things of the same kind* for a quotient; hence, the quotient is 3 elevenths, a number which is one-third of $\frac{9}{11}$; hence,

PROPOSITION III.—*If the numerator of a fraction be divided by any number, the value of the fraction will be divided into as many equal parts as there are units in the divisor*

Examples.

1. Divide $\frac{16}{19}$ by 2, 4, 8, 16.	5. Divide $\frac{18}{19}$ by 2, 3, 6, and 9
2. Divide $\frac{14}{11}$ by 2, 7, and 14.	6. Divide $\frac{24}{25}$ by 3, 6, 8, 12.
3. Divide $\frac{20}{19}$ by 2, 5, 4, and 10.	7. Divide $\frac{27}{29}$ by 3, 9, and 27.
4. Divide $\frac{60}{26}$ by 5, 6, 10, 15, 20.	8. Divide $\frac{54}{59}$ by 6, 9, 27, 54.

142. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9}{11 \times 3} = \frac{9}{33}.$$

ANALYSIS.—In $\frac{9}{11}$ there are 9 fractional units, each of which is $\frac{1}{11}$. Now, if we multiply the denominator by 3, it becomes 33, and the fractional unit becomes $\frac{1}{33}$, which is one-third part of $\frac{1}{11}$. If, then, we take this fractional unit 9 times, the result, $\frac{9}{33}$, is just one-third part of $\frac{9}{11}$; hence, we have divided the fraction $\frac{9}{11}$ by 3: therefore, we have

PROPOSITION IV.—*If the denominator of a fraction be multiplied by any number, the value of the fraction will be divided into as many equal parts as there are units in the multiplier.*

Hence, to divide a fraction, *multiply the denominator.*

Examples.

1. Divide $\frac{3}{4}$ by 6, 7, and 8.	5. Divide $\frac{15}{17}$ by 7, 5, and 3
2. Divide $\frac{4}{9}$ by 5, 4, and 9.	6. Divide $\frac{14}{27}$ by 7, 8, and 6.
3. Divide $\frac{14}{17}$ by 3, 4, and 12.	7. Divide $\frac{25}{19}$ by 3, 7, and 11.
4. Divide $\frac{30}{47}$ by 6, 8, and 11.	8. Divide $\frac{11}{15}$ by 8, 4, and 10

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141. What is proved in Proposition III.?—142. What is proved in Proposition IV.?

143. Multiply both terms of the fraction $\frac{3}{5}$ by 4.

OPERATION.

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

ANALYSIS.—In $\frac{3}{5}$, the fractional unit is $\frac{1}{5}$, and it is taken 3 times. By multiplying the denominator by 4, the fractional unit becomes $\frac{1}{20}$, the value of which is one-fourth of $\frac{1}{5}$. By multiplying the numerator by 4, we increase the *number* of fractional units taken, 4 times; that is, we *increase the number of parts taken just as many times* as we *diminish the value of each part;* hence the value of the fraction is not changed: therefore,

PROPOSITION V.—*If both terms of a fraction be multiplied by the same number, the value of the fraction will not be changed.*

Examples.

1. Multiply both terms of the fraction $\frac{7}{8}$ by 4, by 6, and by 5
2. Multiply both terms of $\frac{8}{11}$ by 5, by 8, by 9, and 11.
3. Multiply both terms of $\frac{16}{19}$ by 7, by 8, and 9.
4. Multiply both terms of $\frac{14}{29}$ by 5, 8, 6, and 12.
5. Multiply both terms of $\frac{23}{25}$ by 2, 3, 4, and 5

144. Divide both terms of the fraction $\frac{6}{15}$ by 3.

OPERATION.

$$\frac{6 \div 3}{15 \div 3} = \frac{2}{5}.$$

ANALYSIS.—In $\frac{6}{15}$, the fractional unit is $\frac{1}{15}$, and it is taken 6 times. By dividing the denominator by 3, the fractional unit becomes $\frac{1}{5}$, the value of which is 3 *times as great as* $\frac{1}{15}$. By dividing the numerator by 3, we diminish the number of fractional units taken 3 times; that is, we *diminish the number of parts taken just as many times as we increase the value of the fractional unit:* hence, the value of the fraction is not changed; therefore,

PROPOSITION VI.—*If both terms of a fraction be divided by the the same number, the value of the fraction will not be changed.*

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143. What is *proved in* Proposition V.?—**144.** What is proved in *Proposition VI.?*

Examples.

1. Divide both terms of $\frac{4}{8}$ by 2 and by 4.
2. Divide both terms of $\frac{3}{6}$ by 3.
3. Divide both terms of $\frac{24}{36}$ by 2, 3, 4, 6, and 12.
4. Divide both terms of $\frac{48}{64}$ by 2, 4, 8, and 16.
5. Divide both terms of $\frac{72}{96}$ by 2, 3, 4, 6, and 12.
6. Divide both terms of $\frac{36}{144}$ by 2, 3, 4, 6, and 36.

REDUCTION OF FRACTIONS.

145. **Reduction of Fractions** is the operation of changing a fractional number from one unit to another without altering its value.

146. **The lowest terms** of a fraction are when the numerator and denominator are prime to each other.

CASE I.

147. To reduce a whole number to a fraction having a given denominator.

1. Reduce 17 to a fraction whose denominator shall be 5.

Analysis.—To reduce 17 to such a fraction is the same as to reduce 17 to fifths. In 17 there are 17 times as many fifths as there are in 1. In 1 there are 5 fifths; therefore, in 17 there are 17 times 5 fifths, or, 85 fifths; hence,

Operation.

$17 \times 5 = 85$

$17 = \frac{85}{5}$

Rule.—*Multiply the whole number by the denominator, and write the product over the required denominator.*

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145 What is reduction of fractions?

146. What are the lowest terms of a fraction?

147. How do you reduce a whole number to a fraction having a given denominator?

Examples.

1. Change 18 to a fraction whose denominator shall be 7.
2. Change 25 to a fraction whose denominator shall be 12
3. Change 19 to a fraction whose denominator shall be 8.
4. Change 29 to a fraction whose denominator shall be 14
5. Change 65 to a fraction whose denominator shall be 37
6. Reduce 145 to a fraction having 9 for its denominator
7. Reduce 450 to twelfths.
8. Reduce 327 to a fraction having 36 for its denominator
9. Reduce 97 to a fraction having 128 for its denominator.
10. Reduce 167 to eighty-ninths.
11. Reduce 325 to a fraction whose denominator shall be 75

CASE II.

148. **To reduce a mixed number to an equivalent improper fraction.**

1. Reduce $12\frac{5}{7}$ to its equivalent improper fraction.

Analysis.—Since in any number there are 7 times as many 7ths as units 1, there will be 84 sevenths in 12: To these add 5 sevenths, and the equivalent fraction becomes 89 sevenths. Hence,

OPERATION.

$12 \times 7 = 84$ sevenths.
add 5 sevenths.
gives $12\frac{5}{7} = 89$ sevenths
Ans. $= \frac{89}{7}$.

Rule.—*Multiply the whole number by the denominator: to the product add the numerator, and place the sum over the given denominator.*

Examples.

1. Reduce $39\frac{7}{8}$ to its equivalent improper fraction.
2. Reduce $112\frac{9}{10}$ to its equivalent improper fraction.

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148. How do you reduce a mixed number to an equivalent improper fraction?

3. Reduce $427\frac{11}{24}$ to its equivalent improper fraction.
4. Reduce $676\frac{37}{51}$ to an improper fraction.
5. Reduce $367\frac{9}{164}$ to an improper fraction.
6. Reduce $847\frac{36}{175}$ to an improper fraction.
7. Reduce $67426\frac{368}{879}$ to an improper fraction.
8. How many 200ths in $675\frac{187}{200}$?
9 How many 151ths in $187\frac{41}{151}$?
10 Reduce $149\frac{5}{9}$ to an improper fraction.
11. Reduce $375\frac{94}{99}$ to an improper fraction.
12. Reduce $17494\frac{543}{99999}$ to an improper fraction.
13. Reduce $4834\frac{57}{95}$ to an improper fraction.
14. Reduce $1789\frac{5}{9}$ to an improper fraction.
15. In $125\frac{6}{7}$ yards, how many sevenths of a yard?
16. In $375\frac{3}{4}$ feet, how many fourths of a foot?
17. In $464\frac{19}{63}$ hogsheads, how many sixty-thirds.
18. In $96\frac{11}{640}$ acres, how many 640ths of an acre?
19. In $984\frac{41}{112}$ pounds, how many 112ths of a pound?
20. In $35\frac{72}{366}$ years, how many 366ths of a year?

21. How many one hundred and thirty-fifths are there in the mixed number $87\frac{41}{135}$?

22. Place 4 sevens in such a manner that they shall express the number 78.

23. By means of 5 threes write a number that is equal to 334.

CASE III

149. To reduce an improper fraction to an equivalent whole or mixed number.

1. In $\frac{278}{5}$ how many entire units?

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149. How do you reduce an improper fraction to an equivalent mixed number?

Analysis.—Since there are 5 fifths in 1 unit, there will be, in 278 fifths, as many units 1 as 5 is contained times in 278, viz., 55 and $\frac{3}{5}$ times. Hence, the following

OPERATION.

$$5)\underline{278}$$
$$55\tfrac{3}{5}.$$

Rule.—*Divide the numerator by the denominator, and the quotient will be the equivalent whole or mixed number.*

Examples.

Reduce the following fractions to whole, or mixed numbers.

1. Reduce $\frac{108}{63}$.
2. Reduce $\frac{576}{48}$.
3. Reduce $\frac{1764}{324}$.
4. Reduce $\frac{19900}{800}$.
5. Reduce $\frac{135}{15}$ pounds.
6. Reduce $\frac{2356}{42}$ days.
7. Reduce $\frac{6384}{56}$ yards.
8. Reduce $\frac{4976}{224}$.
9. Reduce $\frac{102402}{160}$ acres.
10. Reduce $\frac{4478}{841}$.
11. Reduce $\frac{17959}{1256}$.
12. Reduce $\frac{526950}{2342}$.
13. Reduce $\frac{4790}{25}$.
14. Reduce $\frac{1512}{108}$.
15. Reduce $\frac{375941}{999}$.
16. Reduce $\frac{3745174}{349}$.

CASE IV.

150. **To reduce a fraction to its lowest terms.**

1. Reduce $\frac{70}{175}$ to its lowest terms.

Analysis.—By inspection, it is seen that 5 is a common factor of the numerator and denominator. Dividing by it we have $\frac{14}{35}$. We then see that 7 is a common factor of 14 and 35: dividing by it, we have $\frac{2}{5}$. Now, 2 and 5 are prime to each other; therefore, the fraction $\frac{2}{5}$ is in its *lowest terms.*

1ST OPERATION.

$$5)\tfrac{70}{175} = \tfrac{14}{35}.$$
$$7)\tfrac{14}{35} = \tfrac{2}{5}.$$

2d. The greatest common divisor of 70 and 175, is 35 (Art. 119). If we divide both terms of the fraction by it, we obtain $\frac{2}{5}$. The value of the fraction is not changed in either operation, since the numerator and denominator are both divided by the same number (Art. 144): hence,

2D OPERATION.

$$35)\tfrac{70}{175} = \tfrac{2}{5}$$

Rule.

I. *Divide the numerator and denominator, successively, by all their common factors:* Or,

II. *Divide the numerator and denominator by their greatest common divisor.*

Examples.

Reduce the following fractions to their lowest terms:

1. Reduce $\frac{7}{49}$.
2. Reduce $\frac{84}{420}$.
3. Reduce $\frac{104}{312}$.
4. Reduce $\frac{1049}{8392}$.
5. Reduce $\frac{275}{440}$.
6. Reduce $\frac{351}{795}$.
7. Reduce $\frac{172}{1118}$.
8. Reduce $\frac{63}{81}$ by 2d method.
9. Reduce $\frac{315}{405}$ " "
10. Reduce $\frac{1157}{623}$ " "
11. Reduce $\frac{792}{1386}$ " "
12. Reduce $\frac{374}{1030}$.
13. Reduce $\frac{410}{510}$.
14. Reduce $\frac{345}{1745}$.
15. Reduce $\frac{8343}{9747}$.
16. Reduce $\frac{549}{7143}$.
17. Reduce $\frac{2160}{2340}$.
18. Reduce $\frac{315}{1512}$.
19. Reduce $\frac{10560}{35520}$.
20. Reduce $\frac{6048}{38592}$.
21. Reduce $\frac{864}{21600}$.
22. Reduce $\frac{1080}{46420}$.

CASE V.

151. **To reduce a compound fraction to a simple fraction.**

1. What is the equivalent fraction of $\frac{3}{5}$ of $\frac{4}{7}$?

Analysis.—Three-fifths of $\frac{4}{7}$ is three times $\frac{1}{5}$ of $\frac{4}{7}$: 1 fifth of $\frac{4}{7}$ is $\frac{4}{35}$ (Art. 142); and 3 times $\frac{4}{35}$ is $\frac{12}{35}$ (Art. 139); hence, $\frac{3}{5}$ of $\frac{4}{7} = \frac{12}{35}$; hence,

OPERATION.

$$\frac{3 \times 4}{5 \times 7} = \frac{12}{35}$$

Rule.

I. *If there are mixed numbers, reduce them to improper fractions:*

II. *When there are common factors in the numerators and denominators, cancel them:*

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150. How do you reduce a fraction to its lowest terms? **151.** How do you reduce a compound fraction to a simple one?

III. *Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

Examples.

1. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{2}{3}$ to a simple fraction.
2. Reduce $\frac{2}{5}$ of $\frac{7}{9}$ of $\frac{3}{4}$ to a simple fraction.
3 Reduce $\frac{2}{3}$ of $\frac{3}{7}$ of $2\frac{1}{4}$ to a simple fraction.
4 Change $\frac{2}{9}$ of $\frac{3}{5}$ of $\frac{5}{8}$ of $3\frac{1}{5}$ to a simple fraction.
5. Change $\frac{9}{10}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{5}{14}$ to a simple fraction.
6. What is the value of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$?
7. What is the value of $\frac{2}{7}$ of $\frac{5}{6}$ of $4\frac{1}{5}$?
8. What is the value of $\frac{9}{10}$ of $7\frac{1}{3}$ of $5\frac{5}{12}$?
9. Reduce $\frac{7}{8}$ of $9\frac{1}{3}$ of $6\frac{3}{7}$ of $2\frac{4}{5}$ to a whole or mixed number
10. Reduce $\frac{9}{14}$ of $\frac{7}{12}$ of $21\frac{7}{9}$ to a whole or mixed number.
11. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{27}{100}$ of $\frac{5}{13}$ to a simple fraction.
12. Reduce $\frac{41}{110}$ of $\frac{3}{19}$ of $\frac{57}{108}$ of $\frac{3}{7}$ to a simple fraction.
13. Reduce $3\frac{5}{8}$ of $\frac{5}{7}$ of $\frac{32}{301}$ of 49 to a simple fraction.

CASE VI.

152. **To reduce fractions of different denominators to equivalent fractions that shall have a common denominator.**

1. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{4}$ to a common denominator.

Analysis.—Multiplying both terms of the first fraction by 20, the product of the other denominators, gives $\frac{40}{60}$ Multiplying both terms of the second fraction by 12, the product of the other denominators, gives $\frac{48}{60}$. Multiplying both terms of the third by 15, the product the other denominators, gives $\frac{45}{60}$. In each case, both terms of e fraction have been multiplied by the same number; therefore, e value is not changed (Art. 143): hence,

OPERATION.

$2 \times 5 \times 4 = 40$ 1st num.
$4 \times 3 \times 4 = 48$ 2d num.
$3 \times 5 \times 3 = 45$ 3d num.
$3 \times 5 \times 4 = 60$ denom.

Rule.—I. *Reduce mixed numbers to improper fractions, d compound to simple fractions, when necessary;*

II. ***Multiply the numerator of each fraction by all the denominators except its own, for the new numerators, and all the denominators together for a common denominator.***

Note—When the numbers are small, the work may be performed mentally; thus,

$\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{5}$ become, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{24}{50}$; and $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{4}$ become, $\frac{16}{40}$, $\frac{20}{40}$, $\frac{30}{40}$

Examples.

Reduce the following fractions to common denominators:

1. Reduce $\frac{3}{4}$, $5\frac{1}{3}$, and $\frac{6}{7}$.
2. Reduce $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{7}$, and $\frac{1}{2}$ of 5.
3. Reduce $9\frac{1}{2}$, $4\frac{1}{3}$, $2\frac{3}{4}$, and $\frac{4}{5}$.
4. Reduce $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{1}{2}$, and $2\frac{1}{4}$.
5. Reduce $2\frac{1}{2}$ of 3, $\frac{6}{7}$, $\frac{4}{9}$, and $\frac{3}{5}$.
6. Reduce $2\frac{1}{2}$ of $3\frac{1}{7}$, and $4\frac{6}{9}$.
7. Reduce $\frac{3}{7}$ of $\frac{2}{3}$, and $6\frac{4}{5}$.
8. Reduce $4\frac{8}{9}$, $2\frac{1}{3}$, $5\frac{1}{2}$, and 6
9. Reduce $5\frac{1}{5}$, $\frac{6}{5}$, $3\frac{1}{2}$, and $3\frac{2}{3}$.
10. Reduce $\frac{3}{4}$ of $5\frac{1}{3}$, and $4\frac{4}{5}$.
11. Reduce $4\frac{1}{3}$ of $3\frac{1}{2}$, and $7\frac{5}{7}$.
12. Reduce $6\frac{1}{3}$ of 2, $\frac{3}{7}$, $5\frac{8}{7}$, and $\frac{1}{3}$.

Note.—We may often shorten the work of multiplying the numerator and denominator of each fraction by such a number as will make the denominators the same in all.

Reduce the following fractions to common denominators.

1. Reduce $\frac{3}{4}$, $\frac{7}{12}$, $\frac{1}{2}$, and $\frac{5}{6}$ to a common denominator.
2. Reduce $\frac{6}{7}$, $\frac{8}{21}$, and $\frac{2}{3}$ to a common denominator.
3. Reduce $4\frac{1}{5}$, $\frac{9}{10}$, and $7\frac{1}{4}$ to a common denominator.
4. Reduce $10\frac{5}{9}$, $\frac{5}{6}$, and $7\frac{1}{3}$ to a common denominator.
5. Reduce $6\frac{1}{5}$, $\frac{5}{6}$, and $7\frac{1}{3}$ to a common denominator.
6. Reduce $\frac{4}{5}$, $\frac{7}{8}$, $14\frac{1}{2}$, and $3\frac{3}{4}$ to a common denominator.
7. Reduce $\frac{7}{12}$, $\frac{8}{9}$, $2\frac{5}{6}$, and $1\frac{3}{8}$ to a common denominator.
8. Reduce $\frac{6}{7}$, $\frac{1}{6}$, $\frac{16}{21}$, and $\frac{2}{3}$ to a common denominator.
9. Reduce $\frac{9}{11}$, $\frac{3}{4}$, $\frac{19}{22}$, and $\frac{1}{2}$ to a common denominator.
10. Reduce $2\frac{1}{2}$, $5\frac{1}{6}$, $\frac{9}{10}$, and $4\frac{5}{12}$ to a common denominator.

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152. How do you reduce fractions to a common denominator?

CASE VII.

153. To reduce fractions to their least common denominator.

The least common denominator is the least common multiple of the denominators.

1. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{4}{9}$, to their least common denominator.

ANALYSIS.—If there are mixed numbers or compound fractions, they must be reduced. We then find the least common multiple of the denominators 4, 6, and 9, which is 36. This number is divided by each denominator, to ascertain by what the terms of the fraction must be multiplied to reduce it to 36ths.

OPERATION.

2)4 .. 6 .. 9
3)2 .. 3 .. 9
2 .. 1 .. 3

LEAST COMMON DENOMINATOR.

$2 \times 3 \times 2 \times 3 = 36.$

$(36 \div 4) \times 3 = 27$ 1st numerator.
$(36 \div 6) \times 5 = 30$ 2d numerator.
$(36 \div 9) \times 4 = 16$ 3d numerator.

Therefore, the fractions, reduced to their least common denominator, are

$\frac{27}{36}$, $\frac{30}{36}$, and $\frac{16}{36}$.

Rule.

I. *Find the least common multiple of the denominators: this will be the least common denominator of the fractions:*

II. *Divide the least common denominator by the denominator of each fraction, separately; multiply the quotient by the numerator and place the product over the least common denominator; the results will be the new and equivalent fractions.*

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153. How do *you reduce fractions* to their least common denominator?

Examples.

1. Reduce $\frac{3}{8}$, $\frac{4}{7}$, and $\frac{5}{12}$ to their least common denominator.
2. Reduce $\frac{5}{14}$, $\frac{3}{7}$, and $\frac{16}{21}$ to their least common denominator.
3. Reduce $2\frac{3}{4}$, $\frac{5}{16}$, and $\frac{9}{32}$ to their least common denominator
4. Reduce $5\frac{3}{8}$, $4\frac{5}{12}$, and $\frac{7}{24}$ to their least common denominator
5. Reduce $8\frac{7}{15}$, $\frac{2}{5}$, and $\frac{7}{30}$ to their least common denominator
6. Reduce $9\frac{8}{11}$, $\frac{3}{22}$, and $\frac{5}{33}$ to their least common denominator
7. Reduce $2\frac{1}{2}$, $3\frac{5}{21}$, and $\frac{1}{14}$ to their least common denominator
8. Reduce $3\frac{5}{12}$, $\frac{7}{6}$, $\frac{3}{8}$, and $\frac{9}{16}$ to their least common denominator
9. Reduce $\frac{8}{9}$, $\frac{5}{27}$, and $\frac{7}{36}$ to their least common denominator.
10. Reduce $4\frac{6}{13}$, $7\frac{3}{26}$, and $\frac{5}{39}$ to their least common denominator
11. Reduce $6\frac{2}{5}$, $8\frac{7}{10}$, and $2\frac{9}{20}$ to their least common denominator
12. Reduce $\frac{9}{17}$, $2\frac{3}{34}$, and $1\frac{5}{68}$ to their least common denominator
13. Reduce $5\frac{7}{9}$, $6\frac{5}{18}$, $\frac{7}{36}$, and $\frac{1}{72}$ to their least denominator.

DENOMINATE FRACTIONS.

154. A DENOMINATE FRACTION is one whose unit is denominate. Thus, $\frac{5}{7}$ of a yard is a denominate fraction.

CASE VIII.

155. To change a denominate fraction from a greater unit to a less.

1. In $\frac{7}{8}$ of a yard, how many inches?

OPERATION.

$$\frac{7}{\underset{2}{\cancel{8}}} \times \frac{3}{1} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{63}{2}$$
$$= 31\tfrac{1}{2}.$$

ANALYSIS.—Since 3 feet make a yard, $\frac{7}{8}$ yd. = $\frac{7}{8}$ of $\frac{3}{1}$ feet; and since 12 inches make one foot, $\frac{7}{8}$ yard = $\frac{7}{8}$ of $\frac{3}{1}$ of $\frac{12}{1}$ inches = $\frac{63}{2}$ = $31\frac{1}{2}$ inches.

Rule.

Multiply the fraction by the units of the scale, in succession, till you reach the required unit.

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154. What is a denominate fraction?—**155.** How do you change a denominate fraction from a greater to a less unit?

CASE IX.

156. To change a denominate fraction from a less unit to a greater.

1. Reduce $\frac{4}{9}$ of a pound to a fraction of a ton.

ANALYSIS.—Since one pound is $\frac{1}{25}$ of a quarter, $\frac{4}{9}$ lb. $=\frac{4}{9}$ of $\frac{1}{25}$ qr.; and since one quarter is $\frac{1}{4}$ of a cwt., $\frac{4}{9}$ lb. $=\frac{4}{9}$ of $\frac{1}{25}$ of $\frac{1}{4}$ cwt.; and since one cwt. is $\frac{1}{20}$ of a ton,

$$\frac{4}{9}\text{ lb.} = \frac{\cancel{4}}{9} \text{ of } \frac{1}{25} \text{ of } \frac{1}{4} \text{ of } \frac{1}{\underset{5}{\cancel{20}}} = \frac{1}{4500}\text{ ton.}$$

Rule.—*Divide the fraction, that is, multiply the denominator by the units of the scale, in succession, till the required unit is reached.*

CASE X.

157. To find the value of a denominate fraction in integers of lower denominations.

1. What is the value of $\frac{7}{9}$ lb. Troy?

ANALYSIS.—$\frac{7}{9}$ lb. $=\frac{7}{9}$ of $\frac{12}{1}=\frac{84}{9}=$ $9\frac{3}{9}$ oz.: $\frac{3}{9}$ oz. $=\frac{3}{9}$ of $\frac{20}{1}=\frac{60}{9}=6\frac{6}{9}$ pwt.: $\frac{6}{9}$ pwt. $=\frac{6}{9}$ of $\frac{24}{1}=\frac{144}{9}=16$ gr.: hence,

OPERATION.

```
       7
      12
    9)84
oz.  9 . . 3
          20
        9)60
   pwt. 6 . . 6
             24
          9)144
         gr. 16
```

Ans. 9 oz., 6 pwt., 16 gr

Rule.—*Multiply the numerator of the fraction by the units of the scale, and divide the product by the denominator; if there is a remainder, treat it in the same way, till the required denomination is reached. The quotients of the several operations will form the answer.*

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156 How do you change a denominate fraction from a less to a greater unit?—*157. How do you find the value of a denominate fraction in integers of lower denominations.*

Examples.

1. Reduce £$\frac{3}{4}$ to the fraction of a farthing.
2 Reduce ton to the fraction of a pound.
3. Reduce week to the fraction of a minute
4 Reduce $\frac{9}{16}$ lb. Troy to the fraction of a grain.
5 Reduce $\frac{2}{3}$ inch to the fraction of a rod.
6 Reduce $\frac{4}{5}$ inch to the fraction of a yard.
7 Reduce $\frac{11}{20}$ of a second to the fraction of a degree.
8. Reduce $\frac{15}{26}$ of a cubic foot to the fraction of a cord.
9. What is the value of £$\frac{7}{16}$? of £$\frac{7}{15}$?
10. Find the value of $\frac{7}{9}$ mile: the value of $\frac{5}{7}$ mile.
11. What is the value of $\frac{5}{6}$ furlong?
12. Reduce $\frac{3}{4}$ penny to the fraction of a guinea.
13. Reduce $\frac{7}{9}$ farthing to the fraction of 6 guineas.
14. Reduce $\frac{9}{11}$ hour to the fraction of 5 seconds.

CASE XI.

158. To reduce a compound denominate number to a fraction of a given denomination.

1. Reduce 3 oz. 14 pwt. 15 gr. to the fraction of a pound.

ANALYSIS.—3 oz. 14 pwt. 15 gr. = 1791 gr. In 1 lb. there are 5760 gr.; therefore 3 oz. 14 pwt. 15 gr. is $\frac{1791}{5760}$ lb.

Rule.—*Reduce the compound number and the unit of the given denomination to the lowest unit named in either, and then divide the first result by the second.*

Examples.

1. Change 7 fur. 28 rd. 2 yd. to the fraction of a mile.
2. Reduce 17s. 6d. 2 far. to the fraction of a £.
3. Reduce 19 cwt. 3 qr. 16 lb. to the fraction of a ton.

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158. How do you reduce a compound number to a fraction of a *given denomination?*

4. Reduce 9 oz. $5\frac{2}{5}$ pwt. to the fraction of 1 lb. Troy.
5. Reduce 5 da. 16 hr. 40 m. to the fraction of a week.
6. Change 3 pk. 7 qt. 1 pt. to the fraction of a bushel.
7. Change 3 qr. 3 na. 1 inch to the fraction of 1 yard.
8. Change 18s. 8d. 3 far. to the fraction of £1 9s. 6d.
9. Change $\frac{7}{8}$s. to the fraction of £$\frac{3}{5}$.
10. Change $4\frac{7}{9}$d. to the fraction of £$\frac{4}{9}$.

ADDITION.

159. **Addition of Fractions** is the operation of finding the sum of two or more fractions.

160. **The sum** of two or more fractions is a number which contains the same fractional unit as many times as it is contained in all the fractions taken together.

CASE I.

161. When the fractions have the same unit.

1. What is the sum of $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$, and $\frac{3}{2}$?

OPERATION.

$$1 + 3 + 6 + 3 = 13$$

hence, $\frac{13}{2} = 6\frac{1}{2}$ sum.

Analysis.—In this example the unit of the fraction is 1, and the fractional unit $\frac{1}{2}$. There is 1 half in the first, 3 halves in the second, 6 in the third, and 8 in the fourth; hence, there are 13 halves in all, equal to $6\frac{1}{2}$.

2. What is the sum of £$\frac{1}{2}$ and £$\frac{2}{3}$?

OPERATION.

$$£\tfrac{1}{2} = £\tfrac{3}{6}$$
$$£\tfrac{2}{3} = £\tfrac{4}{6}$$
$$£\tfrac{3}{6} + £\tfrac{4}{6} = £\tfrac{7}{6} = £1\tfrac{1}{6}.$$

Analysis.—*The unit of both fractions* is £1. In the first, the fractional unit is £$\frac{1}{2}$, and in the second, £$\frac{1}{3}$. These fractional units, being different, cannot be expressed in one collection. But £$\frac{1}{2}$ = £$\frac{3}{6}$ and £$\frac{2}{3}$ = £$\frac{4}{6}$, in each of which expressions the *fractional unit* is £$\frac{1}{6}$: hence, their sum is £$\frac{7}{6}$ = £$1\frac{1}{6}$.

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159. What is Addition of Fractions?—160. What is the sum of two or more fractions?—*161 How do you add fractions which have the same unit?*

Rule.

I. *When the fractions have the same denominator, add their numerators, and place the sum over the common denominator:*

II. *When they have not the same denominator, reduce compound fractions to simple ones, and then reduce all to a common denominator, and add as before.*

NOTE.—1. Reduce each fraction to its lowest terms before adding.

2. After the addition is performed, reduce every result to its simplest form; that is, improper fractions to mixed numbers, and the fractional parts to their lowest terms.

162. When each of two fractions has 1 for a numerator.

1. What is the sum of $\frac{1}{5}$ and $\frac{1}{7}$?

OPERATION.

$$\frac{1}{5}+\frac{1}{7}=\frac{7}{35}+\frac{5}{35}=\frac{12}{35}.$$

ANALYSIS.—Reducing to a common denominator, we find the fractions to be $\frac{7}{35}$ and $\frac{5}{35}$, and their sum to be $\frac{12}{35}$. That is, *the sum of two fractions whose numerators are each* 1, *is equal to the sum of their denominators divided by their product.*

2. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? of $\frac{1}{2}$ and $\frac{1}{5}$? of $\frac{1}{7}$ and $\frac{1}{6}$? of $\frac{1}{9}$ and $\frac{1}{10}$?

3. What is the sum of $\frac{1}{12}$ and $\frac{1}{10}$? of $\frac{1}{15}$ and $\frac{1}{16}$? of $\frac{1}{6}$ and $\frac{1}{9}$? of $\frac{1}{8}$ and $\frac{1}{5}$?

163. When there are mixed numbers.

1. What is the sum of $12\frac{3}{5}$, $11\frac{2}{3}$, and $15\frac{5}{7}$?

OPERATION.

Whole Numbers.	*Fractions.*
$2+11+15=38$	$\frac{3}{5}+\frac{2}{3}+\frac{5}{7}=\frac{63}{105}+\frac{70}{105}+\frac{75}{105}=\frac{208}{105}=1\frac{103}{105}$:
then,	$38+1\frac{103}{105}=39\frac{103}{105}$. *Ans.*

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162. What is the sum of two fractions when each has a numerator 1?—*163. How do you* add mixed numbers?

When there are mixed numbers, add the whole numbers and the fractions separately, and then add their sums.

Examples.

1. Add $\frac{5}{9}$, $\frac{7}{12}$, $\frac{5}{18}$, and $\frac{21}{17}$.
2. Add $\frac{7}{8}$, $\frac{7}{12}$, $\frac{13}{16}$, $\frac{11}{18}$, and $\frac{19}{24}$.
3. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{9}{16}$, $\frac{5}{32}$, and $\frac{15}{64}$.
4. Add $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$.
5. Add $\frac{1}{5}$, $4\frac{1}{3}$, and $\frac{2}{5}$.
6. Add $\frac{3}{11}$, $\frac{5}{12}$, $\frac{13}{4}$, and $\frac{2}{3}$.
7. Add $\frac{9}{17}$, $\frac{5}{12}$, $\frac{2}{5}$, and $\frac{7}{8}$.
8 Add $1\frac{2}{4}$, $3\frac{1}{7}$, and $\frac{1}{2}$ of 7.
9. Add $3\frac{33}{25}$, $7\frac{4}{5}$, $\frac{17}{24}$ and $2\frac{11}{18}$
10. Add $2\frac{3}{5}$, $4\frac{7}{8}$, and $\frac{3}{4}$ of $5\frac{3}{10}$
11. Add $12\frac{3}{4}$, $9\frac{2}{3}$, $\frac{4}{7}$ of $6\frac{1}{2}$.
12. Add $\frac{9}{10}$ of $6\frac{7}{8}$ and $\frac{4}{7}$ of $7\frac{1}{2}$.
13. Add $\frac{1}{5}$ of $9\frac{3}{8}$ and $\frac{2}{3}$ of $4\frac{5}{8}$.
14. Add $\frac{3}{5}$, $\frac{9}{10}$ of $\frac{5}{11}$ of 8, and $2\frac{1}{2}$.
15. Add $4\frac{3}{8}$, $\frac{9}{11}$ of $\frac{1}{6}$ of $15\frac{1}{2}$.
16. Add $3\frac{5}{7}$, $4\frac{5}{8}$, and $\frac{1}{3}$ of 16.

17. Bought a cord of wood for $2\frac{5}{8}$ dollars ; a barrel of flour for \$$9\frac{5}{6}$; and some pork for \$$5\frac{3}{4}$: what was the entire cost ?

18. A person travelled in one day $35\frac{1}{3}$ miles ; the next, $28\frac{4}{7}$ miles ; and the next $25\frac{7}{21}$ miles : how many miles did he travel in the three days ?

19. A grocer bought 4 firkins of butter, weighing respectively $54\frac{3}{4}$, $55\frac{3}{8}$, $51\frac{7}{16}$, and $50\frac{21}{32}$ pounds : what was their entire weight ?

20. I paid for groceries at one time $\frac{7}{12}$ of a dollar ; at another, $3\frac{4}{5}$ dollars ; at another, $7\frac{3}{4}$ dollars ; and at another, $5\frac{1}{6}$ dollars : what was the whole amount paid ?

21. A merchant had three pieces of Irish linen ; the first piece contained $22\frac{5}{6}$ yards ; the second $20\frac{7}{8}$ yards ; and the third $21\frac{4}{5}$ yards : how many yards in the three pieces ?

22. A man sold 5 loads of hay ; the first weighed $18\frac{7}{12}$ cwt. ; the second $19\frac{11}{20}$ cwt. ; the third $19\frac{5}{9}$ cwt. ; the fourth $21\frac{11}{15}$ cwt. ; and the fifth $20\frac{13}{15}$ cwt. ; what was the weight of the whole ?

23. A *farmer has three fields* ; the first contains $17\frac{3}{5}$ acres ;

the second $25\frac{2}{9}$ acres ; and the third $46\frac{8}{15}$ acres : how many in the three fields ?

24. A man sold $112\frac{5}{7}$ bushels of wheat for $250\frac{1}{5}$ dollars ; $9\frac{5}{12}$ bushels of corn for $62\frac{3}{8}$ dollars ; $225\frac{9}{14}$ bushels of oats for $104\frac{7}{9}$ dollars : how many bushels of grain did he sell, and how much did he receive for the whole ?

CASE II.

164. When the fractions have different units.

1. What is the sum of $\frac{4}{5}$ lb. and $\frac{3}{4}$ oz. ?

OPERATIONS.

$$\frac{4}{5}\text{ lb.} = \frac{4}{5} \times 16\text{ oz.} = \frac{64}{5}\text{ oz.}$$
$$\frac{64}{5}\text{ oz.} + \frac{3}{4}\text{ oz.} = \frac{256}{20}\text{ oz.} + \frac{15}{20}\text{ oz.}$$
$$= \frac{271}{20}\text{ oz.} = 13\frac{11}{20}\text{ oz.}$$

ANALYSIS.—In $\frac{4}{5}$ lb. there are $\frac{64}{5}$ oz. (Art. 155.) Then, the units of the fractions being the same, viz., 1 oz., we reduce to a common denominator and add, and obtain $13\frac{11}{20}$ oz.

$$\frac{3}{4}\text{ oz.} = \frac{3}{4} \times \frac{1}{16}\text{ lb.} = \frac{3}{64}\text{ lb.}$$
$$\frac{4}{5}\text{ lb.} + \frac{3}{64}\text{ lb.} = \frac{256}{320}\text{ lb.} + \frac{15}{320}\text{ lb.} = \frac{271}{320}\text{ lb}$$
$$\frac{271}{320}\text{ lb} = 13\frac{11}{20}\text{ oz.} = 13\text{ oz. } 8\frac{4}{5}\text{ dr.}$$

SECOND METHOD.—Three-fourths of an ounce is equal to $\frac{3}{64}$ lb. (Art. 156.) Then, by adding, we find the sum to be

$$\frac{4}{5}\text{ lb.} = \frac{4}{5} \times 16\text{ oz.} = \frac{64}{5}\text{ oz.} = 12\text{ oz. } 12\frac{4}{5}\text{ dr.}$$
$$\frac{3}{4}\text{ oz.} = \frac{3}{4} \times 16\text{ dr.} = \frac{48}{4}\text{ dr.} = \qquad 12$$
$$\text{Sum, } \ldots\ldots\ 13 \qquad 8\frac{4}{5}$$

THIRD METHOD.—Find the value of each fractional part in terms of integers of the lower denominations (Art. 157), and then add

Rule.

I. *Reduce the given fractions to the same unit, and then add as in Case I.* Or,

II. *Reduce the fractions separately to integers of lower denominations, and then add the denominate numbers.*

Examples.

1. Add $\frac{3}{8}$ of yard to $\frac{5}{9}$ of an inch.

2. Add together $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour.

3. Add $\frac{5}{8}$ cwt., $\frac{43}{6}$ lb., 15 oz., $\frac{2}{3}$ cwt., and 7 lb together.

4. Add $\frac{1}{5}$ of a pound troy to $\frac{1}{8}$ of an ounce.

5. Add $\frac{4}{9}$ of a ton to $\frac{5}{12}$ of a hundredweight.

6. Add $\frac{5}{9}$ of a chaldron to $\frac{3}{7}$ of a bushel.

7. What is the sum of $\frac{3}{4}$ of a tun, and $\frac{3}{5}$ of a hogshead of wine?

8. Add $\frac{1}{5}$ of $\frac{3}{4}$ of a common year, $\frac{3}{8}$ of $\frac{5}{9}$ of a day, and $\frac{7}{9}$ of $\frac{2}{3}$ of $\frac{3}{8}$ of $19\frac{1}{2}$ hours, together.

9. Add $\frac{5}{8}$ of an acre, $\frac{2}{5}$ of 19 square feet, and $\frac{3}{7}$ of a square inch, together.

10. What is the sum of $\frac{1}{7}$ of a yard, $\frac{1}{7}$ of a foot, and $\frac{1}{7}$ of an inch?

11. What is the sum of $\frac{2}{3}$ of a £, and $\frac{5}{9}$ of a shilling?

12. Add together $\frac{7}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot.

13. What is the sum of $\frac{3}{5}$ of a leap year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day?

14. Add $\frac{3}{5}$ lb. troy, $\frac{1}{4}$ oz. and $\frac{5}{8}$ pwt.

15. Add together $\frac{3}{19}$ of a circle, $3\frac{5}{8}$ signs, $\frac{2}{3}$ of a degree, and $\frac{2}{9}$ of $5\frac{1}{7}$ minutes.

16. What is the sum of $\frac{7}{8}$ yd., $\frac{3}{5}$ of $\frac{5}{8}$ qr. and $3\frac{1}{3}$ na.?

17. Add $\frac{3}{16}$ of a cord, $\frac{5}{9}$ cubic feet, and $\frac{2}{9}$ of $\frac{1}{2}$ of $24\frac{3}{7}$ cubic feet.

18. What is the sum of $\frac{3}{4}$ of $\frac{1}{2}$ of 4 cords, $\frac{5}{6}$ of $\frac{9}{16}$ of 15 cord feet, and $\frac{5}{9}$ of $31\frac{1}{2}$ cubic feet?

19. Add $\frac{5}{8}$ of 3 ells English to $\frac{5}{12}$ of a yard.

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164. **How** ***do you add fractions*** **when they have different units?**

20. Add together $\frac{4}{5}$ of 3 A. 1 R. 20 P., $\frac{3}{8}$ of an acre, and $\frac{3}{4}$ of 3 R. 15 P.

21. What is the sum of $\frac{7}{12}$ of a ton, $\frac{3}{10}$ of a cwt., and $\frac{5}{12}$ of an ounce?

22. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$ of a mile, $\frac{3}{8}$ of a furlong, $\frac{4}{53}$ of a rod, and $\frac{1}{2}$ of a foot?

SUBTRACTION.

165. Subtraction of Fractions is the operation of finding the difference between two fractional numbers.

166. The difference between two fractions is such a number as added to the less will give the greater.

CASE I.

167. When the unit of the fractions is the same.

1. What is the difference between $\frac{3}{4}$ and $\frac{1}{4}$?

OPERATION.

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

Analysis.—The unit of both fractions is the same, being the abstract unit 1. The *fractional unit* is also the same, being $\frac{1}{4}$ in each; hence, the difference of the fractions is equal to the difference of the fractional units, which is $\frac{2}{4}$.

2. What is the difference between $\frac{4}{5}$ lb. and $\frac{2}{3}$ of a pound?

OPERATION.

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}\text{ lb.}$$

Analysis.—The unit in both fractions is 1 lb. The fractional unit of the first is $\frac{1}{5}$ lb., and of the second $\frac{1}{3}$ lb. Reducing to the same fractional unit, we have $\frac{12}{15}$ lb. and $\frac{10}{15}$ lb., the difference of which is $\frac{2}{15}$ lb.; hence,

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165. What is subtraction of fractions?—166. What is the difference between two fractions?—167. How do you subtract when the unit of *the fractions is the same*?

Rule.

I. *If the fractional unit is the same in both, subtract the less numerator from the greater, and place the difference over the common denominator.*

II. *When the fractional units are different, reduce to common denominator; then subtract the less numerator from the greater, and place the difference over the common denominator.*

Examples.

1. From $\frac{3}{7}$ take $\frac{1}{7}$.
2. From $\frac{14}{19}$ take $\frac{11}{19}$.
3. From $\frac{16}{25}$ take $\frac{12}{25}$.
4. From $\frac{204}{305}$ take $\frac{104}{305}$.
5. From $\frac{6}{7}$ take $\frac{4}{5}$.
6. From $\frac{11}{12}$ take $\frac{13}{16}$.
7. From $\frac{14}{15}$ take $\frac{12}{15}$.
8. From $37\frac{11}{15}$ take $\frac{1}{3}$ of $5\frac{5}{6}$.
9. From $\frac{3}{4}$ take $\frac{5}{9}$.
10. From $\frac{7}{8}$ take $\frac{5}{18}$.
11. From 25 take $\frac{11}{15}$.
12. From $\frac{6}{15}$ of 3 take $\frac{1}{3}$ of $\frac{4}{9}$.
13. From $\frac{1}{7}$ of $\frac{3}{6}$ of 7 take $\frac{3}{8}$.
14. From $3\frac{5}{8}$ take $\frac{2}{3}$ of $\frac{7}{8}$.
15. From $\frac{2}{3}$ of 15 take $\frac{4}{5}$ of 3.
16. From $7\frac{1}{3}$ of 2 take $\frac{1}{5}$ of $\frac{2}{6}$.

17. To what fraction must I add $\frac{2}{5}$ that the sum may be $\frac{5}{8}$?

18. What number added to $1\frac{7}{9}$, will make 5?

19. What number is that to which if $7\frac{3}{5}$ be added the sum will be $17\frac{2}{5}$?

20. From the sum of $3\frac{5}{8}$ and $10\frac{4}{5}$ take the difference of $25\frac{1}{4}$ and $17\frac{11}{20}$.

21. What number is that from which if you subtract $\frac{1}{2}$ of $\frac{1}{3}$ of a unit, and to the remainder add $\frac{3}{5}$ of $\frac{7}{8}$ of a unit, the sum will be 9?

22. If I buy $\frac{3}{5}$ of $\frac{4}{5}$ of a vessel, and sell $\frac{1}{2}$ of $\frac{5}{6}$ of my share, how much of the whole vessel will I have left?

23. A man bought a horse for $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{9}{10}$ of \$500, and sold him again for $\frac{5}{7}$ of $\frac{1}{3}$ of $\frac{2}{3}$ of \$1680: what did he gain by the bargain?

24. Bought wheat at $1\frac{7}{8}$ dollars a bushel, and sold it for $2\frac{1}{4}$ dollars a bushel : what did I gain on a bushel?

25. From a barrel of cider containing $31\frac{1}{2}$ gallons, $12\frac{5}{7}$ gallons were drawn : how much was there left?

26. Bought $10\frac{2}{3}$ cords of wood at one time, and $24\frac{5}{6}$ cords at another; after using $16\frac{7}{9}$ cords, how much remained?

27. A merchant bought two firkins of butter, one containing $54\frac{9}{10}$ pounds, and the other $56\frac{11}{12}$ pounds; he sold $43\frac{13}{18}$ pounds at one time, and $34\frac{4}{5}$ pounds at another: how much had he left?

28. A man having \$$50\frac{1}{2}$, expended \$$15\frac{7}{16}$ for dry-goods, and $12\frac{7}{8}$ for groceries : how much had he left?

29. A boy having $\frac{3}{4}$ of a dollar, gave $\frac{1}{5}$ of a dollar for an inkstand, and $\frac{1}{4}$ of it for a slate : how much had he left?

30. Bought two pieces of cloth, one containing $27\frac{4}{9}$ yards, the other $32\frac{1}{6}$ yards, from which I sold $40\frac{17}{18}$ yards : how much had I left?

168. When each fraction has the numerator 1.

1. What is the difference between $\frac{1}{6}$ and $\frac{1}{8}$?

OPERATION.

$$\frac{1}{6} - \frac{1}{8} = \frac{8}{48} - \frac{6}{48} = \frac{2}{48}.$$

ANALYSIS.—Reducing both fractions to a common denominator and subtracting, we find the difference to be $\frac{2}{48}$; that is,

The difference between two fractions, each of whose numerators is 1, *is equal to the difference of the denominator divided by their product.*

2. From $\frac{1}{9}$ take $\frac{1}{15}$.

3. From $\frac{1}{14}$ take $\frac{1}{15}$.

4. From $\frac{1}{19}$ take $\frac{1}{20}$.

5. From $\frac{1}{27}$ take $\frac{1}{30}$.

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168. What is the difference when the numerator of each fraction *is 1?*

169. When there are mixed numbers.

1. What is the difference between $16\frac{1}{5}$ and $3\frac{1}{3}$?

Analysis.—Since we cannot take $\frac{5}{15}$ from $\frac{3}{15}$, we must add, mentally, $1 = \frac{15}{15}$ to the fraction of the minuend, thus making it $\frac{18}{15}$; then $\frac{5}{15}$ from $\frac{18}{15}$ leaves $\frac{13}{15}$. We must now carry 1 to the next figure of the subtrahend, and say 4 from 16 leaves 12. Hence, to subtract one mixed number from another,

OPERATION.

$$\begin{array}{r} 16\frac{1}{5} = 16\frac{3}{15}. \\ 3\frac{1}{3} = 3\frac{5}{15}. \\ \hline 12\frac{13}{15}. \end{array}$$

Subtract the fractional part from the fractional part, and the integral part from the integral part.

2. What is the difference between $14\frac{4}{7}$ and $12\frac{6}{19}$?
3. What is the difference between $115\frac{3}{8}$ and $39\frac{7}{8}$?
4. What is the difference between $78\frac{3}{16}$ and $4\frac{7}{32}$?
5. What is the difference between $48\frac{5}{19}$ and $41\frac{15}{38}$?
6. What is the difference between $287\frac{5}{25}$ and $104\frac{37}{100}$?

CASE II.

170. When fractions have different units.

1. What is the difference between $\frac{1}{2}$ of £ and $\frac{1}{3}$ of a shilling?

Analysis.—Reducing to the common unit 1s., we find the difference to be $\frac{58}{6}$s. = 9s. 8d.

OPERATION.

$$\pounds\tfrac{1}{2} = \tfrac{1}{2} \times 20\text{s.} = \tfrac{20}{2}\text{s.}$$
$$\tfrac{20}{2}\text{s.} - \tfrac{1}{3}\text{s.} = \tfrac{60}{6}\text{s.} - \tfrac{2}{6}\text{s.} = \tfrac{58}{6}\text{s.}$$
$$= 9\tfrac{4}{6}\text{s.} = 9\text{s. } 8\text{d.}$$

Second Method.—Reducing to the common unit £1, we find the difference to be £$\frac{29}{60}$ = 9s. 8d.

$$\tfrac{1}{3}\text{s.} = \tfrac{1}{3} \times \pounds\tfrac{1}{20} = \pounds\tfrac{1}{60}.$$
$$\pounds\tfrac{1}{2} - \pounds\tfrac{1}{60} = \pounds\tfrac{30}{60} - \pounds\tfrac{1}{60}.$$
$$= \pounds\tfrac{29}{60} = 9\text{s. } 8\text{d.}$$

Third Method.—Reduce the fractions to integral units, and then subtract as in denominate numbers.

$$\begin{array}{rr} \pounds\frac{1}{2} = & 10\text{s.} \\ \frac{1}{3}\text{s.} = & 4\text{d.} \\ \hline & 9\text{s. } 8\text{d} \end{array}$$

Rule.

I. *Reduce the fractions to the same unit, and then subtract as in Case I.:* Or,

II. *Find the value of each fraction in units of lower denominations, and then subtract as in denominate numbers.*

Examples.

1. From $\frac{5}{6}$ of a pound troy, take $\frac{5}{8}$ of an ounce.

2. From $\frac{3}{8}$ of a ton, take $\frac{2}{3}$ of $\frac{3}{4}$ of a pound.

3. From $\frac{2}{3}$ of $\frac{5}{7}$ of a hogshead of wine, take $\frac{3}{4}$ of $\frac{1}{2}$ of a quart.

4. From $\frac{3}{5}$ of a league, take $\frac{5}{8}$ of a mile.

5. What is the difference between $1\frac{2}{3}$s. and $\frac{2}{3}$ of $7\frac{1}{2}$d?

6. What is the difference between $\frac{21}{28}$ of a degree and $\frac{3}{4}$ of $\frac{1}{7}$ of a degree.

7. From $\frac{15}{16}$ of a square mile, take $36\frac{7}{9}$ acres.

8. From $\frac{6}{7}$ of a ton, take $\frac{5}{9}$ of 12 cwt.

9. From $1\frac{3}{4}$ lb. troy, take $\frac{1}{6}$ of an ounce.

10. From $2\frac{3}{8}$ cords, take $\frac{3}{4}$ of a cord foot.

11. From $\frac{1}{6}$ of a yard, take $\frac{2}{3}$ of an inch.

12. From $\frac{1}{2}$ of $\frac{3}{4}$ of a pound, take $\frac{4}{5}$ of $\frac{1}{3}$ of a dram, apothecaries' weight.

13. From a piece of ground containing $2\frac{59}{160}$ acres take 1 A. 1 P. and 9 square yards.

14 A pound avoirdupois is equal to 14 oz. 11 pwt. 16 gr troy: what is the difference, in troy weight, between the ounce avoirdupois and the ounce troy?

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169. How do you subtract when there are mixed numbers?
170. What is the rule when the fractions have different units.

MULTIPLICATION.

171 **Multiplication of Fractions** is the operation of taking one number as many times as there are units in another, when one or both are fractional.

1. If 1 pound of tea cost $\frac{5}{8}$ of a dollar, what will $\frac{3}{7}$ of a pound cost.

Analysis.—The cost will be equal to the price of 1 lb. taken as many times as there are units in the multiplier (Art. 84).

OPERATION.

$$\$\tfrac{5}{8} \times \tfrac{3}{7} = \tfrac{5 \times 3}{8 \times 7} = \$\tfrac{15}{56}.$$

One-seventh of a pound of tea will cost one-seventh as much as 1 lb. Since 1 lb. cost $\$\frac{5}{8}$, $\frac{1}{7}$ of 1 lb. will cost $\frac{1}{7}$ of $\$\frac{5}{8} = \$\frac{5}{56}$ (Art. 142). But 3 sevenths of 1 lb. will cost three times as much as $\frac{1}{7}$; that is, $\$\frac{5}{56} \times 3 = \frac{15}{56}$ (Art. 139). Hence, to multiply one fraction by another:

Rule.

Cancel all factors common to the numerator and denominator; then multiply the numerators together for a new numerator, and the denominators together for a new denominator.

172. Principles of the operation.

1. When the multiplier is less than 1, we do not take the whole of the multiplicand, but only such a part of it as the multiplier is of 1.

2. When the multiplier is a proper fraction, multiplication does not *increase* the multiplicand, as in the multiplication of whole numbers. The product is the same part of the multiplicand as the multiplier is of 1.

3. When either of the factors is a whole number, write 1 under t for a denominator.

4. When either of the factors is a mixed number, reduce it to an improper fraction.

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171. What is multiplication of fractions? What is the rule?—172 What is the *first principle of the* operation? What is the second?

Examples.

1. Multiply $\frac{3}{7}$ by 8.
2. Multiply $\frac{8}{75}$ by 12.
3. Multiply $\frac{32}{40}$ by 9.
4. Multiply $\frac{14}{19}$ by 15.
5. Multiply $\frac{4}{5}$ of $\frac{4}{7}$ by 35.
6. Multiply $1\frac{3}{4}$ of $2\frac{1}{2}$ by 1
7. Multiply $2\frac{1}{5}$ of $\frac{2}{7}$ by 70.
8. Multiply $4\frac{2}{3}$ of 8 by 36.

9. Multiply 67 by $9\frac{1}{12}$.
10 Multiply 842 by $7\frac{1}{2}$.
11. Multiply 360 by $12\frac{3}{5}$.
12. Multiply 460 by $11\frac{3}{4}$.
13. Multiply 620 by $10\frac{5}{4}$.
14. Multiply 1340 by $8\frac{3}{4}$.

15. Multiply $\frac{4}{9}$ by 8.
16. Multiply 15 by $\frac{6}{7}$.
17. Multiply $7\frac{7}{8}$ by 8.
18. Multiply $9\frac{1}{2}$ by $18\frac{3}{4}$.
19. Multiply $3\frac{2}{7}$ by $4\frac{14}{33}$.
20. Multiply $\frac{176}{81}$ by 9.
21. Multiply $\frac{7}{8}$ by $\frac{3}{5}$.
22. Multiply $\frac{1}{4}$ of $\frac{3}{8}$ by $\frac{5}{9}$.
23. Multiply $\frac{5}{12}$ by $\frac{9}{20}$ of $\frac{6}{27}$.
24. Multiply $\frac{1}{2}$ of $\frac{7}{8}$ by $\frac{4}{7}$ of $\frac{6}{1}$
25. Multiply $\frac{7}{8}$ by 16.
26. Multiply 28 by $\frac{9}{14}$.
27. Multiply $8\frac{7}{10}$ by 15.
28. Multiply $\frac{6}{11}$ of $\frac{2}{3}$ by $\frac{10}{24}$.
29. Multiply $5\frac{1}{4}$ by $\frac{4}{5}$ of $3\frac{1}{3}$.
30. Multiply $842\frac{1}{4}$ by $7\frac{1}{2}$.
31. Multiply $\frac{5}{9}$ by $\frac{6}{7}$.
32. Multiply $\frac{9}{10}$ by $7\frac{7}{11}$.

33. Multiply $\frac{7}{11}$, $\frac{22}{23}$, and $\frac{46}{49}$ together.
34. Multiply $\frac{14}{27}$, $\frac{9}{28}$, $\frac{6}{13}$, and $\frac{26}{30}$ together.
35. What is the product of $\frac{12}{17}$ by $\frac{2}{9}$ of 17.
36. What is the product of 6 by $\frac{2}{3}$ of 5.
37. What is the product of $\frac{1}{8}$ of $\frac{1}{6}$ of 3 by $15\frac{1}{7}$?
38. What is the product of $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$?
39. What is the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$, and $4\frac{1}{6}$?
40. What will 7 yards of cloth cost, at \$$\frac{3}{4}$ a yard?
41. What will $12\frac{3}{4}$ bushels of apples cost, at \$$\frac{3}{7}$ a bushel
42. If one bushel of wheat costs \$$1\frac{1}{3}$, what will $\frac{2}{3}$ of *bushel cost?*

43. If one horse eats $\frac{5}{8}$ of a ton of hay in one month, how much will 18 horses eat in the same time?

44. If a man earns $\$\frac{15}{16}$ in one day, how much can he earn in 24 days?

45. What will $3\frac{1}{2}$ yards of cloth cost, at $\frac{7}{8}$ of a dollar a yard?

46. At $16 a ton, what will $\frac{11}{12}$ of a ton of hay cost?

47. If one pound of tea costs $\$1\frac{1}{4}$, what will $6\frac{1}{8}$ pounds cost?

48. What will $3\frac{3}{8}$ boxes of raisins cost, at $\$2\frac{1}{2}$ a box?

49. At 75 cents a bushel, what will $\frac{11}{15}$ of a bushel of corn cost?

50. If a lot of land is worth $\$75\frac{8}{15}$, what will $\frac{5}{11}$ of it be worth?

51. What will $17\frac{1}{2}$ yards of cambric cost, at $2\frac{1}{2}$ shillings a yard?

52. Bought $15\frac{5}{8}$ barrels of sugar, at $\$20\frac{4}{5}$ a barrel: what did the whole cost?

53. If one bushel of corn is worth $\frac{5}{6}$ of a dollar, what is $\frac{4}{5}$ of a bushel worth?

54. If I own $\frac{7}{15}$ of a farm, and sell $\frac{9}{14}$ of my share, what part of the whole farm do I sell?

55. I bought a book for $\frac{9}{10}$ of a dollar, and a knife for $\frac{5}{12}$ the cost of the book: how much did I pay for the knife?

56. At $\frac{2}{3}$ of $\frac{11}{12}$ of a dollar a pound, what will $\frac{4}{5}$ of $\frac{15}{16}$ of a pound of tea cost?

57. If hay is worth $\$9\frac{3}{4}$ a ton, what is $\frac{2}{3}$ of $3\frac{1}{5}$ tons worth?

58. If a man can dig a cellar in $22\frac{1}{2}$ days, how many days would it take him to dig $\frac{5}{9}$ of it?

59. If a railroad train runs 1 mile in $\frac{1}{20}$ of an hour, how long will it be in running $106\frac{2}{3}$ miles?

60 A owned $\frac{7}{8}$ of a farm and sold $\frac{4}{5}$ of his share to B, who sold $\frac{5}{9}$ of what he bought to C, who sold $\frac{6}{7}$ of what he bought to D: what part of the whole did D have?

61. A owned $\frac{3}{5}$ of 200 acres of land, and sold $\frac{2}{3}$ of his share to B, who sold $\frac{1}{4}$ of what he bought to C: how many acres had each?

DIVISION.

173. Division of Fractions is the operation of finding how many times one number is contained in another, when one or both are fractional.

1. What is the quotient of $\frac{7}{8}$ divided by $\frac{14}{5}$?

Analysis.—How many times is $\frac{14}{5}$ contained in $\frac{7}{8}$? If $\frac{7}{8}$ be divided by 14, the quotient will be $\frac{7}{8 \times 14} = \frac{1}{16}$. Since the true divisor is but $\frac{1}{5}$ of 14, the divisor used is 5 times *too large;* hence, the partial quotient $\frac{1}{16}$, is 5 times *too small.* Multiplying this by 5, we have the true quotient, $= \frac{5}{16}$. This result is produced by inverting the terms of the divisor and multiplying.

OPERATION.

$$\frac{7}{8} \div \frac{14}{5} = \frac{7}{8} \times \frac{5}{14}.$$
$$= \frac{5}{16} \quad \textit{Ans.}$$

Rule.—*Invert the terms of the divisor, cancel, and proceed as in multiplication.*

174. Directions for the operation.

1. If either the dividend or divisor is a whole number, make it fractional, by writing 1 under it for a denominator.

2. Cancel all common factors.

3 If the dividend and divisor have a common denominator, they will cancel, and the quotient of the numerators will be the answer.

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173. What is division of fractions? What is the rule?—174. What is the first direction for performing the operation? What the second? What the third? What the fourth? What the fifth?

4. When either term of the fraction is a mixed number, reduce to the form of a simple fraction.

5. If the numerator of the dividend is divisible by the numerator of the divisor, and the denominator by the denominator, divide without inverting.

Examples.

1. Divide $\frac{21}{15}$ by 7.
2. Divide $\frac{9}{14}$ by 6.
3. Divide $\frac{13}{15}$ by 9.
4. Divide $\frac{120}{319}$ by 40.
5. Divide $\frac{23}{64}$ by 13.
6. Divide 5 by $\frac{7}{10}$.
7. Divide 27 by $\frac{3}{4}$.
8. Divide $\frac{1}{8}$ by $\frac{1}{7}$.
9. Divide $\frac{9}{10}$ by $\frac{3}{8}$.
10. Divide $\frac{45}{80}$ by $\frac{5}{14}$.
11. Divide $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{6}{7}$ of $\frac{3}{4}$.
12. Divide $\frac{7}{8}$ of $\frac{6}{7}$ by $\frac{4}{5}$ of $\frac{8}{9}$.
13. Divide $\frac{3}{8}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{5}{6}$.
14. Divide 56 by $\frac{11}{12}$.
15. Divide 1000 by $\frac{80}{183}$.
16. Divide 725 by $\frac{25}{47}$.
17. Divide $4\frac{3}{8}$ by 5.
18. Divide $9\frac{5}{11}$ by 12.
19. Divide $\frac{1}{3}$ of $16\frac{1}{2}$ by $4\frac{1}{7}$.
20. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7.
21. Divide $\frac{5}{6}$ of 50 by $4\frac{1}{3}$.
22. Divide $300\frac{5}{28}$ by $6\frac{1}{4}$.
23. Divide $\frac{4}{7}$ of $3\frac{3}{4}$ by $\frac{19}{20}$ of $7\frac{1}{2}$.
24. Divide $9\frac{7}{8}$ by $8\frac{1}{3}$.
25. Divide $\frac{5}{9}$ of $\frac{7}{11}$ by $6\frac{1}{5}$.
26. Divide $\frac{12}{17}$ by 4.
27. Divide $\frac{20}{27}$ by 5.
28. Divide $\frac{60}{75}$ by 8.
29. Divide $\frac{432}{521}$ by 48
30. Divide $\frac{42}{125}$ by 21
31. Divide 36 by $\frac{9}{10}$.
32. Divide 420 by $\frac{3}{8}$.
33. Divide $\frac{9}{20}$ by $\frac{3}{8}$.
34. Divide $\frac{14}{25}$ by $\frac{7}{15}$.
35. Divide $\frac{2}{3}$ of $\frac{27}{50}$ by $\frac{20}{27}$.
36. Divide $\frac{7}{9}$ by $\frac{15}{16}$.
37. Divide $\frac{3}{5}$ of $\frac{8}{9}$ by $\frac{6}{7}$ of $\frac{3}{4}$
38. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{4}{9}$.
39. Divide 650 by $\frac{100}{127}$.
40. Divide 1273 by $\frac{17}{56}$.
41. Divide 4324 by $\frac{128}{475}$.
42. Divide $6\frac{2}{9}$ by 8.
43. Divide $12\frac{4}{9}$ by 42.
44. Divide $3\frac{1}{6}$ by $9\frac{1}{2}$.
45. Divide 100 by $4\frac{3}{8}$.
46. Divide $44\frac{1}{33}$ by $\frac{213}{333}$.
47. Divide $111\frac{1}{9}$ by $33\frac{1}{3}$.
48. Divide $191\frac{1}{5}$ by $159\frac{1}{3}$.
49. Divide $5\frac{3}{8}$ by $\frac{3}{8}$ of $1\frac{1}{2}$
50. Divide $5205\frac{1}{5}$ by $\frac{4}{5}$ of 90.

51. At $\frac{1}{8}$ of a dollar a pound, how much butter can be bought for $\frac{21}{32}$ of a dollar?

52. At $\frac{4}{5}$ of a dollar a yard, how much cloth can be bought for $\frac{7}{8}$ of a dollar?

53. If a bushel of potatoes cost $\frac{3}{8}$ of a dollar, how many bushels can be bought for $\frac{9}{16}$ of a dollar?

54. If $\frac{1}{5}$ of a ton of hay will feed 1 horse one week, how many horses will $\frac{8}{10}$ of a ton feed, the same time?

55. If $\frac{6}{7}$ of a bushel of apples cost $\frac{3}{8}$ of a dollar, what will 1 bushel cost?

56. What will a barrel of flour cost, if $\frac{5}{18}$ of a barrel cost $\frac{6}{7}$ of a dollar?

57. If $\frac{3}{8}$ of a bushel of apples cost $\frac{2}{5}$ of a dollar, what will 1 bushel cost?

58. How much molasses at $\frac{2}{7}$ of a dollar a gallon, can be bought for $1\frac{5}{7}$ dollars?

59. A man sold $\frac{35}{54}$ of a mill, which was $\frac{7}{9}$ of his share: what part of the mill did he own?

60. What number multiplied by $\frac{3}{4}$, will give a product of $15\frac{3}{4}$?

61. What number multiplied by $5\frac{1}{3}$, will give a product of 146?

62. The dividend is $520\frac{1}{3}$, and the quotient $36\frac{9}{10}$: what is the divisor?

63. What number is that, which if multiplied by $\frac{5}{8}$ of $\frac{3}{7}$ of $15\frac{1}{8}$, will produce $\frac{5}{8}$?

64. If 7 lb. of sugar cost $\frac{42}{75}$ of a dollar, what will 1 pound cost?

65. If $10\frac{1}{2}$ lb. of nails cost $\frac{3}{7}$ of a dollar, what is the price per pound?

66. If $\frac{4}{7}$ of a yard of cloth cost $3, what will 1 yard cost?

67. A family consumes $165\frac{3}{5}$ pounds of butter in $8\frac{1}{2}$ weeks: how much do they consume in 1 week?

68. At $\$9\frac{3}{8}$ a barrel, how much flour can be bought for $\$138\frac{3}{4}$?

69. If a man divides $\$3\frac{5}{9}$ equally among 8 beggars, how much does he give them apiece?

70. If 8 pounds of tea cost $\$7\frac{5}{8}$, what is the price per pound?

71. If $\frac{4}{5}$ of a ton of hay sells for $\$10\frac{2}{3}$, what is the price of 1 ton?

72. If $\frac{7}{9}$ of an acre of ground produces $84\frac{7}{16}$ bushels of potatoes, how many bushels will 1 acre produce?

73. What quantity of cloth may be purchased for $\$5\frac{1}{16}$, at the rate of $\$6\frac{3}{4}$ a yard?

74. How long would a person be in traveling $125\frac{5}{7}$ miles, if he traveled $31\frac{6}{14}$ miles per day?

75. How many bottles, each holding $1\frac{2}{7}$ gallons, can be filled from a barrel of wine, containing $31\frac{1}{2}$ gallons?

76. How long will it take 11 men to do a piece of work, that 1 man can do in $15\frac{8}{9}$ days?

77. If $\frac{4}{7}$ of a barrel of flour costs 6 dollars, what is the price per barrel?

78. Eighty-one is $\frac{9}{4}$ of how many times 8?

79. Five-eighths of 48 is $\frac{5}{9}$ of how many times 9?

80. How many times can a vessel, containing $\frac{4}{5}$ of a gallon be filled from $\frac{1}{3}$ of a barrel of $31\frac{1}{2}$ gallons?

81. If $5\frac{1}{2}$ lb. of tea cost $\$4\frac{2}{5}$, what is the price of 1 pound?

82. If $\frac{3}{4}$ of $\frac{5}{9}$ of a ship is worth $2540, what is the whole vessel worth?

83. If $\frac{5}{7}$ of $\frac{3}{4}$ of a barrel of flour will last a family 1 week, how long *will* $9\frac{5}{14}$ *barrels last* them?

COMPLEX FRACTIONS.

175. A COMPLEX FRACTION is only another form of expression for the division of fractions: thus, $\frac{\frac{7}{9}}{\frac{5}{6}}$, is the same as $\frac{7}{9}$ divided by $\frac{5}{6}$; and may be written, $\frac{7}{9} \div \frac{5}{6} = \frac{42}{45}$.

176. To reduce a complex fraction to a simple fraction.

1. Reduce $\frac{6\frac{2}{3}}{1\frac{1}{7}}$ to a simple fraction.

ANALYSIS.—Reducing the divisor and dividend each to a simple fraction, we have $\frac{20}{3}$ and $\frac{8}{7}$. Then $\frac{20}{3}$ divided by $\frac{8}{7}$ is equal to $\frac{20}{3} \times \frac{7}{8} = \frac{35}{6} = 5\frac{5}{6}$.

OPERATION.

$$6\frac{2}{3} = \frac{20}{3}, \text{ and } 1\frac{1}{7} = \frac{8}{7}.$$

$$\frac{20}{3} \div \frac{8}{7} = \frac{20}{3} \times \frac{7}{8} = \frac{35}{6} = 5\frac{5}{6}.$$

Rule.—*Reduce both terms of the fraction to simple fractions: then divide as in division of fractions.*

Examples.

Reduce the following to simple fractions:

1. Reduce $\frac{\frac{5}{6}}{\frac{4}{5}}$.

2. Reduce $\frac{\frac{8}{9}}{\frac{15}{16}}$.

3. Reduce $\frac{1\frac{5}{9}}{\frac{9}{16}}$.

4. Reduce $\frac{87\frac{1}{2}}{\frac{7}{8}}$.

5. Reduce $\frac{\frac{8}{9}}{4\frac{1}{2}}$.

. Reduce $\frac{8\frac{4}{7}}{12}$.

7. Reduce $\frac{11\frac{3}{32}}{8\frac{7}{8}}$.

8. Reduce $\frac{20}{\frac{4}{7}}$.

9. Reduce $\frac{\frac{5}{9} \text{ of } 7\frac{3}{11}}{\frac{4}{11} \text{ of } 17\frac{3}{7}}$.

10. Reduce $\frac{26\frac{8}{35}}{\frac{3}{5} \text{ of } 17}$.

11. Reduce $\frac{55\frac{1}{5}}{\frac{1}{8} \text{ of } 8\frac{2}{3}}$.

12. Reduce $\frac{5}{8}$ of $\frac{3}{10}$ of $\frac{9\frac{3}{4}}{13}$.

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175. What is a complex fraction?—176. How do you reduce a complex to a simple fraction?

Applications in Fractions.

1. What will $5\frac{1}{4}$ cords of wood cost, at $\frac{1}{6}$ of $\frac{3}{7}$ of $\frac{4}{5}$ of $50 a cord?

2. A farmer sold $\frac{3}{8}$ of a ton of hay for $$6\frac{3}{5}$: what would be the price of a ton at the same rate?

3. A person walks $77\frac{3}{8}$ miles in $10\frac{1}{2}$ hours: at what rate is that per hour?

4. From the product of $\frac{2}{5}$ and $11\frac{1}{3}$, take $\frac{6}{13}$, and multiply the remainder by $20\frac{3}{4}$.

5. How much greater is $\frac{3}{4}$ of the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, and $\frac{1}{9}$, than the sum of $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$?

6. If $\frac{3}{5}$ of a ton of hay is worth $$7\frac{1}{8}$, what is $2\frac{2}{3}$ tons worth?

7. If $\frac{2}{3}$ of a dollar will pay for $\frac{7}{8}$ of a yard of cloth, how many yards can be bought for $$11\frac{3}{7}$?

8. What is the value of $3\frac{1}{2}$ cords of wood, at $$4\frac{2}{3}$ a cord?

9. At $\frac{1}{5}$ of a dollar a peck, how many bushels of apples can be bought for $$6\frac{2}{3}$?

10. What is the difference between $\frac{2}{3}$ of a league and $\frac{7}{10}$ of a mile?

11. What is the sum of $4\frac{9}{10}$ miles, $\frac{2}{7}$ of a furlong, and $\frac{3}{8}$ of $1\frac{1}{3}$ yards?

12. At $$1\frac{4}{5}$ per day, how many days' labor can be obtained for $$36\frac{3}{5}$?

13. Bought $5\frac{1}{3}$ yards of cloth at $$4\frac{1}{8}$ a yard, and paid for it in wheat at $$1\frac{4}{7}$ a bushel: how many bushels were required?

14. What number must be taken from $27\frac{3}{4}$, and the remainder multiplied by $14\frac{2}{3}$, that the product shall be 100?

15. Three persons, A, B, and C, purchase a piece of property for $6300; A pays $\frac{3}{7}$ of it, B $\frac{4}{9}$, and C the remainder: what is the value of each one's share?

16. What number diminished by the difference between [illegible] and $\frac{3}{5}$ of itself, leaves a remainder equal to 34?

17. What is the sum of $\frac{2}{7}$ of £15, £3$\frac{3}{7}$, $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1, and $\frac{2}{3}$ of $\frac{3}{7}$ of a shilling?

18. If $\frac{1}{6}$ of John's marbles is equal to $\frac{1}{8}$ of James', and together they have 56, how many has each?

19. A person owning $\frac{3}{7}$ of 2000 acres of land, sold $\frac{2}{3}$ of his share: how many acres did he retain?

20. A boy having 240 marbles, divided them in the following manner: he gave to A, $\frac{1}{3}$, to B, $\frac{1}{10}$, to C, $\frac{1}{8}$, and to D, $\frac{1}{6}$, keeping the remainder himself: what number of marbles had each?

21. A man having engaged in trade with $3740, found, at the end of 3 years, that he had gained 156\frac{1}{3}$ more than $\frac{1}{3}$ of his capital: what was his average annual gain?

22. Two boys having bought a sled, one paying $\frac{3}{4}$ of a dollar, and the other $\frac{7}{8}$ of a dollar, sold it for $\frac{7}{16}$ of a dollar more than they gave for it: what did they sell it for, and what was each one's share of the gain?

23. A farmer having 126$\frac{6}{7}$ bushels of wheat, sold $\frac{5}{8}$ of it at 2\frac{1}{5}$ a bushel, and the remainder at 1\frac{3}{4}$ a bushel: how much did he receive for his wheat?

24. A man having 19\frac{1}{8}$, expended it for wheat and corn, of each an equal quantity; for the wheat he paid 1\frac{4}{5}$ a bushel, and for the corn $$\frac{3}{4}$ a bushel: how much of each did he buy?

25. Two persons engage in trade: A furnished $\frac{7}{12}$ of the capital, and B, $\frac{5}{12}$: if B had furnished 492\frac{2}{3}$ more, their shares would have been equal: how much did each furnish?

26. A man being asked how many sheep he had, said, he had them in three fields: in the first he had 63, which was $\frac{7}{8}$ of what he had in the second; and $\frac{5}{9}$ of what he had in the second was 4 times what he had in the third: how many had he in all?

DUODECIMALS.

177. **Duodecimals** are a system of numbers, which arise from dividing a unit according to the scale of 12. The units divided are, the foot in length, the square foot, and the cubic foot.

If the unit 1 foot be divided into 12 equal parts, each part is called an *inch* or *prime*, and marked ′. If a prime be divided into 12 equal parts, each part is called a *second*, and marked ″. If a second be divided, in like manner, into 12 equal parts, each part is called a *third*, and marked ‴; and so on for divisions still smaller: hence,

$\frac{1}{12}$ of a foot = 1 inch, or prime, ... $1'$.

$\frac{1}{12}$ of $\frac{1}{12}$ of a foot = $\frac{1}{144}$ of a foot, or 1 second, . $1''$.

$\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of a foot = $\frac{1}{1728}$ of a foot, or 1 third, . $1'''$

If the square foot, and the cubic foot, be divided according to the same scale, the primes, seconds, thirds, &c., will have the same relation to the unit and to each other, as in the foot of length.

Table.

12‴	make	1″	second.
12″	"	1′	inch or prime.
12′	"	1	foot.

Hence: Duodecimals are denominate fractions, in which the primary unit is 1 *foot*, and the *scale* uniform, the units of the scale, at every point, being 12.

Notes.—1. The marks ′, ″, ‴, &c., which denote the *fractional units*, are called *indices*.

2. Duodecimals are chiefly used in measuring *Lengths*, *Surfaces Volumes*, or *Solids*.

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177. What are *duodecimals?* What are the units divided? If the unit 1 foot be divided into 12 equal parts, what is each part called?

ADDITION AND SUBTRACTION.

178. THE OPERATIONS of Reduction, Addition, Subtraction, Multiplication, and Division of Duodecimals, correspond so nearly with those of denominate numbers, that additional rules are deemed unnecessary.

Examples.

1. In 86′ how many feet?
2. In 750″ how many ft.?
3 In 37000‴ how many ft.?
4. In 67′ how many feet?
5. In 470‴ how many ft.?
6. In 375″ how many ft.?
7. What is the sum of 8 ft. 9′ 7″ and 6 ft. 7′ 3″ 4‴?
8. Find the difference between 32 ft. 6′ 6″ and 29 ft. 7‴
9. Add together 9 ft. 6′ 4″ 3‴, 12 ft. 2′ 9″ 10‴, 26 ft. 0′ 5″, and 40 ft. 1′ 0″ 3‴.

10. What is the sum of 125 ft. 0′ 6″, 45 ft. 11′ 0″ 2‴, and 12 ft. 6′?

11. What is the sum of 84 ft. 7′, 96 ft. 0′ 11″, 42 ft. 6′ 9′ 10‴, and 5′ 7″ 11‴?

12. From 127 ft. 3′ 6″ 4‴ 11⁗, take 40 ft. 0′ 10″ 7‴ 5⁗

13. What is the difference between 425 ft. 9′ 10″ and 107 ft 10′ 9″ 8‴?

14. What is the sum and difference of 325 ft. 7′ 6″ 2‴ and 217 ft. 10′ 9″?

MULTIPLICATION.

179. MULTIPLICATION OF DUODECIMALS is the operation of finding the superficial contents and the contents of volume, when the linear dimensions are known.

If 1 inch be divided into 12 equal parts, what is each part called? If the second be divided in like manner, what is each part called? What are indices? For what are duodecimals used?—178. How are the fundamental operations performed?—179. What is *multiplication of duodecimals*? How are the areas of figures found? How are the *contents of volume* found?

The superficial contents, or area of figures, are found by multiplying the length and breadth together.

The contents of volume or cubical contents, are found by multiplying together the length, breadth, and height.

180. Principles of the Multiplication.

1. Feet multiplied by feet, give square feet.
2. Feet × Primes = 1 ft. × $\frac{1}{12}$ ft. = $\frac{1}{12}$ sq. ft., or primes.
3. Primes × Primes = $\frac{1}{12}$ ft. × $\frac{1}{12}$ ft. = $\frac{1}{144}$ sq. ft., or seconds.
4. Primes × Seconds = $\frac{1}{12}$ ft. × $\frac{1}{144}$ ft. = $\frac{1}{1728}$ sq. ft., or thirds.
5. Seconds × Seconds = $\frac{1}{144}$ ft. × $\frac{1}{144}$ ft. = $\frac{1}{20736}$ sq. ft., or fourths.

From the foregoing, we have the following principles:

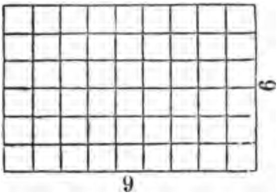
The index of any product is equal to the sum of the indices of the factors.

Note.—The denominator of primes is 12, of seconds, 144, of thirds 1728, of fourths, 20736, &c.

181. To find the square measure, or area of a surface.

1. Find the square measure of a floor that is 9 feet long and 6 feet wide.

Note.—A **Square** is a figure bounded by four equal sides at right angles to each other.



Analysis.—Draw an horizontal line and lay off 9 equal parts, each denoting a foot. Then draw a second horizontal line perpendicular to it, and lay off 6 equal parts, each denoting a foot. Through the points of division of the first line draw parallels to the second, and through the points of division of the

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180. What are the five principles of multiplication? What is the rule for the indices? What is the rule for the multiplication of duodecimals?—181. What is the rule for finding the square measure of a surface?

second line draw parallels to the first: there will thus be formed a number of small squares.

The number of squares in the first row will be equal to 9, the number of linear units in the first line; and the number of rows will be equal to six, the number of units in the second line: therefore, the whole number of squares will be equal to 9 × 6 = 54. Hence, to find the area, or measure,

Multiply the length by the breadth, and the product will be the number of squares.

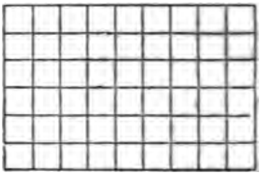
Note.—The square which is the unit of surface, is the square described on the unit of length. If the unit of length is a foot, the unit of surface is 1 square foot—if 1 yard, the unit of surface is 1 square yard. &c.

182. To find the Cubic Measure of a Volume or Solid.

1. What is the cubic measure of a block of marble that is 9 ft. long, 6 ft. wide, and 4 ft. thick?

Note.—A Cube is a figure bounded by six equal squares at right angles to each other, called *faces*; and the sides of the squares are the *edges* of the cube.

Analysis.—The face on which the block stands, is called its *base*, the area of which is equal to 9 × 6 = 54 sq. ft.



If now you take 54 equal cubes, of 1 foot each, they can be placed side by side on the base, and will form a block of marble 9 ft. long, 6 ft. wide, and 1 foot thick. If you place a second tier, the block will be 2 feet thick; a third tier will make it 3 feet thick, and so on, for any number of tiers: hence, the contents of the block, that is four feet thick, are 9 × 6 × 4 = 216 cu. ft.

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182. How do you find the cubic measure of a volume or solid?

Rule.—*Multiply the length, breadth, and thickness together*

183. When the Dimensions are in feet and 12ths of a foot.

Multiply 8 ft. 9′ 5″ by 3 ft. 6′, and then the product by 2 ft. 6′.

Analysis.—First multiply 8 ft. 9′ 5′ by 6′. Since 5″ $= \frac{5}{144}$ ft., and 6′ $= \frac{6}{12}$ ft., 5″ × 6′ $= \frac{5}{144} \times \frac{6}{12} = \frac{30}{1728}$ sq. ft., or 30 thirds. Since 12‴ = 1″, 30‴ ÷ 12 = 2″ and 6‴ over, which write down.

Then 9′ × 6′ $= \frac{9}{12} \times \frac{6}{12} = \frac{54}{144}$ sq. ft., or 54″, to which add the 2″ found in the last product, making 56″. Then, since 12″ = 1′, 56″ ÷ 12 = 4′ and 8″ over, which write down.

1st operation.

8 ft.	9′	5″	
3	6′		
4	4′	8″	6‴
26	4	3	
30 sq. ft.	8′	11″	6‴

Then 8 feet × 6′ = 8 ft. × $\frac{6}{12}$ ft. $= \frac{48}{12}$ sq. ft., to which add the 4′ from the last product, making 52′. Then, since 12′ = 1 square foot, 52′ ÷ 12 = 4 sq. ft. and 4′, both of which set down.

We next multiply, in the same manner, by 3 feet, giving a product of 26 sq. ft. 4′ 8″. The sum of the partial products, 30 sq. ft. 8′ 11″ 6‴, is the first required product.

Now, multiply by 2 ft. 6′.

First, 6‴ × 6′ $= \frac{6}{1728}$ sq. ft. × $\frac{6}{12}$ ft. $= \frac{36}{20736}$ cu. ft. = 36⁗ cu. ft. = 3‴. Then 11″ × 6′ $= \frac{11}{144}$ sq. ft. × $\frac{6}{12}$ ft. $= \frac{66}{1728}$ cu. ft. and three added from the last product gives 69‴ = 5″ and 9‴ over, which write down.

Then 8′ × 6′ $= \frac{8}{12}$ sq. ft. × $\frac{6}{12}$ ft. $= \frac{48}{144}$ cu. ft. = 48″, to which add 5″ from the last product, gives 53″ = 4′ and 5″ over, which write down.

2d operation.

30 sq. ft.	8′	11″	6‴
2	6′		
15	4′	5″	9‴
61	5	11	
76 cu. ft.	10′	4″	9‴

Then, 30 sq. ft. × $\frac{6}{12}$ ft. $= \frac{180}{12}$ = 180′, to which add 4 from the last product, *making 184′* = 15 cu. ft. and 4′. Next, multiply by

2 feet, giving the partial product 61 cu. ft. 5′ 11″; and the sum 76 cu. ft. 10′ 4″ 9‴ is the entire product, in cubic feet and 12ths of cubic feet.

Rule.—I. *Place the multiplier under the multiplicand, so that units of the same order shall fall in the same column*

II. *Multiply the multiplicand by each term of the multiplier in succession, beginning with the lowest unit of each, and make the index of each product equal to the sum of the indices of the factors:*

III. *Reduce each product, as it arises, to the next higher unit; write down the remainder, and carry the quotient to the next product:*

IV. *Find the sum of the several products.*

Examples.

1. How many cubic feet in a stick of timber 12 feet 6 inches long, 1 foot 5 inches broad, and 2 feet 4 inches thick?

2. Multiply 9 ft. 6′ by 4 ft. 7′.

3. Multiply 12 ft. 5′ by 6 ft. 8′.

4. Multiply 35 ft. 4′ 6″ by 9 ft. 10′.

5. What is the product of 45 ft. 4′ 3″ by 12 ft. 2′ 9″?

6. What is the product of 140 ft. 0′ 2″ 4‴ by 20 ft. 10′?

7. What is the product of 279 ft. 10′ 6″ by 8′ 4″?

8. What are the contents of a board 14 ft. 6′ 3″ long, and 2 ft. 9′ wide?

9. How many square feet in a floor 18 ft. 9′ long, and 15 ft. 10′ wide?

10. How many square yards in a ceiling 70 ft. 9′ long, and 12 ft. 3′ wide?

11. How many square feet are there in a ceiling whose length is 75 feet, and width 42 feet?

12. How many square yards are there in a lot of ground whose length is 118 feet, and width 25 feet?

13. How many square feet are there in a board whose length is 18 feet, and breadth 14 inches?

14. What is the cost of painting the side of a house that is 27 feet high and 22 feet wide, at 40 cents per square yard?

15. How many acres are there in a field whose length is 45 rods, and width 37 rods?

16. What is the area of a piece of ground that is 112 ft. 5 in. long, and 27 ft. 9 in. wide?

17. How many flagstones, that are 4 ft. 6 in. by 4 ft., will be required to cover a walk which is 6 ft. 9 in. wide and 264 ft. long?

18. What will be the cost of paving a yard 64 ft. 6 square, at 5 cents a square foot?

19. What are the cubic contents of a block of marble 6 ft. 9′ long, 4 ft. 8′ wide, and 2 ft. 10′ thick?

20. There is a room 97 feet 4′ around it; it is 9 feet 6′ high: what will it cost to paint the walls, at 18 cents a square yard?

21. What is the cubic measure of a pile of wood that is 18 ft. long, 7 ft. high, and 4 ft. wide?

22. How many cords are there in a pile of wood that is 48 ft. long, 9 ft. high, and 3 ft. 6 in. wide?

23. A gallon contains 231 cubic inches: how many gallons of air are contained in a room, which is 21 ft. 6 in. long, 15 ft. wide, and 10 ft. high?

24. A common brick is 8 in. long, 4 in. wide, and 2 in. thick: how many bricks are there in a pile, whose height is 12 ft. 4 in., width 8 ft. and length 15 ft. 9 in., supposing no waste space?

25. A ditch surrounds a plot of ground which is 240 ft long, and 164 ft. wide. The ditch is 3 ft. 6 in. wide, and 6 ft in. deep. What is the cubic measure of the ditch?

26. How many cubic feet of wood in a pile 36 ft. 5′ long 6 ft. 8′ high, and 3 ft. 6′ wide?

27. What will a pile of wood 26 ft. 8′ long, 6 ft. 6 in. high, and 3 ft. 3′ wide cost at $3.50 a cord?

28 How many cubic yards of earth were dug from a cellar

which measured 38 ft. 10′ long, 20 ft. 6′ wide, and 9 ft 4′ deep?

29. At 16 cents a yard, what will it cost to plaster a room 22 ft. 8′ long, 18 ft. 9′ wide, and 11 ft. 6′ high? There are to be deducted 8 windows, each 6 ft. 4′ high, and 2 ft. 9′ wide · 2 doors, each 7 ft. 6′ high, and 3 ft. 2′ wide; and the base moulding, which is 1 foot wide?

DIVISION.

184. Division of Duodecimals is the operation of finding from two duodecimal numbers a third, which multiplied by the first, will give the second?

1. The floor of a hall contains 103 sq. ft. 4′ 5″ 8‴ 4^{iv}, and is 6 ft. 11′ 8″ wide: what is its length?

OPERATION.

$$
\begin{array}{lllllllllll}
\text{ft.} & & & \text{sq. ft.} & & & & & \text{ft.} & & \\
6 & 11' & 8'') & 103 & 4' & 5'' & 8''' & 4^{iv} (& 14 & 9' & 11'' \\
 & & & 97 & 7' & 4'' & & & & & \\
\cline{4-6}
 & & & 5 & 9' & 1'' & 8''' & & & & \\
 & & & 5 & 2' & 9'' & 0''' & & & & \\
\cline{4-8}
 & & & & 6' & 4'' & 8''' & 4^{iv} & & & \\
 & & & & 6' & 4'' & 8''' & 4^{iv} & & &
\end{array}
$$

Analysis. — The units of the dividend are square feet and fractions of a square foot. The units of the divisor are linear feet and fractions of a linear foot.

First, consider how often the first two parts of the divisor are contained in the first part of the dividend. The first two parts of the divisor are nearly equal to 7 feet, and this is contained in 103 sq. ft. 14 times and something over. Multiplying the divisor by this term of the quotient and subtracting, we find the remainder 5 ft. 9′ 1″, to which bring down 8‴.

Next, consider how many times the first two parts of the divisor, (equal to 7 feet nearly) are contained in the first two parts of the remainder, reduced to the next lower unit; that is 5 ft. 9′ = 69′. Multiplying the divisor by the quotient figure 9′, and making the subtraction, we have 6′ 4″ 8‴, to which bring down 4^{iv}.

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184. What is the division of duodecimals? How is it performed?

Consider, again, how often, *nearly*, 7 feet is contained in 6′ 4″ = 76″. Multiplying the divisor by the quotient 11″, we find a product equal to the last remainder. Hence,

The process of division is the same as that in other denominate numbers, except in the manner of selecting the quotient figure.

185. Principles of the operation.

NOTES.—1 If the integral unit of the dividend and divisor is the same, *the unit of the quotient will be abstract.*

2. If the unit of the dividend is a superficial unit, and the unit of the divisor a linear unit, *the unit of the quotient will be linear.*

3. If the unit of dividend is a unit of volume, and the unit of the divisor linear, *the unit of the quotient will be superficial.*

4. If the unit of the dividend is a unit of volume, and the unit of the divisor superficial, *the unit of the quotient will be linear*

Examples.

1. Divide 29 sq. ft. 0′ 4″ by 6 ft. 4′.

2. Divide 50 sq. ft. 0′ 10″ 6‴ by 9 ft. 6′.

3. What is the length of a floor whose area is 1176 sq. ft. 1′ 6″, and breadth 24 ft. 3′?

4. A load of wood, containing 119 cu. ft. 2′ 6″ 8‴, is 3 ft. 4′ high, and 4 ft. 2′ wide: what is its length?

5. In a granite pillar there are 105 cu. ft. 5′ 7″ 6‴; it is 3 ft. 9′ wide, and 2 ft. 3′ thick: what is its length?

6. There are 394 sq. ft. 2′ 9″ in the floor of a hall that is 10 ft. 7′ wide: what is its length?

7. A board 17 ft. 6′ long, contains 27 sq. ft. 8′ 6″: what is its width?

8. From a cellar 42 ft. 10′ long, 12 ft. 6′ wide, were thrown 158 cu. yd. 17 cu. ft. 4′ of earth: how deep was it?

9. A block of marble contains 86 cu. ft. 2′ 7″ 9‴ 6ⁱᵛ. It is 4 ft. 8′ wide and 2 ft. 10′ thick: what is its length?

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185. What are the *principles* of the operation?

DECIMAL FRACTIONS.

186. There are two kinds of Fractions in general use: Common Fractions and Decimal Fractions.

A Common Fraction is one whose unit is divided into any number of equal parts.

A Decimal Fraction is one whose unit is divided according to the *scale of tens.*

187. If the unit 1 be divided into 10 equal parts, each part is called *one-tenth.*

If the unit 1 be divided into one hundred equal parts, or each tenth into ten equal parts, each part is called *one-hundredth.*

If the unit 1 be divided into one thousand equal parts, or each hundredth into ten equal parts, the parts are called *thousandths*, and we have like expressions for the parts, when the unit is further divided according to the scale of tens.

These fractions may be written thus:

Three-tenths, - - - - - - -	$\frac{3}{10}$
Seventh-tenths, - - - - - - -	$\frac{7}{10}$.
Sixty-five hundredths, - - - -	$\frac{65}{100}$.
215 thousandths, - - - - -	$\frac{215}{1000}$.
1275 ten-thousandths, - - - -	$\frac{1275}{10000}$.

From which we see, that the fractional unit of a decimal is one of the equal parts arising from dividing the unit 1 according to the scale of tens: hence, it is one-tenth, one-hundredth one-thousandth, &c.

188. A Decimal Number, or *decimal,* is one which contains a decimal unit.

189. A Mixed Decimal, is one composed of a whole number and a decimal.

Notation and Numeration.

190. The denominators of decimal fractions are seldom written. The fractions are expressed by means of a period, placed at the left of the numerator, called the decimal point (.)

Thus, $\frac{3}{10}$ . . . . is written .3
$\frac{65}{100}$ . . . . " .65
$\frac{215}{1000}$ . . . . " .215
$\frac{1275}{10000}$ . . . . " .1275

The denominator, however, of every decimal, is always understood :

It is the unit 1, *with as many ciphers annexed as there are places of figures in the decimal.*

The place next to the decimal point is called the place of *tenths*, and its unit is 1 tenth ; the next place, at the right, is the place of *hundredths*, and its unit is 1 hundredth ; the next is the place of *thousandths*, and its unit is 1 thousandth ; and similarly for places still to the right.

Decimal Numeration Table.

Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.		
4	.	.	.	.	.	.	is read	4 tenths.
.5	4	.	.	.	.	.	"	54 hundredths.
0	6	4	.	.	.	.	"	64 thousandths.
.6	7	5	4	.	.	.	"	6754 ten-thousandths.
.0	1	2	3	4	.	.	"	1234 hundred-thousandths.
.0	0	7	6	5	4	.	"	7654 millionths.
.0	*0*	*4*	*3*	*6*	*0*	*4*	"	43604 ten-millionths.

NOTE.—Decimal fractions are numerated from left to right; thus, *tenths*, *hundredths*, *thousandths*, &c. They are read 4 tenths, 54 hundredths, 64 thousandths, &c.

Whole numbers and decimals written together.

Whole numbers.									Decimals.						
Tens of millions.	**Millions.**	**Hundreds of thousands.**	**Tens of thousands.**	**Thousands.**	**Hundreds.**	**Tens.**	**UNITS.**	**Decimal point.**	**Tenths.**	**Hundredths.**	**Thousandths.**	**Ten-thousandths.**	**Hundred-thousandths.**	**Millionths.**	**Ten-millionths.**
4	2	0	3	1	4	5	1	.	2	0	4	3	0	7	8

191. Principles.

1. That the denominator belonging to any decimal fraction is 1, with as many ciphers annexed as there are places of figures in the decimal.

2. That the unit of any place is ten times as great as the unit of the next place to the right—the same as in whole numbers: hence, whole numbers and decimals may be written together, by placing the decimal point between them.

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186. How many kinds of fractions are there? What are they? What is a common fraction? What is a decimal fraction?—187. When the unit 1 is divided into 10 equal parts, what is each part called? What is each part called when it is divided into 100 equal parts? When into 1000? Into 10,000? &c. How are decimal fraction formed?—188. What is a decimal number?—189. What is a mixed decimal?—190. Are the denominators of decimal fractions generally set down? How are the fractions expressed? Is the denominator understood? What is it? What is the place next the decimal point called? What is its unit? What is the next place called? What is its unit? What is the third place called? What is its unit? Which way are decimals numerated and read?—191. What are the two principles of decimals?

192. Rule for Writing Decimals.

Write the decimal as if it were a whole number, prefixing as many ciphers as are necessary to make its right-hand figure of the required name.

193. Rule for Reading Decimals.

Read the decimal as though it were a whole number adding the denomination indicated by the lowest decimal unit

Examples.

Write the following common fractions decimally:

(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{[illegible]6}{100}$	$\frac{17}{10}$	$\frac{5}{1000}$	$\frac{27}{100}$	$\frac{47}{1000}$.
(6.)	(7.)	(8.)	(9.)	(10.)
$6\frac{41}{100}$	$7\frac{8}{1000}$	$9\frac{5}{100}$	$10\frac{158}{100}$	$12\frac{327}{10}$.

Write the following numbers in figures, and numerate them:

1. Twenty-seven, and four-tenths.
2. Thirty-six, and fifteen-thousandths.
3. Ninety-nine, and twenty-seven ten-thousandths.
4. Three hundred and twenty thousandths.
5. Two hundred, and three hundred and twenty millionths.
6. Three thousand six hundred ten-thousandths.
7. Five, and three-millionths.
8. Forty, and nine ten-millionths.
9. Forty-nine hundred ten-thousandths.
10. Fifty-nine, and sixty-seven ten-thousandths.
11. Four hundred and sixty-nine ten-thousandths.
12. Seventy-nine, and four hundred and fifteen millionths.
13. Sixty-seven, and two hundred and 27 ten-1000ths
14. One hundred and five, and ninety-five ten-millionths.
15. Forty, and 204 thousand millionths.

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192. What is the rule for writing decimals?—193. What is the ule for reading *decimals*

UNITED STATES MONEY.

194. The denominations of United States Money correspo to the decimal division, if we regard one dollar as the un

For, the dimes are tenths of the dollar, the cents are h dredths of the dollar, and the mills, being tenths of a ce are thousandths of the dollar.

Examples.

1. Express $37 and 26 cents and 5 mills, decimally.
2. Express $17 and 5 mills, decimally.
3. Express $215 and 8 cents, decimally.
4. Express $275 5 mills, decimally.
5. Express $9 8 mills, decimally.
6. Express $15 6 cents 9 mills, decimally.
7. Express $27 18 cents 2 mills, decimally.
8. Express $3 5 cents 9 mills, decimally.

ANNEXING AND PREFIXING CIPHERS.

195. Annexing a cipher is placing it on the right of number.

If a cipher is annexed to a decimal it makes *one more cimal place*, and therefore, a cipher must also be *added the denominator* (Art. 190).

The numerator and denominator will therefore have b

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194. If the denominations of Federal Money be expressed decim what is the unit? What part of a dollar is 1 dime? What par a dime is 1 cent? What part of a cent is a mill? What part dollar is 1 cent? 1 mill?—195. When is a cipher annexed to a n ber? Does the annexing of ciphers to a decimal alter its value? V *not?* What does five-tenths become by annexing a cipher? W *by annexing two* ciphers? Three ciphers?

multiplied by the same number, and consequently the value of the fraction will not be changed (Art. 143): hence,

Annexing ciphers to a decimal does not alter its value.

Take as an example, $.5 = \frac{5}{10}$.

If we annex a cipher to the decimal, we at the same time annex one to the denominator; thus,

.5 becomes $.50 = \frac{50}{100}$ by annexing one cipher.
.5 becomes $.500 = \frac{500}{1000}$ by annexing two ciphers.
.5 becomes $.5000 = \frac{5000}{10000}$ by annexing three ciphers.

196. PREFIXING a cipher is placing it on the left of a number.

If ciphers are *prefixed* to a decimal, the same number of ciphers must be *annexed* to the denominator; for, the denominator must always contain as many ciphers as there are decimal places in the numerator. Now, the numerator will remain unchanged while the denominator will be increased ten times for every cipher annexed; and hence the value of the fraction will be *diminished* ten times for every cipher prefixed to the decimal (Art. 142): hence,

Prefixing ciphers to a decimal diminishes its value ten times for every cipher prefixed.

Take, for example, the decimal $.3 = \frac{3}{10}$.

.3 becomes $.03 = \frac{3}{100}$ by prefixing one cipher;
.3 becomes $.003 = \frac{3}{1000}$ by prefixing two ciphers;
.3 becomes $.0003 = \frac{3}{10000}$ by prefixing three ciphers:

in which the fraction is diminished ten times for every cipher prefixed.

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196. When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? *What effect, then, has it* on the value of the decimal?

197. Analysis of decimals.

Analyze 62.25. It is composed of 6 tens, 2 units, 2 tenths, and 5 hundredths; or it is composed of 62 units and 25 hundredths; or of 622 tenths and 5 hundredths; or 6225 hundredths.

NOTE.—Let it be remembered that a fractional unit of any one place is $\frac{1}{10}$ of the unit of the place next on the left, or $\frac{1}{100}$ of the unit which is 2 places to the left, or $\frac{1}{1000}$ of the fractional unit, which is three places to the left.

ADDITION OF DECIMALS.

198. ADDITION OF DECIMALS is the operation of finding the sum of two or more decimal numbers.

It must be remembered, that only units of the same value can be added together. Therefore, in setting down decimal numbers for addition, figures having the same unit value must be placed in the same column.

The addition of decimals is then made in the same manner as that of whole numbers.

1. Find the sum of 87.06, 327.3, and .0567.

OPERATION.

$$\begin{array}{r} 87.06 \\ 327.3 \\ .0567 \\ \hline 414.4167 \end{array}$$

ANALYSIS.—Place the decimal points in the same column: this brings units of the same value in the same column: then add as in whole numbers: hence,

Rule.

I. *Set down the numbers to be added so that figures of the same unit value shall stand in the same column:*

---

198. What is addition? What parts of a unit may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

II. *Add as in simple numbers, and point off in the sum, from the right hand, a number of places for decimals equal to the greatest number of places in any of the numbers added.*

PROOF.—The same as in simple numbers.

Examples.

1. Add 6.035, 763.196, 445.3741, and 91.5754 together.

2. Add 465.103113, .78012, 1.34976, .3549, and 61.11.

3. Add 57.406 + 97.004 + 4 + .6 + .06 + .3.

4. Add .0009 + 1.0436 + .4 + .05 + .047.

5. Add .0049 + 49.0426 + 37.0410 + 360.0039.

6. Add 5.714, 3.456, .543, 17.4957 together.

7. Add 3.754, 47.5, .00857, 37.5 together.

8. Add 54.34, .375, 14.795, 1.5 together.

9. Add 71.25, 1.749, 1759.5, 3.1 together.

10. Add 375.94, 5.732, 14.375, 1.5 together.

11. Add .005, .0057, 31.008, .00594 together.

12. Required the sum of 9 tens, 19 hundredths, 18 thousandths, 211 hundred-thousandths, and 19 millionths.

13. Find the sum of two, and twenty-five thousandths, five and twenty-seven ten-thousandths, forty-seven, and one hundred twenty-six millionths, one hundred fifty, and seventeen ten-millionths.

14. Find the sum of three hundred twenty-seven thousandths, fifty-six ten-thousandths, four hundred, eighty-four millionths, and one thousand five hundred sixty hundred-millionths.

15. What is the sum of 5 hundredths, 27 thousandths, 476 hundred-thousandths, 190 ten-thousandths, and 1279 ten-millionths?

16. *What is the sum of* 25 dollars 12 cents 6 mills, 9 dol-

lars 8 cents, 12 dollars 7 dimes 4 cents, 18 dollars 5 dimes 8 mills, and 20 dollars 9 mills?

17. What is the sum of 126 dollars 9 dimes, 420 dollars 75 cents 6 mills, 317 dollars 6 cents 1 mill, and 200 dollars 4 dimes 7 cents 3 mills?

18. A man bought 4 loads of hay, the first contained 1 ton 25 thousandths; the second, 997 thousandths of a ton; the third, 88 hundredths of a ton; and the fourth, 9876 ten-thousandths of a ton: what was the entire weight of the four loads?

19. Paid for a span of horses, $225.50; for a carriage, $127.055; and for harness and robes, $75.28: what was the entire cost?

20. Bought a barrel of flour, for $9.375; a cord of wood, for $2.12½; a barrel of apples, for $1.62½; and a quarter of beef, for $6.09: what was the amount of my bill?

21. A farmer sold grain as follows: wheat, for $296.75; corn, for $126.12½; oats, for $97.37½; rye, for $100.10; and barley, for $50.62½: what was the amount of his sale?

22. A person made the following bill at a store: 5 yards of cloth, for $16.408; 2 hats, for $4.87½; 4 pairs of shoes, for $6; 20 yards of calico, for $2.378; and 12 skeins of silk, for $0.62½: what was the amount of his bill?

23. What is the sum of $99 87 cents 5 mills; $87 6 cents 18 mills; $59 42 cents 20 mills; $60 49 cents 16 mills; and $21 29 cents 13 mills?

24. What is the sum of $97 4 mills; $25 19 mills; $65 95 cents 6 mills; $4 87½ cents 3 mills; and $55 14½ cents 9 mills?

25. Mr. James bought of Mr. Squires, the grocer, the following articles: a bag of coffee, for $37.874; a chest of tea, for $50.009; a barrel of sugar, for $19 4 cents and 6 mills; and 9 gallons of wine, for $27 69 cents and 15 mills: what was the amount of his bill?

SUBTRACTION

199. Subtraction of Decimals is the operation of finding the difference between two decimal numbers.

1. From 6.304 take .0563

Analysis.—In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (Art. 195): hence,

OPERATION.

```
6.3040
 .0563
------
6.2477
```

Rule.

I. *Write the less number under the greater, so that figures of the same unit value shall fall in the same column:*

II. *Subtract as in simple numbers, and point off the decimal places in the remainder, as in addition.*

Proof.—Same as in simple numbers.

Examples.

1. From 875.05 take .0467.
2. From 410.0591 take 41.496.
3. From 7141.604 take .09046.
4. Required the difference between 57.49 and 5.768.
5. What is the difference between .3054 and 3.075?
6. Required the difference between 1745.3 and 173.45.
7. What is the difference between seven-tenths and 54 ten-thousandths?
8. What is the difference between .105 and 1.00075?
9. What is the difference between 150.43 and 754.355?
10. From 1754.754 take 375.49478.

---

199. What is subtraction of decimal fractions? How do you set lown the numbers for subtraction? How do you then subtract? low many decimal places do you point off in the remainder? What s the *proof?*

11. Take 75.304 from 175.01.

12. Required the difference between 17.541 and 35 49.

13. Required the difference between 7 tenths and 7 millionths.

14. From 396 take 67 and 8 ten-thousandths.

15. From 1 take one-thousandth.

16. From 6374 take fifty-nine and one-tenth.

17. From 365.0075 take 5 millionths.

18. From 21.004 take 98 ten-thousandths

19 From 260.3609 take 47 ten-millionths.

20. From 10.0302 take 19 millionths.

21. From 2.03 take 6 ten-thousandths.

22. From one thousand, take one-thousandth.

23. From twenty-five hundred, take twenty-five hundredths.

24. From two hundred, and twenty-seven thousandths, take ninety-seven, and one hundred twenty ten-thousandths.

25. A man owning a vessel, sold five thousand seven hundred sixty-eight ten-thousandths of her: how much had he left?

26. A farmer bought at one time 127.25 acres of land; at another, 84.125 acres; at another, 116.7 acres. He wishes to make his farm amount to 500 acres: how much more must he purchase?

27. Bought a quantity of lumber for $617.37½, and sold it for $700: how much did I gain by the sale?

28. Having bought some cattle for $325.50; some sheep for $97.12½; and some hogs for $60.87½; I sold the whole for $510.10: what was my entire gain?

29. A dealer in coal bought 225.025 tons of coal: he sold to A, 1.05 tons; to B, 20.007 tons; to C, 40.1255 tons; and to D. 37.00056 tons: how much had he left?

30. A man owes $2346.865: and has due him, from A, $1240.06; and from B, $1867.98½: how much will he have left after paying his debts?

MULTIPLICATION.

200. **Multiplication** of decimals is the operation of taking one number as many times as there are units in another, when one or both of the factors contain decimals.

1. Multiply 8.03 by 6.102.

Analysis.—If we change both factors to common fractions, the product of the numerators will be the same as that of the decimal numbers, and *the number of decimal places will be equal to the number of ciphers in the two denominators;* hence,

OPERATION

$$8.03 = 8\tfrac{3}{100} = \tfrac{803}{100}$$

$$6.102 = 6\tfrac{102}{1000} = \tfrac{6102}{1000}$$

$$\tfrac{803}{100} \times \tfrac{6102}{1000} = \tfrac{4899906}{100000}$$

$$\begin{array}{r} 8.03 \\ 6.102 \\ \hline 1606 \\ 803 \\ 4818 \\ \hline 48.99906 \end{array}$$

Rule.—*Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as there are decimal places in both factors; and if there be not so many in the product, supply the deficiency by prefixing ciph*[illegible]

Proof.—The same as in whole numbers.

Examples.

1. Multiply 2.125 by 375 thousandths.
2. Multiply .4712 by 5 and 6 tenths.
3. Multiply .0125 by 4 thousandths.
4. Multiply 6.002 by 25 hundredths.
5. Multiply 473.54 by 57 thousandths.
6. Multiply 137.549 by 75 and 437 thousandths.
7. Multiply 3, .7495, and 73487, together.

---

200. What is multiplication of decimals? After multiplying, how many decimal places will you point off in the product? When there are not so many in the product, what do you do? Give the rule for the multiplication of decimals.

8. Multiply .04375 by 47134 hundred-thousandths.
9. Multiply .371343 by seventy-five thousand 493.
10. Multiply 49.0754 by 3 and 5714 ten-thousandths.
11. Multiply .573005 by 754 millionths.
12. Multiply .375494 by 574 and 375 hundredths.
13. Multiply .000294 by one millionth.
14. Multiply 300.27 by 62.
15. Multiply 93.01401 by 10.03962.
16. Multiply 596.04 by 0.000012.
17. Multiply 38049.079 by 0.000016.
18. Multiply 1192.08 by 0.000024.
19. Multiply 76098.158 by 0.000032.
20. Multiply thirty-six thousand by thirty-six thousandths.
21. Multiply 125 thousand by 25 ten-thousandths.
22. Find the product of 50 thousand by 75 ten-millionths.
23. Find the product of 48 hundredths by 75 ten-thousandths.

24. What are the contents of a lot of land, 16.25 rods long, and 9.125 rods wide?

25. What are the contents of a board 12.07 feet long, and 1.005 feet wide?

26. What will 27.5 yards of cloth cost, at .875 dollars per yard?

27. At $25.125 an acre, what will 127.045 acres of land cost?

28. Bought 17.875 tons of hay, at $11.75 a ton: what was the cost of the whole?

29. A gentleman purchased a farm of 420.25 acres, at $35.08 an acre; he afterwards sold 196.175 acres to one man at $37.50 an acre, and the remainder to another person, at $36.125 an acre: what did he gain?

30. A merchant bought two pieces of cloth, one containing 37.5 yards, at $2.75 a yard, and the other, containing 27.35 yards, at $3.125 a yard; he sold the whole at an average price of $2.94 a yard: did he gain or lose by the bargain, and how much?

CONTRACTIONS IN MULTIPLICATION

201. Contractions, in the multiplication of decimals, are short methods of finding the product.

CASE I.

202. To multiply by 10, 100, 1000, &c.

1. Multiply 267.496 by 100.

Analysis.—Removing the decimal point one place to the right, increases the value of the decimal ten times; removing it two places, one hundred times, &c. To multiply by 10, 100, &c., we remove the decimal point as many places to the right as there are ciphers in the multiplier: hence,

OPERATION.

267.496
100
26749.6

Rule.—*Remove the decimal point as many places to the right as there are ciphers in the multiplier; annexing ciphers, if necessary.*

Examples.

1. Multiply 479.64 by 10; also, by 100.
2. Multiply 69.4729 by 1000; also, by 10.
3. Multiply 41.53 by 10000; also, by 100.
4. Multiply 27.04 by 100; also, by 1000.
5. Multiply 129.072 by 1000; also, by 10.
6. Multiply 87.1 by 10000; also, by 100.
7. Multiply 140.1 by 1000; also, by 10.

CASE II.

203. To multiply two decimals, and retain in the product a certain number of decimal places.

1. Let it be required to find the product of 2.38645 multiplied by 38.2175, in such a manner that it shall contain but four *decimal places.*

ANALYSIS.—Write the unit figure of the multiplier under that place of the multiplicand which has the same number, counted from the decimal point, as the figures to be retained in the product, and write the other figures in their proper places. Now, the product of the unit figure of the multiplier, by the figure of the multiplicand directly over it, will have the unit value of the required product. The product of the next figure at the right, in the multiplicand, by the tens of the multiplier, will also give a product of the required unit value; and the same will be true for the product of any two figures equally distant from the unit figure of the multiplier and the figure of the multiplicand directly over it.

OPERATION.

```
2.38645
  38.2175
---------
715935
190916
  4773
   239
   167
    12
-------
91.2042
```

In regard to the decimals, we observe, that the tenths multiplied by the figure at the *left* of the one standing over the unit figure of the multiplier, will give a product of the required unit value; and the same will be true for any two figures equally distant from the decimal point and from the figure standing over the unit place of the multiplier.

We therefore begin the operation with the highest unit figure of the multiplier, and the corresponding figure of the multiplicand, and then multiply in succession by the figures at the right. We must remember that the *whole of the multiplicand* should be multiplied by every figure of the multiplier. Hence, to compensate for the parts omitted, we begin with one figure to the right of that which gives the true unit, and carry one when the product is greater than 5 and less than 15; 2, when it falls between 15 and 25; 3, when it falls between 25 and 35; and so on for the higher numbers.

For example, when we multiply by the 8, instead of saying 8 times 4 are 32, and writing down the 2, we say first, 8 times 5 are 40, and then carry 4 to the product 32, which gives 36

---

201. What is contraction in the multiplication of decimals?—202 How do you multiply by 10, 100, &c? If there are not as many decimal places in the product as there are ciphers, what do you do?—203. Explain the manner of multiplying two decimals together so as to retain a given number of places in the product.

So, when we multiply by the last figure 5, we first say, 5 times 3 are 15, then 5 times 2 are 10 and 2 to carry, make 12, which is written down.

Examples.

1. Multiply 36.74637 by 127.0463, retaining three decimal places in the product.

CONTRACTION.	COMMON WAY.
36.74637	36.74637
127.0463	127.0463
3674637	11023911
734927	22047822
257225	14698548
1470	25722459
220	7349274
11	3674637
4668.490	4668.490346931

2. Multiply 54.7494367 by 4.714753, reserving five places of decimals in the product.

3. Multiply 475.710564 by .3416494, retaining three decimal places in the product.

4. Multiply 3754.4078 by .734576, retaining five decimal places in the product.

5. Multiply 4745.679 by 751.4549, and reserve only whole numbers in the product.

DIVISION.

204. Division of Decimals is the operation of finding how many times one number is contained in another, when one or both are decimals

---

204. What is division of decimals? How is division performed? How does the number of decimal places in the dividend compare with those in the divisor and quotient? How do you determine the number of decimal *places in the* quotient? Give the rule for the division *of decimals.*

1. Divide the decimal .71505 by 2.043.

Analysis.—Division of decimals is performed in the same manner as division of whole numbers. Since the dividend must be equal to the product of the divisor and quotient, it must contain as many decimal places as both of them. Art. 200.) Therefore,

OPERATION.

```
2.043).71505(35
        6129
        -----
        10215
        10215
        -----
```

Ans. 0.35

There must be as many decimal places in the quotient as the number of decimal places in the dividend exceeds that in the divisor: Hence,

Rule.—*Divide as in simple numbers, and point off in the quotient, from the right hand, as many places for decimals as the number of decimal places in the dividend exceeds that in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.*

Examples.

1. Divide 4.6842 by 2.11.
2. Divide 12.82561 by 1.505.
3. Divide 33.66431 by 1.01.
4. Divide .010001 by .01.
5. Divide 24.8410 by .002.
6. Divide .0125 by 2.5.
7. Divide .051 by .012.
8. Divide .063 by 9.
9. Divide 1.05 by 14.
10. Divide 5.1435 by 4.05.
11. Divide .46575 by 31.05.
12. Divide 2.46616 by .145.

13. What is the quotient of 75.15204, divided by 3? By 3? By .03? By .003? By .0003.

14. What is the quotient of 389.27688, divided by 8? By .08? By .008? By .0008? By .00008?

15. What is the quotient of 374.598, divided by 9? By .9? By .09? By .009? By .0009? By .00009?

16 What is the quotient of 1528.4086488, divided by 6? By .06? By .006? .0006? By .00006? By 000006?

17. Divide 17.543275 by 125.7.

18. Divide 1437.5435 by .7493

19. Divide .000177089 by .0374.

20. Divide 1674.35520 by 9.60.

21. Divide 120463.2000 by 1728.

22. Divide 47.54936 by 34.75.

23. Divide 74.35716 by .00573.

24. Divide .37545987 by 75.714.

25 If 25 men remove 154.125 cubic yards of earth in a day, how much does each man remove?

26. If 167 dollars 8 dimes 7 cents and 5 mills be equally divided among 17 men, how much will each receive?

27. Bought 45.22 yards of cloth for $97.223: how much was it a yard?

28. If 375.25 bushels of salt cost $232.655, what is the price per bushel?

29. At $0.125 per pound, how much sugar can be bought for $2.25?

30. How many suits of clothes can be made from 34 yards of cloth, allowing 4.25 yards for each suit?

31. If a man travel 26.18 miles a day, how long will it take him to travel 366.52 miles?

32. A miller wishes to purchase an equal quantity of wheat, corn, and rye; he pays for the wheat, $2.225 a bushel; for the corn, $0.985 a bushel; and for the rye, $1.168 a bushel; how many bushels of each can he buy for $242.979?

33. A farmer purchased a farm containing 56 acres of woodland, for which he paid $46.347 per acre; 176 acres of meadow land, at the rate of $59.465 per acre; besides which there was a swamp on the farm that covered 37 acres, for which he was charged $13.836 per acre. What was the area of the land; what its cost; and what was the average price per acre?

34. A person dying has $8345 in cash, and 6 houses, valued at $4379.837 each; he ordered his debts to be paid, amounting to $3976.480, and $120 to be expended at his funeral; the residue was to be divided among his five sons in the fol-

lowing manner: the eldest was to have a fourth part, and each of the other sons to have equal shares. What was the share of each son?

205. When the decimal places of the divisor exceed those of the dividend.

When there are more decimal places in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places equal to those of the divisor; *all the figures of the quotient will then be whole numbers.* And always bear in mind that, *the number of decimal places in the quotient, is equal to the excess of the number in the dividend over the number in the divisor.*

Examples.

1. Divide 4397.4 by 3.49.

OPERATION.

3.49)4397.40(1260 *Ans.*
349
907
698
2094
2094

ANALYSIS.—We annex one 0 to the dividend. Had it contained no decimal place, we should have annexed two.

2. Divide 1097.01097 by .100001.
3. Divide 9811.0047 by .1629735.
4. Divide .1 by one ten-thousandth.
5. Divide 10 by one-tenth.
6. Divide 6 by .6. By .06. By .006. By .2. By .3. By .003. By .5. By .005. By .000012.

206. When the division does not terminate.

When it is necessary to continue the division further than the figures of the dividend will permit, we may annex ciphers to it, and consider them as decimal places

---

205. What do you do when the decimal places of the divisor exceed those of the dividend? What will the quotient then be?

Examples.

1. Divide 4.25 by 1.25.

OPERATION.

```
1.25)4.25(3.4
     3.75
     ----
      500
      500
      ---
          Ans. 3.4
```

Analysis.—In this example, after having exhausted the decimals of the dividend, we annex a 0, and then the decimal places used in the dividend will exceed those in the divisor by 1.

2. Divide .2 by .06.

OPERATION.

```
.06).20(3.333 +
     18
     --
      20
      18
      --
       20
       18
       --
        2
     Ans. 3.333 +
```

Analysis.—We see, that in this example, the division will never terminate. In such cases, the division should be carried to the third or fourth place, which will give the answer true enough for all practical purposes, and the sign + should then be written, to show that the division may still be continued.

3. Divide 37.4 by 4.5.
4. Divide 586.4 by 375.
5. Divide 94.0369 by 81.032.
6. Divide 36.2678 by 2.25

207. United States Currency.

If we regard 1 dollar as the unit of United States Currency, all the lower denominations,—dimes, cents, and mills,—are decimals of the dollar. Hence, all the operations upon United States Money are the same as the corresponding operations on decimal fractions.

---

206. How do you continue the division after you have brought down all the figures of the dividend? When the division does not terminate, what sign do you place after the quotient? What does it show?

207. What is the unit of United States Currency? What parts of this unit are dimes? What parts are cents? Mills?

CONTRACTIONS IN DIVISION.

208. Contractions in Division of Decimals, like that of whole numbers, are short methods of finding the quotients.

CASE I.

209. **To divide by 10, 100, &c.**

1. Divide 479.256 by 10.

Analysis.—Removing the decimal point one place to the left, diminishes the value of the decimal ten times; two places, 100 times, &c.; therefore, to divide by 10, 100, 1000, &c., we remove the decimal point as many places to the left as there are ciphers in the divisor.

OPERATION.

$10\overline{)479.256}$

47.9256

Rule.—*Remove the decimal point as many places to the left as there are ciphers in the divisor.*

Examples.

1. Divide 3169.274 by 100; by 1000.
2. Divide 57135.62 by 1000; by 100; by 10.
3. Divide 67.5 by 100; by 1000; by 1000000.

Note.—If there are not as many figures at the left of the decimal point as there are 0's in the divisor, prefix ciphers before writing the decimal point.

4. Divide 4.9 by 100; by 1000; by 10000.
5. Divide .30467 by 10; by 100; by 1000.
6. Divide .4741 by 100, by 1000; by 10000
7. Divide 4.97 by 10; by 100; by 1000.

CASE II.

210. **To divide so that the quotient may contain a given number of decimals.**

---

208. What are contractions in division of decimals?

1 Divide 754.347385 by 61.34775, and find a quotient which shall contain three places of decimals.

Rule.

I *Note the unit of the first quotient figure, and then note the number of figures which the quotient must contain:*

II. *Select, from the left, as many figures of the divisor as you wish places in the quotient, and multiply the figures so selected by the first quotient figure, observing to carry for the figures cast off, as in the contraction of multiplication:*

III. *Use each remainder as a new dividend, and in each following division omit one figure at the right of the divisor.*

CONTRACTED METHOD.	COMMON METHOD.	
61.34775)754.347385(12.296	61.34775)754.34738500(12.296	
61348	61347	75
14086	14086	988
12269	12269	550
1817	1817	4385
1227	1226	9550
590	590	48350
552	552	12975
38	38	353750
37	36	808650
1	1	545100

Analysis.—In this example the order of the first quotient figure is tens; hence, there are two places of whole numbers in the quotient; and as there are three decimal places required, there will be five places in all; hence, *five figures* of the divisor must be used.

In the operation, by the common method, the figures at the right of the vertical line, do not affect the quotient figures.

---

209. How do you divide by 10, 100, &c.?—210. Explain the manner of dividing, so that the quotient shall contain a given number of decimal *places.*

Examples.

1. Divide 59 by .74571345, and let the quotient contain four places of decimals.

2. Divide 17493.40777704962 by 495.783269, and let the quotient contain four places of decimals

3. Divide 98.187437 by 8.4765618, and let the quotient contain seven places of decimals.

4. Divide 47194.379457 by 14.73495, and let the quotient contain as many decimal places as there will be integers in it.

REDUCTION.

211. A Denominate Decimal is one in which the unit of the fraction is denominate. Thus, .3 of a dollar, .7 of a shilling, .8 of a yard, &c., are denominate decimals, in which the units are, 1 dollar, 1 shilling, 1 yard.

CASE I.

212. To change a common to a decimal fraction.

The value of a fraction is the quotient of the numerator divided by the denominator (Art. 133).

1. Reduce $\frac{7}{8}$ to a decimal.

Analysis.—If we place a decimal point after the 7, and then write any number of 0's after it, the value of the numerator will not be changed.

If then, we divide by the denominator, the quotient will be the decimal number: Hence,

OPERATION.

$8\overline{)7.000}$

$.875$

Rule.—*Annex decimal ciphers to the numerator, and then divide by the denominator, pointing off as in division of decimals*

---

211. What is a denominate decimal?

Examples.

Reduce the following common fractions to decimals.

1. Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.
2. Reduce $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{5}{16}$.
3. Reduce $\frac{3}{8}$ and $\frac{1}{25}$.
4. Reduce $\frac{3}{192}$ and $\frac{4}{15}$
5. Reduce $\frac{1}{8}$ and $\frac{3}{1000}$.
6. Reduce $\frac{9}{35}$ and $\frac{15}{34}$.
7. Express $\frac{1412}{5907}$ decimally.
8. Express $\frac{275}{3842}$ decimally.
9. Reduce $\frac{7}{16}$ and $\frac{5}{64}$.
10. Express $\frac{14}{3125}$ decimally.
11. Reduce $\frac{67}{125}$ and $\frac{93}{250}$.
12. Reduce $\frac{3}{8}$ of $\frac{2}{5}$ of 6.
13. Reduce $\frac{4}{5}$ of $\frac{11}{12}$.
14. Reduce $\frac{9}{16}$ of $\frac{43}{50}$.
15. Reduce $\frac{2}{3}$ of $\frac{2}{3\frac{1}{4}}$.
16. Reduce $\frac{10\frac{1}{4}}{20}$ and $\frac{4\frac{1}{4}}{758}$.

17. What is the decimal value of $\frac{2}{3}$ of $\frac{3}{5}$ multiplied by $\frac{5}{12}$?

18. What is the value, in decimals, of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$ divided by $\frac{3}{8}$ of $\frac{2}{4}$?

19. A man owns $\frac{7}{8}$ of a ship; he sells $\frac{4}{22}$ of his share: what part is that of the whole, expressed in decimals?

20. Bought $\frac{11}{30}$ of $87\frac{3}{11}$ bushels of wheat for $\frac{9}{20}$ of 7 dollars a bushel: how much did it come to, expressed in decimals?

21. If a man receives $\frac{8}{5}$ of a dollar at one time, \$$7\frac{1}{2}$ at another, and \$$8\frac{3}{4}$ at a third: how much in all, expressed in decimals?

22. What mixed decimal is equal to the sum of $\frac{5}{4}$ of 18, $\frac{8}{11}$ of $\frac{11}{12}$, and $7\frac{4}{9}$?

23. What decimal is equal to $\frac{2}{3}$ of $3\frac{1}{2}$ taken from $\frac{3}{5}$ of $8\frac{3}{4}$

24. What decimal is equal to the sum of $\frac{16}{21}$, $\frac{13}{7}$, and $\frac{2}{5}$?

CASE II.

213. To change a decimal to the form of a common fraction.

Analysis.—A decimal fraction may be changed to the form of a common *fraction by simply* writing its denominator (Art. 190).

Examples.

Express the following decimals in common fractions.

1. Reduce .25 and .75.
2. Reduce .125 and .625.
3. Reduce .105 and .0025.
4. Reduce .8015 and .6042.
5. Reduce .68375.
6. Reduce .01875.
7. Reduce .22575.
8. Reduce .265625.
9. Reduce .33$\frac{1}{3}$.
10. Reduce .5714$\frac{2}{7}$.

CASE III.

214. To reduce a compound number to a decimal of a given denomination.

1. Reduce £1 4s. $9\frac{3}{4}$d. to the decimal of a £.

Analysis.—We first reduce 3 farthings to the decimal of a penny, by dividing by 4. We then annex the quotient .75d. to the 9 pence. We next divide by 12, giving .8125, which is the decimal of a shilling. This we annex to the shillings, and then divide by twenty

OPERATION.

$\frac{3}{4}$d. = .75d.; hence,
$9\frac{3}{4}$d. = 9.75d.
12)9.75d.
.8125s., and
20)4.8125s.
£.240625; therefore,
£1 4s. $9\frac{3}{4}$d. = £1.240625

Rule.—I. *If the lowest denomination contains a fraction, reduce it to a decimal and prefix the integral part:*

II. *Then divide by the units of the ascending scale, annex the quotient to the next higher denomination, and proceed in the same manner through all the denominations, to the required unit.*

Note.—When any denomination, between the lowest and the highest is wanting, the number to be prefixed to the corresponding quotient, is 0.

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212. How do you change a common to a decimal fraction?
213. How do you change a decimal to the form of a common fraction?
214. How do you reduce a compound number to a decimal of a *ven denomination*?

Examples.

1. Reduce 14 drams to the decimal of a lb. Avoirdupois.
2. Reduce 78d. to the decimal of £.
3. Reduce 63 pints to the decimal of a peck.
4. Reduce 9 hours to the decimal of a day.
5. Reduce 375678 feet to the decimal of a mile.
6. Reduce 7 oz. 19 pwt. of silver to the decimal of a pound
7. Reduce 3 cwt. 7 lb. 8 oz. to the decimal of a ton.
8. Reduce 2.45 shillings to the decimal of a £.
9. Reduce 1.047 roods to the decimal of an acre.
10. Reduce 176.9 yards to the decimal of a mile.
11. Reduce 2 qr. 14 lb. to the decimal of a cwt.
12. Reduce 10 oz. 18 pwt. 16 gr. to the decimal of a lb.
13. Reduce 3 qr. 2 na. to the decimal of a yard.
14. Reduce 1 gal. to the decimal of a hogshead.
15. Reduce 17 h. 6 m. 43 sec. to the decimal of a day.
16. Reduce 4 cwt. $2\frac{3}{5}$ qr. to the decimal of a ton.
17. Reduce 19s. 5d. 2far. to the decimal of a pound.
18. Reduce 1 R. 37 P. to the decimal of an acre.
19. Reduce 2 qr. 3 na. to the decimal of an English ell.
20. Reduce 2 yd. 2 ft. $6\frac{1}{2}$ in. to the decimal of a mile.
21. Reduce $15' \, 22\frac{1}{2}''$ to the decimal of a degree.
22. Reduce 1 cwt. 1 qr. 1 lb. to the decimal of a ton
23. Reduce 3 bush. 3 pk. to the decimal of a chaldron
24. Reduce 17 yd. 1 ft. 6 in. to the decimal of a mile.
25. What decimal part of a year is $9\frac{1}{2}$ months?
26. What decimal part of an acre is 1 R. 14 P.?
27. What decimal part of a chaldron is 45 pk.?
28. What decimal part of a mile is 72 yards?
29. What part of a ream of paper is 9 sheets?
30. What part of a rod in length is 4.0125 inches?
31. Reduce 10 wk. 2 da. to the decimal of a leap year.
32. Reduce 4 ℥ 1 ʒ 1 ℈ 10 gr. to the decimal of a lb.
33. Reduce 3 qt. 1.75 pt. to the decimal of a hhd.
34. Reduce *24 sq. yd. 1.8* sq. ft. to the decimal of an acre.

CASE IV.

215. To find the value of a decimal in integers of lower denominations.

1. What is the value of .832296 of a £?

Analysis.—First multiply the decimal by 20, which brings it to the denomination of shillings, and after cutting off from the right as many places for decimals as there are in the given number, we have 16s. and the decimal .645920 over. This is reduced to pence by multiplying by 12, and then to farthings by multiplying by 4.

OPERATION.

$$\begin{array}{r} .832296 \\ 20 \\ \hline 16.645920 \\ 12 \\ \hline 7.751040 \\ 4 \\ \hline 3.004160 \end{array}$$

Ans. 16s. 7d. 3far.

Rule

I. *Multiply the decimal by the units of the descending scale, and point off as in the multiplication of decimals:*

II. *Multiply the decimal part of the product as before, and continue the operations to the lowest denomination. The integers cut off at the left, form the answer.*

Examples.

1. What is the value of .6725 of a hundredweight?
2. What is the value of .61 of a pipe of wine?
3. What is the value of .83229 of a £?
4. Required the value of .0625 of a barrel of beer.
5. Required the value of .42857 of a month.
6. Required the value of .05 of an acre.
7. Required the value of .3375 of a ton.
8. Required the value of .875 of a pipe of wine.
9. What is the value of .375 of a hogshead of beer?

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215. How do you find the value of a decimal in integers of lower *denominations?*

10. What is the value of .911111 of a pound troy?
11. What is the value of .675 of an English ell?
12. What is the value of .001136 of a mile in length?
13. What is the value of .000242 of a square mile?
14. Required the value of .4629 degrees.
15. Required the value of 875 of a yard.
16. Required the value of .3489 of a pound, apothecaries.
17. Required the value of .759 of an acre.
18. Required the value of 01875 of a ream of paper.
19. Required the value of .0055 of a ton.
20. Required the value of .625 of a shilling.
21. Required the value of .3375 of an acre.
22. Required the value of .785 of a year of 365¼ days.

REPEATING DECIMALS.

216. In changing a common to a decimal fraction, there are two general cases:

1st. When the division terminates; and

2d. When it does not terminate.

In the first case, the quotient will contain a *limited number* of decimal places, and the exact value of the common fraction will be expressed decimally.

In the second case, the quotient will contain an *infinite number* of decimal places, and the exact value of the common fraction cannot be expressed decimally.

CASE I.

217. When the division terminates.

When a common fraction is reduced to its lowest terms (which we suppose to be done in all cases that follow), there will be no factor common to its numerator and denominator.

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216. How many cases are there in changing a common to a decimal fraction? What are they? What distinguishes one of these cases from the *other?*

1. Reduce $\frac{17}{50}$ to its equivalent decimal.

ANALYSIS.—Annexing one 0 to the numerator multiplies it by 10, or by 2 and 5; hence, 2 and 5 become *prime factors* of the numerator every time that a 0 is annexed. But if the division is exact, these prime factors, and *none others*, must also be found in the denominator.

OPERATION.

50)17.00(.34
150
200
200

2. Reduce $\frac{5}{36}$ to its equivalent decimal.

ANALYSIS.—$36 = 18 \times 2 = 9 \times 2 \times 2 = 3 \times 3 \times 2 \times 2$; in which we see that the denominator contains *other factors* than 2 and 5; hence, *the fraction cannot be exactly expressed decimally.*

OPERATION.

36)5.0(.1388+
36
140
108
320
288
320
288

Rule.—I. *Reduce the fraction to its lowest terms, then decompose the denominator into its prime factors; and if there are no factors other than* 2 *and* 5, *the exact division can be made:*

II. *If there are other prime factors, the exact division cannot be made.*

NOTE.—Every 0 annexed to the numerator, introduces the two factors 2 and 5; and these factors must be introduced until we have as *many of each* as there are in the denominator after it shall have been decomposed into its prime factors 2 and 5. But the quotient will contain as many decimal places as there are decimal 0's in the dividend. Hence,

The number of decimal places in the quotient will be equal to the greatest number of factors, 2 *or* 5, *in the divisor.*

3. Can $\frac{7}{25}$ be exactly expressed decimally? How many places?

ANALYSIS.—$25 = 5 \times 5$; hence, the fraction can be exactly expressed decimally, and by two decimals, because 5 is taken twice as a factor in the divisor.

OPERATION.

25)7.0(.28
50
200
200

Examples.

Find the decimals and *number of places* in the following:

1. Express $\frac{9}{150}$ decimally
2. Express $\frac{13}{160}$ decimally.
3. Express $\frac{11}{320}$ decimally.
4. Express $\frac{17}{1280}$ decimally.
5. Express $\frac{11}{640}$ decimally.
6. Express $\frac{17}{500}$ decimally.
7. Express $\frac{7}{250}$ decimally.
8. Express $\frac{31}{1280}$ decimally.

CASE II.

218. When the division does not terminate.

1. Let it be required to reduce $\frac{1}{3}$ to its equivalent decimal.

ANALYSIS.—By annexing decimal ciphers to the numerator 1, and making the division, we find the equivalent decimal to be .3333+, &c., giving 3's as far we choose to continue the division.

OPERATION.

$$\begin{array}{r} 3)1.0000 \\ \hline .3333+ \end{array}$$

The further the *division is continued*, the *nearer* the value of the decimal will approach to $\frac{1}{3}$, the *exact value* of the common fraction. We express this approach to equality of value, by saying, that if the division be continued *without limit*, that is, to *infinity*, the value of the decimal will then become *equal* to that of the common fraction; thus,

.3333 . . . , continued to *infinity* = $\frac{1}{3}$;

for, each succeeding 3 brings the value *nearer* to $\frac{1}{3}$.

Also, .9999 . . . , continued to *infinity* = 1;

for, each succeeding 9 brings the value *nearer* to 1.

2 Find the decimal corresponding to the common fraction $\frac{2}{9}$.

ANALYSIS.—Annexing decimal ciphers and dividing, we find the decimal to be .2222+, in which we see that the figure 2 is continually repeated.

OPERATION.

$$\begin{array}{r} 9)2.0000 \\ \hline .2222 \end{array}$$

Examples.

1. Express the fraction $\frac{5}{7}$ decimally.
2. Change $\frac{4}{13}$ into a decimal fraction.
3 Reduce $\frac{5}{11}$ to a decimal fraction.
4. Reduce $\frac{7}{18}$ to a decimal fraction.

219. Definitions.

1. A REPEATING DECIMAL is a decimal in which a *sin* *figure*, or a *set of figures*, is constantly repeated.

2. A REPETEND is a *single figure*, or a set of figures, w is constantly repeated.

3. A SINGLE REPETEND is one in which only a single fi is repeated ; as

$\frac{2}{9}$ = .2222+, or $\frac{3}{9}$ = .3333+.

Such repetends are expressed by simply putting a mark the first figure ; thus,

.2222+, is denoted by .ˋ2, and .3333+ by .ˋ3.

4. A COMPOUND REPETEND has a set of figures repea thus,

$\frac{19}{33}$ = .57 57+, and $\frac{5723}{9999}$ = .5723 5723+

are compound repetends, and are distinguished by marking first and last figures of the set. Thus, 57 57+ is wr .ˋ57′, and .5723 5723 + is written .ˋ5723′.

5. A PURE REPETEND is one which begins with the decimal figure ; as,

.ˋ3, .ˋ5, .ˋ473′, &c.

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217. How do you determine when a common fraction can b actly expressed decimally? How many decimal places will the in the quotient?—218. Can one-third be exactly expressed decim What is the form of the quotient? To what does the value of quotient approach? When does it become equal to one-third?— 1. What is a repeating decimal? 2. What is a repetend? 3. *is a single repetend?* 4. What is a compound repetend?

6. A **Mixed Repetend** is one which has significant figures or ciphers between the decimal point and the repetend; or which has whole numbers on the left hand of the decimal point · such figures are called *finite figures*. Thus,

.0‵733′, .4‵73′, .3‵573′, 6.‵5,

are all mixed repetends; .0, .4, .3, and 6, are the *finite figures*

7. **Similar Repetends** are such as begin at equal distances from the decimal points; as .3‵54′, 2.7‵534′.

8. **Dissimilar Repetends** are such as begin at different distances from the decimal points; as .‵253′, .47‵52′.

9. **Conterminous Repetends** are such as end at equal distances from the decimal points; as .1‵25′, .‵354′.

10. **Similar and Conterminous Repetends** are such as begin and end at the same distances from the decimal point; thus, 53 2‵753′, 4.6‵325′, and .4‵632′, are similar and conterminous.

REDUCTION OF REPETENDS TO COMMON FRACTIONS.

CASE I.

220. To reduce a pure repetend to its equivalent common fraction.

Analysis.—This proposition is to be analyzed by examining the law of forming the repetends.

1st. $\frac{1}{9}$ = .1111 + &c. = .‵1; and $\frac{4}{9}$ = 4444 + &c. = .‵4:
2d. $\frac{1}{99}$ = .010101 + &c. = .‵01′; and $\frac{27}{99}$ = .2727 + &c. = .‵27′:
3d. $\frac{1}{999}$ = .001001 + &c. = .‵001′; and $\frac{324}{999}$ = .324324 + &c. = .‵324′:
&c., &c., &c., &c.

The above law for the formation of repetends does not depend on the multipliers 4, 27, and 324, but would be the same for *any other figures*.

Rule.—*Divide the number denoting the repetend by as many 9's as there are figures and reduce the fraction to its lowest terms.*

Examples.

1. What is the equivalent common fraction of the repetend 0.`3 ?

We have, $\frac{3}{9}=\frac{1}{3}=0.33333+=.`3$.

2. What is the equivalent common fraction of the repetend 162 ?

We have, $\frac{162}{999}=\frac{18}{111}$. *Ans.*

3. What are the simplest equivalent common fractions of the repetends .`6, .`162′, 0.`769230′, .`945′, and .`09′?

4. What are the least equivalent common fractions of the repetends .`594405′, .`36′, and .`142857′?

CASE II.

221. To reduce a mixed repetend to its equivalent common fraction.

ANALYSIS.—A mixed repetend is composed of the finite figures which precede, and of the repetend itself; hence, its value must be equal to such finite figures plus the repetend.

When the repetend begins at the decimal point, the unit of the first figure is .1. But if the repetend begins at any place at the right of the decimal point, the unit value of the first figure will be diminished ten times for each place at the right, and hence, 0's must be annexed to the 9's which form the divisor.

Rule.—*To the finite figures, add the repetend divided by as many 9's as it contains places of figures, with as many 0's annexed to them as there are places of decimal figures preceding the repetend; the sum reduced to its simplest form will be the equivalent fraction sought.*

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219. 5. What is a pure repetend? 6. What is a mixed repetend? 7 What are similar repetends? 8. What are dissimilar repetends? 9 What are conterminous repetends? 10. What are similar and conterminous repetends?—220. How do you reduce a pure repetend to an equivalent common fraction?—221. How do you find the value of *a mixed repetend?*

Examples.

1. Required the least equivalent common fraction of the mixed repetend, $2.4\text{`}18'$.

Now,

$2.4\text{`}18' = 2 + \frac{4}{10} + \text{`}18' = 2 + \frac{4}{10} + \frac{18}{990} = 2\frac{23}{55}$. *Ans.*

2. Required the least equivalent common fraction of the mixed repetend $.5\text{`}925'$.

We have, $.5\text{`}925' = \frac{5}{10} + \frac{925}{9990} = \frac{16}{27}$. *Ans.*

3. What is the least equivalent common fraction of the repetend $.008\text{`}497133'$?

We have, $.008\text{`}497133' = \frac{8}{1000} + \frac{497133}{999999000} = \frac{83}{9768}$.

4. Required the least equivalent common fractions of the mixed repetends $.13\text{`}8$, $7.5\text{`}43'$, $.04\text{`}354'$, $37.5\text{`}4$, $.6\text{`}75'$, and $.7\text{`}54347'$.

5. Required the least equivalent common fractions of the mixed **repetends** $0.7\text{`}5$, $0.4\text{`}38'$, $.09\text{`}3$, $4.7\text{`}543'$, $.009\text{`}87'$, and $.4\text{`}5$

CASE III.

222. To find the finite figures and the repetends corresponding to any common fraction.

1. Find the finite figures and the repetend corresponding to the fraction $\frac{6}{560}$.

ANALYSIS.—1st. Reduce the fraction to its lowest terms, and then find all the factors 2 and 5 of the denominator.

2d. Add decimal ciphers to the numerator and make the division.

OPERATION.

$$\frac{6}{560} = \frac{3}{280}$$

$$\frac{3}{280} = \frac{3}{2 \times 2 \times 2 \times 5 \times 7}$$

$$280)3.000+ \ (.010\text{`}714285'$$

3d. The number of *finite decimals* preceding the first figure of the repetend will be equal to the greatest number of factors 2 or 5: *in this example it is 3.*

4th. When a remainder is found which is the same as a *previous dividend*, the second repetend begins.

5th. The number of figures in any repetend will never exceed the number, less 1, of the *units* in that factor of the denominator which does not contain 2 or 5. In the example, that number is 7 and the number of figures of the repetend. is 6.

Rule.—*Divide the numerator of the common fraction, reduced to its lowest terms, by the denominator, and point off in the quotient the finite decimals, if any, and the repetend.*

Examples.

1. Find whether the decimal, equivalent to the common fraction $\frac{249}{29304}$, is finite or repeating: required the finite figures, if any, and the repetend.

ANALYSIS.—We first reduce the fraction to its lowest terms, giving $\frac{83}{9768}$. We then search for the factors 2 and 5 in the denominator, and find that 2 is a factor 3 times; hence, we know that there are three finite decimals preceding the repetend. We next divide the numerator 83 by the denominator 9768, and note that the repetend *begins* at the fourth place. After the ninth division, we find the remainder 83; at this point the figures of the quotient begin to repeat; hence, the repetend has 6 places.

OPERATION.

$$\frac{249}{29304} = \frac{83}{9768}$$

$$\frac{83}{9768} = \frac{83}{2 \times 2 \times 2 \times 1221}$$

9768)83.00 ... (.008'497133'

2. Find the finite decimals, if any, and the repetend, if any, of the fraction $\frac{210}{1120}$.

3. Find the finite decimals, if any, and the repetend, if any, of the fraction $\frac{4}{1160}$.

4. Find the finite decimals, if any, and the repetend, if any, *of the fractions* $\frac{12}{123}$, $\frac{80}{135}$, $\frac{72}{135}$.

223. Properties of the Repetends.

There are some properties of repetends which it is important o remark.

1. Any finite decimal may be considered as a repeating lecimal by making ciphers recur; thus,

.35 = .35‵0 = .35‵00′ = .35‵000′ = .35‵0000′, &c.

2. Any repeating decimal, whatever its number of figures, uay be changed to one having twice or thrice that number of igures, or any multiple of that number.

Thus, a repetend 2.3‵57′ having two figures, may be changed o one having 4, 6, 8, or 10 places of figures. For,

2.3‵57′ = 2.3‵5757′ = 2.3‵575757′ = 2.3‵57575757′, &c.;

so, the repetend 4.16‵316′ may be written

4.16‵316′ = 4.16‵316316′ = 4.16‵316316316′, &c.;

and the same may be shown of any other. Hence, two or more repetends, having a different number of places in each, may be reduced to repetends having the same number of places, in the following manner:

Find the least common multiple of the number of places in each repetend, and reduce each repetend to such number of places.

3. Any repeating decimal may be transformed into another having finite decimals and a repetend of the *same* number of figures as the first. Thus,

.‵57′ = .5‵75′ = .57‵57′ = .575‵75′ = .5757‵57′; and
3.4‵785′ = 3.47‵857′ = 3.478‵578′ = 3.4785‵785′;

nd hence, *any two repetends may be made similar.*

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222. How do you find the finite figures and the repetend correponding to any common fraction?—223. 1. How may a finite decimal e made a repeating decimal? 2. When a repetend has a given umber of places, to what other form may it be reduced? How *Into what form may* any repeating decimal be transformed?

These properties may be proved by changing the repetends into their equivalent common fractions.

4. Having made two or more repetends similar by the last article, they may be rendered conterminous by the previous one; thus, *two or more repetends may always be made similar and conterminous.*

5 If two or more repeating decimals, having several repetends of equal places, be added together, their sum will have a repetend of the same number of places; for, *every two sets of repetends will give the same sum.*

6. If any repeating decimal be multiplied by any number, the product will be a repeating decimal having the same number of places in the repetend; for, *each repetend will be taken the same number of times, and consequently must produce the same product.*

Examples.

1. Reduce .13‵8, 7.5‵43′ .04‵354′, to repetends having the same number of places.

Since the number of places are now 1, 2, and 3, the least common multiple is 6, and hence each new repetend will contain 6 places; that is,

.13‵8 = .13‵888888′; 7.5‵43′ = 7.5‵434343′; and

.04‵354′ = .04‵354354′.

2. Reduce 2.4‵18′, .5‵925′, .008‵497133′, to repetends having the same number of places.

3. Reduce the repeating decimals 165.‵164′, .‵04′, .03‵7 to such as are similar and conterminous.

4. Reduce the repeating decimals .5‵3, .4‵75′, and 1.‵751′ to such as are similar and conterminous.

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223. 4. To what form may two or more repetends be reduced?

ADDITION.

224. To add repeating decimals.

I. *Make the repetends, in each number to be added, similar and conterminous:*

II. *Write the places of the same unit value in the same column, and so many figures of the second repetend in each as shall indicate with certainty, how many are to be carried from one repetend to the other: then add as in whole numbers*

Note.—If all the figures of a repetend are 9's, omit them and add 1 to the figure next at the left.

Examples.

1. Add .12‵5, 4.‵163′, 1.‵7143′, and 2.‵54′, together.

DISSIMILAR.	SIMILAR.	SIMILAR AND CONTERMINOUS.	
.12‵5 =	.12‵5 =	.12‵555555555555′ - - -	5555
4.‵163′ =	4.16‵316′ =	4.16‵316316316316′ - - -	3163
1.‵7143′ =	1.71‵4371′ =	1.71‵437143714371′ - - -	4371
2.‵54′ =	2.54‵54′ =	2.54‵545454545454′ - - -	5454
	The true sum =	8.54‵854470131697′	1 to carry.

2. Add 67.3‵45′, 9.‵651′, .‵25′, 17.4‵7, .‵5, together.
3. Add .‵475′, 3.75‵43′, 64.‵75′, .‵57′, .1‵788′, together.
4. Add .‵5, 4.3‵7, 49.4‵57′, .4‵954′, .‵7345,′ together.
5. Add .‵175′, 42.‵57′, .3‵753′, .4‵954′, 3.7‵54′, together.
6. Add 165, .‵164′, 147.‵04′, 4.‵95′, 94.3‵7 4.‵712345′.

SUBTRACTION.

225. To subtract one repeating decimal from another.

I *Make the repetends similar and conterminous:*

II. *Subtract as in finite decimals, observing that when the repetend of the lower line is the larger,* 1 *must be carried to the first right-hand figure.*

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224. How do you add repeating decimals?—225. How do you subtract repeating decimals.

Examples

1. From 11.4‵75′ take 3.45‵735′.

DISSIMILAR. SIMILAR. SIMILAR AND CONTERMINOUS.

11.4‵75′ = 11.47‵57′ = 11.47‵575757′ - - - - 575

3.45‵735′ = 3.45‵735′ = 3.45‵735735′ - - - - 735

The true difference = 8.01‵840021′ 1 to carry.

2. From 47.5‵3 take 1.‵757′.
3. From 17.‵573′ take 14.5‵7.
4. From 17.4‵3 take 12.34‵3.
5. From 1.12‵754′ take .4‵7384′.
6. From 4.75 take .37‵5.
7. From 4.794 take .1‵744′.
8. From 1.45‵7 take .3654.
9. From 1.4‵937′ take .1475.

MULTIPLICATION.

226. To multiply one repeating decimal by another.

Change the repeating decimals into their equivalent common fractions, then multiply them together, and reduce the product to its equivalent repeating decimal.

Examples.

1. Multiply 4.25‵3 by .257.

OPERATION.

4.25‵3 $= 4 + \frac{25}{100} + \frac{3}{900} = 4 + \frac{225}{900} + \frac{3}{900} = \frac{228}{900} = \frac{3828}{900}$

$= \frac{1914}{450} = \frac{957}{225}$

Also, $.257 = \frac{257}{1000}$; hence,

$\frac{957}{225} \times \frac{257}{1000} = \frac{245949}{225000} =$ 1.09310‵6 ;

and since $225000 = 5 \times 5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 9$, there will be five places of finite decimals, and one figure in the repetend.

NOTE.—Much labor will be saved in this and the next rule by keeping every fraction in its lowest terms; and when two fractions are to be multiplied together, cancel all the factors common to both terms before making the multiplication.

2. Multiply .375‵4 by 14.75.
3. Multiply .4‵253′ by 2.57.
4. Multiply .437 by 3.7‵5.
5. *Multiply* 4.573 by .3‵75′.
6. Multiply 3.45‵6 by 42‵5.
7. Multiply 1.‵456′ by 4.2‵3.
8. Multiply 45.1‵3 by .‵245′
9. Multiply .4705‵3 by 1.7‵35′

DIVISION.

227. To divide one repeating decimal by another.

Change the decimals into their equivalent common fractions, and find the quotient of these fractions. Then change the quotient into its equivalent decimal.

Examples.

1. Divide 56.‵6 by 137.

OPERATION.

$$56.{}^{\backprime}6 = 56 + \tfrac{6}{9} = \tfrac{510}{9} = \tfrac{170}{3}.$$

Then, $\tfrac{170}{3} \div 137 = \tfrac{170}{3} \times \tfrac{1}{137} = \tfrac{170}{411} = .{}^{\backprime}41362530'$.

2 Divide 24.3‵18′ by 1.792.

3 Divide 8.5968 by .2‵45′.

4. Divide 2.295 by .‵297′.

5 Divide 47.345 by 1.‵76′.

6. Divide 13.5‵169533′ by 4.‵297′

7. Divide .‵45′ by .‵118881′.

8. Divide .‵475′ by .3‵753′.

9. Divide 3.‵753′ by .‵24′.

CONTINUED FRACTIONS.

228. A CONTINUED FRACTION has 1 for its numerator, and for its denominator a whole number plus a fraction, which also has a numerator of 1, and for a denominator, a whole number plus a similar fraction, and so on.

1. If we take any irreducible fraction, as $\tfrac{15}{29}$, and divide both terms by the numerator, it will take the form,

$$\frac{15}{29} = \frac{1}{\tfrac{29}{15}} = \frac{1}{1 + \tfrac{14}{15}},$$ by making the division.

If, now, we divide both terms of $\tfrac{14}{15}$ by 14, we have,

$$\frac{14}{15} = \frac{1}{1 + \tfrac{1}{14}}.$$

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226. How do you multiply repeating decimals?—227. How do you divide *repeating decimals*?—228. What are continued fractions? What *is the rule for finding* the approximate value?

If, now, we replace $\frac{14}{15}$ by its value, $\dfrac{1}{1 + \frac{1}{14}}$, we shall have

$$\frac{15}{29} = \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{14}}};$$

hence, this is a *continued fraction.*

2. Reduce $\frac{15}{19}$ to the form of a continued fraction.

$$\frac{15}{19} = \frac{1}{1 + \frac{4}{15}}; \quad \frac{4}{15} = \frac{1}{3 + \frac{3}{4}}; \quad \frac{3}{4} = \frac{1}{1 + \frac{1}{3}}$$

hence,

$$\frac{15}{19} = \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \frac{1}{3}}}}.$$

Analysis.—Let us analyze this example. If we neglect what comes after 1, the first term of the first denominator, we shall have, $\frac{1}{1} = 1$, which is called *the first approximating fraction.* If we neglect what comes after 3, the first term of the second denominator, we shall have,

$$\frac{1}{1 + \frac{1}{3}} = \frac{3}{4}$$

the second approximating fraction.

If we neglect what comes after 1, the first term of the third denominator, we shall have,

$$\cfrac{1}{1 + \cfrac{1}{3 + 1}} = \frac{4}{5}$$

the third approximating fraction; and so on, for fractions which follow.

If we stop at the first approximating fraction, the denominator 1 will be *less* than the *true* denominator; for, the true denominator is 1 plus a fraction; hence, the value of the first approximating fraction will be *too great;* that is, it will exceed the value of the given fraction.

If we stop at the second, the denominator 3 will be *less* than the *true denominator;* hence, $\frac{1}{3}$ will be greater than the number to be *added to 1;* therefore, $1 + \frac{1}{3}$ is too large, and $1 \div 1 + \frac{1}{3}$, which is $\frac{3}{4}$, is *too small:* that is, it is *less* than the value of the given fraction. Thus, every *odd* approximating fraction gives a value *too large*, and every *even* one, gives a value *too small.*

Rule.—*Write the given fraction in the form of a continued fraction, using several terms when a near approximation is desired; then take a mean between the last and the preceding approximating fractions.*

Examples.

1 Reduce $\frac{829}{347}$ to the form of a continued fraction.

$$\frac{829}{347} = 2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \frac{1}{19}.}}}}$$

2. Place $\frac{21}{36}$ under the form of a continued fraction, and find the value of each of the approximating fractions.

3. Place $\frac{47}{65}$ under the form of a continued fraction, and find the value of each of the approximating fractions.

4. Place $\frac{17}{27}$ under the form of a continued fraction, and find the value of each approximating fraction.

5. Place $\frac{67}{85}$ under the form of a continued fraction, and find the value of each approximating fraction.

6. Place $\frac{37}{87}$ under the form of a continued fraction, and find the value of each approximating fraction.

7. The solar year contains 365 da. 5 hr. 48 m. 48 sec. Find what fractional part of a day the excess of the solar year is above the common year, when the operation is carried to the fifth approximating fraction.

RATIO AND PROPORTION.

229. Two numbers, of the same kind, may be compared in two ways:

1st. By considering *how much* one is greater or less than the other, which is shown by their difference; and,

2d. By considering *how many times* one number is greater or less than another, which is shown by their quotient.

In comparing two numbers, by means of their difference, the less is always taken from the greater.

In comparing two numbers by their quotient, one is regarded as a *standard* which *measures* the other; hence, to measure a number, is to find how many times it contains the *standard*.

230. A Ratio is the quotient arising from dividing one number by another.

The Terms of a ratio are the divisor and dividend; hence, *every ratio has two terms.*

The *Divisor* is called the ANTECEDENT; and the *Dividend* is called the CONSEQUENT.

The ANTECEDENT and CONSEQUENT, taken together, are called a COUPLET.

231. The ratio of one number to another is expressed in two ways:

1st. By a colon; thus, 4 : 16; and is read, 4 is to 16; or, 16 divided by 4.

2d. In a fractional form, as $\frac{16}{4}$; or, 16 divided by 4.

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229. In how many ways may two numbers of the same kind be compared with each other? If you compare by their difference, what do you do? If you compare by the quotient, how do you regard one of the numbers? How do you measure a number?

230. What is a ratio? What are its terms? How many terms has every ratio? What is the divisor called? What the dividend?—231. In how many ways is the ratio expressed? What are they? How is it read?

Since every ratio may be expressed under the form of a fraction, and since the numerator and denominator may be multiplied or divided by the same number, without altering the value (Arts. 143 and 144), it follows that,

If both terms of a ratio be multiplied or divided by the same number, the ratio will not be changed.

232. A SIMPLE RATIO is when both terms are simple numbers; thus,

7 : 12, is a simple ratio.

233. A COMPOUND RATIO is one which arises from the multiplication of two simple ratios: thus, in the simple ratios

5 : 10, and 3 : 12,

if we multiply the corresponding terms together, we have

$$5 \times 3 : 10 \times 12,$$

which is compounded of the ratios of 5 to 10, and of 3 to 12.

234. The ELEMENTS of a term are its factors: thus, 5 and 3 are the elements of the first term, and 10 and 12 of the second.

These elements are generally written in a column, thus,

$\left.\begin{matrix}5\\3\end{matrix}\right\} : \left.\begin{matrix}10\\12\end{matrix}\right\}$; and read, 5 multiplied by 3, to 10 multiplied by 12

NOTE.—A compound ratio may be reduced to a simple ratio, by multiplying the elements; thus the last ratio is that of 15 to 120

235. To find the ratio of one number to another.

When the antecedent is less than the consequent, the ratio shows *how many times* the consequent is greater than the antecedent.

When the antecedent is greater than the consequent, the ratio shows *what part* the consequent is of the antecedent The phrase, "what part," implies the quotient of a less number divided by a greater.

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232. What is a simple ratio?—233. What is a compound ratio?—234. What are the elements of a term?

Examples.

1. What is the ratio of 9 tons to 15 tons?

ANALYSIS.—In this example the antecedent is 9 tons, and the consequent is 15 tons; the ratio is therefore expressed by the fraction $\frac{15}{9} = \frac{5}{3} = 1\frac{2}{3}$.

2. What is the ratio of 6 inches to 24 inches?
3. What is the ratio of 7 feet to 35 feet?
4. What is the ratio of fifteen dollars to 6 dollars?
5. What is the compound ratio of 5 : 6 and 4 : 10?
6. What is the compound ratio of 6 : 9 and 3 : 4?
7. What is the compound ratio of 4 : 5, 9 : 8, and 3 : 5?

8. What part of 6 is 4?
9. What part of 10 is 5?
10. What part of 34 is 17?
11. What part of 450 is 300?
12. What part of 96 is 16?
13. 8 is what part of 12?
14. 16 is what part of 48?
15. 18 is what part of 90?
16. 15 is what part of 165?
17. 9 is what part of 11?

236. To find the antecedent or consequent, when the ratio and one of the terms are given.

1. The ratio of two numbers is 5; and the antecedent is 4 dollars: what is the consequent?

ANALYSIS.—Since the ratio is equal to the quotient of the consequent divided by the antecedent, it follows:

1st. *That the consequent is equal to the antecedent multiplied by the ratio:*

2d. *That the antecedent is equal to the consequent divided by the ratio.*

OPERATION.

$$\begin{array}{c} \text{Ratio} \\ 5 \end{array} = \frac{\text{consequent.}}{\text{antecedent.}}$$

$$5 \times \text{ant.} = \text{cons.}$$

$$\$4 \times 5 = \$20 = \text{cons.}$$

Examples.

1. The ratio of two numbers is 7, and the antecedent is 16 cwt.: what is the consequent?

2. The consequent is 30 tons, and the ratio is 6: what is *the antecedent?*

3. The antecedent is 15, and the ratio is 4: what is the consequent?

4. The ratio of two numbers is $1\frac{2}{3}$, and the consequent is 7: what is the antecedent?

5. The ratio of two numbers is $\frac{7}{8}$, and the antecedent is $\frac{4}{5}$: what is the consequent?

6. The ratio of the monthly wages of two men is 8: the greater wages of one is $256: what is the wages of the other?

7. The ratio is 25, and the consequent is 14 × 5 × 10; what is the antecedent?

8. The value of a horse is $2\frac{1}{5}$ times that of an ox: the value of the horse is $143: what is the value of the ox?

SIMPLE PROPORTION.

237. A Simple Proportion is an expression of equality between two simple and equal ratios. Thus, the two couplets,

$$4 : 20 \text{ and } 1 : 5,$$

having the same ratio 5, form a proportion, and are written,

$$4 : 20 :: 1 : 5,$$

by simply placing a double colon between the couplets. The terms are read,

4 is to 20 as 1 is to 5,

and taken together, they are called a *proportion.*

238. The 1st and 4th terms of a proportion are called the *extremes;* the 2d and 3d terms, the *means.* Thus, in the proportion,

$$6 : 24 :: 8 : 32,$$

6 and 32 are the *extremes*, and 24 and 8 the *means:*

Since

$$\frac{24}{6} = \frac{32}{8}$$

we shall have, by reducing to a common denominator,

$$\frac{24 \times 8}{6 \times 8} = \frac{32 \times 6}{6 \times 8}$$

But since the fractions are equal, and have the same denominators, their numerators must be equal, viz. :

$$24 \times 8 = 32 \times 6; \text{ that is,}$$

In any proportion, the product of the extremes is equal to the product of the means.

Thus, in the proportions,

$1 : 8 :: 2 : 16$; we have $1 \times 16 = 2 \times 8$.
$4 : 12 :: 8 : 24$; " " $4 \times 24 = 12 \times 8$.

239. Since, in any proportion, the product of the extremes is equal to the product of the means, it follows that,

1st. *If the product of the means be divided by one of the extremes, the quotient will be the other extreme.*

Thus, in the proportion,

$4 : 16 :: 6 : 24$, and $4 \times 24 = 16 \times 6 = 96$;

then, if 96, the product of the means, be divided by one of the extremes, 4, the quotient will be the other extreme, 24; or, if the product be divided by 24, the quotient will be 4.

2d. *If the product of the extremes be divided by either of the means, the quotient will be the other mean.*

Thus, if $4 \times 24 = 16 \times 6 = 96$ be divided by 16, the quotient will be 6; or if it be divided by 6, the quotient will be 16.

NOTE.—We shall denote the required term of a proportion by the letter x

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235. When the antecedent is less than the consequent, what does the ratio express? What does it express when the antecedent is greater than the consequent?—236. To what is the consequent equal, in any ratio? To what is the antecedent equal?—237. What is a simple proportion?—238. Which are the extremes of a proportion? Which the means? What is the product of the means equal to?—239. If the product of the means be divided by one of the extremes, what is the quotient? If the product of the extremes be divided by *one of the means*, what is the quotient?

Examples.

Find the required term in each of the following examples:

1. $5 : 30 :: 10 : x$.
2. $9 : x :: 12 : 36$.
3. $\$15 : \$45 :: x : 27$.
4. $\frac{1}{3} : \frac{1}{20} :: x : \frac{1}{32}$

5. The first three terms of a proportion are 5, 10, and 19: what is the fourth term?

6. The first three terms of a proportion are 6, 24, and 14: what is the fourth term?

7. The first, second, and fourth terms of a proportion are 9, 12, and 16: what is the third term?

8. The first, third, and fourth terms of a proportion are 16, 8, and 20: what is the second term?

DIRECT AND INVERSE PROPORTION.

240. It often happens, that two numbers which are compared together, may undergo certain *changes of value*, in which case they represent *variable* and not *fixed* quantities. Thus, when we say that the amount of work done, in a single day, will be proportional to the number of men employed, we mean, that if we *increase* the number of men, the amount of work done will also be *increased;* or, if we *diminish* the number of men, the work done will also be *diminished.* This is called *Direct Proportion.*

If we say that a barrel of flour will serve 12 men a certain time, and ask how long it will serve 24 men, the time will be less: that is, the time will *decrease* as the number of men is *increased,* and will *increase* as the number of men is *decreased.* This is called, *Inverse Proportion;* hence,

1. *Two numbers are directly proportional, when they increase or decrease together; in which case their ratio is always the same.*

2. *Two numbers are inversely proportional, when one increases as the other decreases; in which case their product is always the same.*

Note.—This is sometimes called, *Reciprocal Proportion.*

First Illustration.

If we refer to the numeration table of integral and decimal numbers (Art. 190), we see that the unit of the first place, at the left of 1, is 1 ten; that is, a number *ten times* as great as 1. The unit of the first decimal place at the right, is 1 tenth, a number only one-tenth of 1. The unit of the second place, at the left, is one hundred times as great as 1; while the unit of the second place, at the right, is only one hundredth of 1; and similarly for all other corresponding places. Hence,

The units of place, taken at equal distances from the unit 1, are inversely proportional.

Second Illustration.

The floor of a room is 20 feet long: what must be its breadth, in order that it may contain 360 square feet?

OPERATION.

$$\frac{360}{20} = 18 \text{ ft. breadth.}$$

Analysis.—The length of the floor, multiplied by its breadth, will give the area or contents; hence, the area, divided by the length, will give the breadth. If the contents remain the same, the length will increase as the breadth diminishes; and the reverse. Hence, *when the contents are the same, the length and breadth are inversely proportional.*

COMPOUND PROPORTION.

241. A Compound Proportion is the comparison of the terms of two equal ratios, when one or both are compound.

$$\text{Thus,}\quad \left.\begin{matrix}5\\4\end{matrix}\right\} : \left.\begin{matrix}8\\3\end{matrix}\right\} :: 5 : 6;$$

$$\text{Or,}\quad \left.\begin{matrix}5\\4\end{matrix}\right\} : \left.\begin{matrix}8\\3\end{matrix}\right\} :: \left.\begin{matrix}3\\5\end{matrix}\right\} : \left.\begin{matrix}6\\8\end{matrix}\right\}.$$

Any compound proportion may be reduced to a simple one, by multiplying the elements of each term together; thus, by multiplying together the elements of the last proportion, we have,

$$20 : 24 :: 15 : 18.$$

Hence, in any compound proportion,

The product of the extremes is equal to the product of the means; and the required term may be found as in Art. 239.

What are the required terms in the following proportions?

1. $3 \times 9 : 12 \times 6 :: 15 : x.$
2. $5 \times 9 : 10 \times 9 :: 18 : x.$

242. If an element is unknown, denote it by x. Then, if all the parts are known except one element, as in the following proportion,

$$\left.\begin{matrix}2\\21\end{matrix}\right\} : \left.\begin{matrix}5\\6\end{matrix}\right\} :: \left.\begin{matrix}3\\7\end{matrix}\right\} : \left.\begin{matrix}5\\x\end{matrix}\right\}$$

that element is equal to the product of the means divided by the product of all the elements of the first term and the known elements of the fourth term; thus,

$$x = \frac{5 \times 6 \times 3 \times 7}{2 \times 21 \times 5} = 3$$

1. What is the required element in the proportion,

$$\left.\begin{matrix}3\\2\\5\end{matrix}\right\} : \left.\begin{matrix}5\\3\\8\end{matrix}\right\} :: \left.\begin{matrix}7\\3\\2\end{matrix}\right\} : \left.\begin{matrix}4\\2\\x\end{matrix}\right\} ?$$

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240. When are two numbers directly proportional? When are two numbers inversely proportional? What is then said of their product? Give the first illustration of inverse proportion. Give the second.—**241.** What is a compound proportion? What is the product of the extremes equal to?—*242. How* do you find the unknown element?

SINGLE RULE OF THREE.

243. THE SINGLE RULE OF THREE is the process of finding from three given numbers, a fourth, to which one of them shall have the same ratio as exists between the other two.

1. If 8 barrels of flour cost $56, what will 9 barrels cost, at the same rate.

NOTE.—We shall denote the required term of the proportion by the letter x.

ANALYSIS.—The condition, "at the same rate," requires that the *quantity*, 8 barrels of flour, have the same ratio to the quantity, 9 barrels, as $56, the cost of 8 barrels, to x dollars, the cost of 9 barrels.

STATEMENT.

$$\begin{array}{ccccccc} \textit{bar.} & & \textit{bar.} & & \$ & & \$ \\ 8 & : & 9 & :: & 56 & : & x. \end{array}$$

$$x = \frac{56 \times 9}{8} = \$63.$$

NOTE.—It is plain that 8 barrels of flour will cost less than 9 barrels: hence, the 3d term is less than the 4th, and these terms are *directly* proportional.

2. If 36 men, in 12 days, can do a certain work, in what time will 48 men do the same work?

OPERATION.

$$48 : 36 :: 12 : x.$$

$$x = \frac{36 \times 12}{48} = 9 \text{ days.}$$

ANALYSIS.—Write the required term, x, in the 4th place, and the term 12, having the same unit value, in the third place.

Then, analyzing the question, we see that 48 men will do the work in a less time than 36 men; therefore, the 3d term will be greater than the 4th, which requires that the 1st shall be greater than the 2d. This brings 48 men into the first place, and 36 men into the 2d. The reason of this is obvious: for, the work done by 36 men in 12 days, is a *fixed quantity*. If a less number of men were employed, it will require more time to do the work: if a greater number were employed, it will require less time; hence, the *men* and *time* are *inversely proportional.*

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243. What is the single rule of three? Give the rule.

Rule – I. *Write the required term, x, in the 4th place, and the term having the same unit value in the 3d place:*

II. *Then analyze the question, and see whether the third term is greater or less than the fourth; when greater, write the greater of the remaining terms in the first place, and when less, write the less there, and the remaining term in the second place:*

III. *Then multiply the second and third terms together and divide the product by the first.*

This rule gives, when quantity and cost are considered;

quantity : quantity : : cost : cost.

When labor and time are considered,

labor : labor : : time : time.

When labor and work done are considered,

labor : labor : : work done : work done.

NOTES.—1. If the first and second terms have different units, they must be reduced to the same unit.

2. If the third term is a compound denominate number, it must be reduced to the smallest unit.

3. The preparation of the terms, and writing them in their proper places, is called the *statement*.

4. When the unknown term and the term named in connection with it, form the *extremes*, the proportion between them is *Inverse*.

Examples.

1. If 8 hats cost $24, what will 110 hats cost, at the same rate?

2. If 2 barrels of flour cost $15, what will 12 barrels cost?

3. If I walk 168 miles in 6 days, how far can I walk, at the same rate, in 18 days?

4. If 8 lb. of sugar cost $1.28, how much will 13 lb. cost?

5. If 300 barrels of flour cost $2100, what will 125 barrels cost?

6. If 120 sheep yield 330 pounds of wool, how many pounds will 36 sheep yield?

7. If 80 yards of cloth cost $340, what will 650 yards cost?

8. What is the value of 4 cwt. of sugar, at 5 cents a pound?

9. If 6 gallons of molasses cost $1.95, what will 6 hogsheads cost?

10. If 16 men consume 560 pounds of bread in a month how much will 40 men consume?

11. If a man travels at the rate of 630 miles in 12 days, how far will he travel in a leap year, Sundays excepted?

12. If 2 yards of cloth cost $3.25, what will be the cost of 3 pieces, each containing 25 yards?

13. If 3 yards of cloth cost 18s. New York currency, what will 36 yards cost?

14. If it requires eight shillings and four pence to buy eight ounces of laudanum, how many ounces can be purchased for 7s. 6d.?

15. If 5 A. 1 R. 16 P. of land, cost $150.5, what will 125 A. 2 R. 20 P. cost?

16. If 13 cwt. 2 qr. of sugar cost $129.93, what will be the cost of 9 cwt.?

17. The clothing of a regiment of 750 men cost £2834 5s.: what will it cost to clothe a regiment of 10500 men?

18. If 3¾ yards of cloth will make a coat and vest, when the cloth is 1½ yards wide, how much cloth will be needed when it is ⅝ of a yard in width?

19. If I have a piece of land 16⅘ rods long and 3⅛ rods wide, what is the length of another piece that is 7 rods wide and contains an equal area?

20. How many yards of carpeting that is three-fourths of a yard wide will carpet a room 36 feet long and 30 feet in breadth?

21. If a man can perform a journey in 8 days, walking 9 hours a day, how many days will it require if he walks 10 hours a day?

22. If a family of 15 persons have provisions for 8 months, by how many must the family be diminished that the provisions may last 2 years?

23 A garrison of 4600 men has provisions for 6 months: to what number must the garrison be diminished that the provisions may last 2 years and 6 months?

24. A certain amount of provisions will subsist an army of 9000 men for 90 days: if the army be increased by 6000, how long will the same provisions subsist it?

25. If 3 yd. 2 qr. of cloth cost $15.75, how much will 8 yd. 3 qr. of the same cloth cost?

26. If .5 of a house cost $201.5, what will .95 cost?

27. What will 26.25 bushels of wheat cost, if 3.5 bushels cost $8.40?

28. If the transportation of 2.5 tons of goods 2.8 miles costs $1.80, what is that per cwt.?

29. If $\frac{3}{4}$ of a yard of cloth cost $2.16, what will be the cost of 5$\frac{1}{2}$ pieces, each containing 447 yards?

30. If $\frac{5}{7}$ of an ounce cost $$\frac{1}{12}$, what will 1$\frac{1}{2}$ oz. cost?

31. What will be the cost of 16$\frac{4}{5}$ lb. of sugar, if 14$\frac{2}{3}$ lb. cost 1\frac{5}{6}$?

32. If 19\frac{1}{3}$ will buy 14$\frac{1}{2}$ yards of cloth, how much will 39$\frac{3}{8}$ yards cost?

33. If $\frac{7}{8}$ of a barrel of cider cost $\frac{9}{11}$ of a dollar, what will $\frac{11}{12}$ of a barrel cost?

34. If $\frac{3}{16}$ of a ship cost $2880, what will $\frac{15}{32}$ of her cost?

35. What will 116$\frac{1}{4}$ yards of cloth cost, if 462 yards cost $150.66?

36. If 6 men and 3 boys can do a piece of work in 330 ays, how long will it take 9 men and 4 boys to do the me work, under the supposition that each boy does half as uch as a man?

37. If 4 men can do a piece of work in 80 days, how ny days will 16 *men require* to do the same work?

38. If 21 sappers make a trench in 18 days, how many days will 7 men require to make a similar trench?

39. A certain piece of grass was to be mowed by 20 men in 6 days; one-half the workmen being called away, it is required to find in what time the remainder will complete the work?

40. If a field of grain be cut by 10 men in 12 days, in how many days would 20 men have cut it?

41. If 90 barrels of flour will subsist 100 men for 120 days, how long will they subsist 75?

42. If a traveller perform a journey in 35.5 days, when the days are 13.566 hours long, in how many days of 11.9 hours, will he perform the same journey?

43. If 50 persons consume 600 bushels of wheat in a year, how long would they last 5 persons!

44. A certain work can be done in 12 days, by working 4 hours each day: how many days would it require to do the same work, by working 9 hours a day?

45. If $7\frac{7}{11}$ barrels of fish cost \$$31\frac{1}{4}$, what will $32\frac{3}{8}$ barrels cost?

46. How much wheat can be bought for \$$96\frac{7}{8}$, if 2 bu. 1 pk. cost \$$1.93\frac{3}{4}$?

47. If $\frac{5}{8}$ of a yard of cloth cost \$$1\frac{5}{8}$, what will $7\frac{1}{2}$ yards cost?

48. What will be the cost of 37.05 square yards of pavement, if 47.5 yards cost \$72.25?

49. If 3 paces or common steps be equal to 2 yards, how many yards will 160 paces make?

50. If a person pays half a guinea a week for his board how long can he board for £21?

51. If 12 dozen copies of a certain book cost \$54.72 what will 297 copies cost at the same rate?

52. If an army of 900 men require \$3618 worth of provisions for 90 days, what will be the cost of subsistence.

for the same time, when the army is increased to 4500 men?

53. A grocer bought a hogshead of rum for 80 cents a gallon, and after adding water sold it for 60 cents a gallon, when he found that the selling and buying prices were proportional to the original quantity and the mixture: how much water did he add?

54. A man failing in business, pays 60 cents for every dollar which he owes; he owes A $3570, and B $1875: how much does he pay to each?

55. A bankrupt's effects amount to $2328.75, his debts amount to $3726: what will his creditors receive on a dollar?

56. If a person drinks 80 bottles of wine in 3 months of 30 days each, how much does he drink in a week?

57. If $4\frac{5}{7}$ yards of cloth cost 14s. 8d. New York currency, what will $40\frac{4}{5}$ yards cost?

58. If a grocer uses a false balance, giving only $14\frac{3}{4}$ oz. for a pound, how much will $154\frac{7}{8}$ lb. of just weight give, when weighed by the false balance?

59. If a dealer in liquors uses a gallon measure which is too small by $\frac{1}{2}$ of a pint, what will be the true measure of 100 of the false gallons?

60. After A has travelled 96 miles on a journey, B sets out to overtake him, and travels 23 miles as often as A travels 19 miles: how far will B travel before he overtakes A?

61. A person owning $\frac{5}{7}$ of a coal mine, sold $\frac{3}{4}$ of his share for $9345: what was the value of the whole mine?

62 At what time, between 6 and 7 o'clock, will the hour and minute hands of a clock be exactly together?

63 If a staff, 5 feet long, casts a shadow of 7 feet, what is the height of a steeple, whose shadow is 196 feet, at the same time *of day?*

64. A can do a piece of work in 3 days, B in 4 days, and C in 6 days: in what time will they do it, working together?

65. A can build a wall in 15 days, but with the assistance of C, he can do it in 9 days: in what time can C do it alone?

66. If 120 men can build ½ mile of wall in 15¼ days, how many men would it require to build the same wall in 40⅔ days?

67. If 3 horses, or 5 colts, eat a certain quantity of oats in 40 days, in what time will 7 horses and 3 colts consume the same quantity?

68. If a person can perform a journey in 24 days of 10½ hours each, in what time can he perform the same journey, when the days are 12¼ hours long?

69. A piece of land, 40 rods long and 4 rods wide, is equivalent to an acre: what is the breadth of a piece 15 rods long that is equivalent to an acre?

70. If a person travelling 12 hours a day finishes one-half of a journey in ten days, in what time will he finish the remaining half, travelling 9 hours a day?

71. How many pounds weight can be carried 20 miles, for the same money that 4½ cwt. can be carried 36 miles?

72. If 72 horses eat a certain quantity of hay in 7½ weeks, how many horses will consume the same in 90 weeks?

73. A watch, which is 10 minutes too fast at 12 o'clock, on Monday, gains 3 min. 10 sec. per day: what will be the time, by the watch, at a quarter-past ten in the morning of the following Saturday?

74. Two persons, A and B, are on the opposite sides of a wood, which is 536 yards in circumference; they begin to travel in the same direction at the same moment; A goes at the rate of 11 yards per minute, and B at the rate of 34 yards in 3 minutes: how many times must A go round the wood before he is overtaken by B?

DOUBLE RULE OF THREE

244. THE DOUBLE RULE OF THREE is an application of the principles of Compound Proportion.

1. If 8 men in 12 days can build 80 rods of wall, how much will 6 men build in 18 days?

ANALYSIS. — We write the required term in the 4th place, and 80 rods, having the same unit, in the 3d place. Then, since 8 men in 12 days (or 96 days' work) will build *less* wall than 6 men in 18 days (or 108 days' work), the third term will be *less* than the 4th: hence, 8 × 12 is written in the first place, and 6 × 18 in the second.

STATEMENT.

$$\left.\begin{matrix}8\\12\end{matrix}\right\} : \left.\begin{matrix}6\\18\end{matrix}\right\} :: 80 : x.$$

OPERATION.

$$x = \frac{18 \times 6 \times 80}{\cancel{12} \times \cancel{8}} = 90 \text{ rods}$$

2. If 20 men can perform a piece of work in 12 days, working 9 hours a day (that is, in 108 hours), how many men will accomplish the same work in 6 days, working 10 hours a day (that is, in 60 hours)?

ANALYSIS.—Write the required term, x, in the 4th place, and 20 men, having the same unit, in the 3d place. Since 20 men require 108 hours to do the work, *more* men will be required to do the same work in 60 hours; therefore, the third term will be *less* than the 4th; and hence, the first term must be less than the 3d: therefore, 6 × 10 is written in the first place, and 12 × 9 in the second.

STATEMENT.

$$\left.\begin{matrix}6\\10\end{matrix}\right\} : \left.\begin{matrix}12\\9\end{matrix}\right\} :: 20 : x.$$

OPERATION.

$$x = \frac{12 \times 9 \times 20}{6 \times 10} = 36 \text{ men.}$$

3. If 24 men, in 6 days, working 7 hours a day, can build a wall 115 feet long, 3 feet thick, and 4 feet high, how long a wall can 36 men build in 12 days, working 14 hours a day, if the wall is 4 feet thick and 5 feet high?

---

244. What is the double rule of three?

ANALYSIS.—In this example, an element, viz., length of wall is required. This element, denoted by x, is put in the 4th place with the other elements composing the 4th term.

STATEMENT.

$$\left.\begin{matrix}24\\6\\7\end{matrix}\right\} : \left.\begin{matrix}36\\12\\14\end{matrix}\right\} :: \left.\begin{matrix}115\\3\\4\end{matrix}\right\} : \left.\begin{matrix}x\\4\\5\end{matrix}\right\}$$

OPERATION.

$$x = \frac{36 \times 12 \times 14 \times 115 \times 3 \times 4}{24 \times 6 \times 7 \times 4 \times 5} = 414$$

Rule.—I. *Denote the required term, or element, by* x*:*

II. *Write the required term, or the term containing the required element, in the 4th place, and the term having like units with it, in the third place:*

III. *Then analyze the question, and see whether the third term is greater or less than the 4th:*

IV. *When greater, write the greater of the two remaining terms in the first place, and when less, write the less term there, and the remaining term in the second place: then find the required element as in* Art. 242.

NOTE.—Review page 215.

Examples.

1. If 2 men can dig 125 rods of ditch in 75 days, in how many days can 18 men dig 243 rods?

2. If 400 soldiers consume 5 barrels of flour in 12 days, how many soldiers will consume 15 barrels in 2 days?

3. If a person can travel 120 miles in 12 days of 8 hours each, how far will he travel in 15 days of 10 hours each?

4. If a pasture of 16 acres will feed 6 horses for 4 months how many acres will feed 12 horses for 9 months?

5. If 60 bushels of oats will feed 24 horses 40 days, how long will 30 bushels feed 48 horses?

6. If 82 men build a wall 36 feet long, 8 feet high, and 4 feet thick, in 4 days; in what time will 48 men build a wall *864 feet long* 6 feet high, and 3 feet wide?

7. If the freight of 80 tierces of sugar, each weighing 3½ hundredweight, for 150 miles, is $84, what must be paid for the freight of 30 hogsheads of sugar, each weighing 12 hundredweight, for 50 miles?

8. A family consisting of 6 persons, usually drink 15.6 gallons of beer in a week: how much will they drink in 12 5 weeks, if the number be increased to 9?

9. If 12 tailors in 7 days can finish 14 suits of clothes. how many tailors in 19 days can finish the clothes of a regiment of 494 men?

10. If a garrison of 3600 men eat a certain quantity of bread in 35 days, at 24 ounces per day to each man, how many men, at the rate of 14 ounces per day, will eat twice as much in 45 days?

11. A company of 100 men drank £20 worth of wine at 2s. 6d. per bottle: how many men, at the same rate, will £7 worth supply, when wine is worth 1s. 9d. per bottle?

12. A garrison of 3600 men has just bread enough to allow 24 oz. a day to each man for 34 days; but a siege coming on, the garrison was reinforced to the number of 4800 men: how many ounces of bread a day must each man be allowed, to hold out 45 days against the enemy?

13. Bought 5000 planks, 15 feet long and 2½ inches thick; how many planks are they equivalent to, of 12½ feet long and 1¼ inches thick?

14. If 12 pieces of cannon, eighteen-pounders, can batter down a castle in 3 hours, in what time would nine twenty-four-pounders batter down the same castle, both pieces of cannon being fired the same number of times, and their balls flying with the same velocity?

15. If the wages of 13 men for 7½ days, be $149.76, what will be the wages of 20 men for 15⅓ days?

16. If a footman travel 264 miles in 6¾ days of 12¼ hours

each, in how many days of $10\frac{2}{7}$ hours each will he travel $129\frac{3}{8}$ miles ?

17. If 120 men in 3 days, of 12 hours each, can dig a a trench of 30 yards long, 2 feet broad, and 4 feet deep, how many men would be required to dig a trench, 50 yards long, 6 feet deep, and $1\frac{1}{2}$ yards broad, in 9 days of 15 hours each?

18. If a stream of water running into a pond of 175 acres, raises it 10 inches in 15 hours, how much would a pond of 80 acres be raised by the same stream in 9 hours?

19. A person having a journey of 500 miles to perform, walks 200 miles in 8 days, walking 12 hours a day: in how many days, walking 10 hours a day, will he complete the remainder of the journey ?

20. If 1000 men, besieged in a town, with provisions for 28 days, at the rate of 18 ounces per day for each man, be reinforced by 600 men, how many ounces a day must each man have that the provisions may last them 42 days ?

21. If a bar of iron 5 ft. long, $2\frac{1}{2}$ in. wide, and $1\frac{3}{4}$ in. thick weigh 45 lb., how much will a bar of the same metal weigh that is 7 ft. long, 3 in. wide, and $2\frac{1}{4}$ in. thick ?

22. If 5 compositors in 16 days, working 14 hours a day. can compose 20 sheets of 24 pages each, 50 lines in a page, and 40 letters in a line, in how many days, working 7 hours a day, can 10 compositors compose 40 sheets of 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

23. Fifty thousand bricks are to be removed a given distance in 10 days. Twelve horses can remove 18000 in 6 days: how many horses can remove the remainder in 4 days?

24. If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep, in how many days, of 9 hours long, will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{5}{8}$ wide, and $3\frac{1}{3}$ deep ?

PARTNERSHIP.

245. A **Partnership**, or **Firm**, is an association of two or more persons, under an agreement to share the profits and losses of business.

Partners are the persons thus associated.

246. **Capital**, or **Stock**, is the amount of money or property contributed by the partners, and used in the business.

Profit is the increase of capital between two given dates.

Loss is the decrease of capital between two given dates.

Dividend is the amount of profit apportioned to each partner.

247. **Assets** of a Firm, are its cash on hand, property, and all debts due to it.

248. **Liabilities** of a Firm, embrace all the debts which it owes, and all its indorsements.

249. **Solvency** is when the assets exceed the liabilities.

250. **Insolvency** is when the liabilities exceed the assets.

251. An **Assignment** is a transfer of the assets of an insolvent person or firm to others, for the benefit of creditors.

252. **Assignees** are the persons to whom such transfer is made.

253. **When the capital of each partner is employed for the same time.**

Since the profit arises from the use of the capital, each man's share of it should be proportional to his amount of stock. Hence,

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245. **What is a partnership or firm?** What are partners?—246. What is capital or stock? What is profit? What is loss? What is a dividend?—247. What are assets?—248. What are liabilities?—249. What is solvency?—250. What is insolvency?—251. What is an assignment? 252. *What are assignees?—253.* What is the rule, when each man's *capital is employed for the same time?*

Rule.—*As the whole stock is to each man's stock, so is the whole gain or loss to each man's share of the gain or loss.*

Examples.

1. Mr. Jones and Mr. Wilson form a copartnership, the former putting in $1250, and the latter $750: at the end of the year there is a profit of $720: what is the share of each?

$$\begin{array}{r} 1250 \\ 750 \\ \hline 2000 \end{array}$$

STATEMENT.

2000 : 1250 : : 720 : x = Jones' share = $450.

2000 : 750 : : 720 : x = Wilson's share = $270.

OPERATION.

$$\frac{\overset{25}{\cancel{1250}} \times \overset{18}{\cancel{720}}}{\underset{40}{\cancel{2000}}} = x = \$450. \qquad \frac{\overset{15}{\cancel{750}} \times \overset{18}{\cancel{720}}}{\underset{40}{\cancel{2000}}} = x = \$270.$$

2. A, B, and C, entered into partnership with a capital of $7500, of which A put in $2500, B put in $3000, and C put in the remainder; at the end of the year their gain was $3000: what was each one's share of it?

3. A and B have a joint stock of $4200, of which A owns $3600, and B, $600; they gain, in one year, $2000: what is each one's share of the profits?

4. A, B, C, and D, have $40000 in trade, each an equal share; at the end of six months their profits amount to $16000: what is each one's share, allowing A to receive $50, and D, $30, out of the profits, for extra services?

5. Three merchants loaded a vessel with flour; A loaded 500 barrels, B, 700 barrels, and C, 1000 barrels; in a storm at sea it became necessary to throw overboard 440 barrels: what was each one's share of the loss?

6. A man bequeathed his estate to his four sons, in the following manner, viz.: to his first, $5000, to his second, $4500,

to his third, \$4500, and to his fourth, \$4000. But on settling the estate, it was found that after paying the debts and expenses, only \$12000 remained to be divided: how much should each receive?

7. A widow and her two sons receive a legacy of \$4500 of which the widow is to have $\frac{1}{2}$, and the sons, each $\frac{1}{4}$. But the elder son dying, the whole is to be divided in the same proportion between the mother and younger son: what will each receive?

8. Four persons engage jointly in a land speculation; D puts in \$5499 capital. They gain \$15000, of which A takes \$4320.50, B, \$5245.75, and C, \$3600.75: how much capital did A, B, and C put in, and what is D's share of the gain?

9. A steam-mill, valued at \$4300, was entirely destroyed by fire. A owned $\frac{1}{4}$ of it, B $\frac{1}{3}$, and C the remainder; supposing it to have been insured for \$2500, what was each one's share of the loss?

10. A copartnership is formed with a joint capital of \$16970. A puts in \$5 as often as B puts in \$7, and as often as C puts in \$8; their annual gain is equal to C's stock: what is each person's stock and gain?

11. A man failing in business is indebted to A, \$475.50, to B, \$362.12½, to C, \$250.87½, and to D, \$140. He is worth only \$614.25: to how much is each entitled?

12. Four persons, A, B, C, and D, agreed to do a piece of work for \$270. They were to do the work in the proportions of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{7}{15}$: what should each receive for his work?

13. A, B, and C, form a copartnership, with a capital of \$50000, of which A puts in \$18500, B, \$24650, and C, the remainder. C, on account of his superior knowledge of the business, was to receive $\frac{1}{16}$ of all the profits, exclusive of his share. At the end *of the* year, the net profit is \$7360: *what should each receive?*

14. Two merchants, A and B, form a copartnership. A contributes $10500, and B, $16500. At the end of the year, the assets are $29400, and the liabilities $4750. Now, supposing the partnership to continue, with what capital does each partner commence the new year?

15. Three persons buy a piece of land for $4569, and the parts for which they pay bear the following proportions to each other, viz.: the sum of the first and second, the sum of the first and third, and the sum of the second and third, are to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{7}{10}$: how much did each pay, and what part did each own?

254. When the capital is employed for unequal times.

When the partners employ their capital for *unequal times*, the profits of each will depend on two circumstances:

1st, *On the amount of capital he puts in*; and

2dly, *On the length of time it is continued in business*:

Therefore, the profit of each will depend on the product of these two elements. The whole profit will be proportional to the sum of these products. Hence, the following

Rule.—*Multiply each man's capital by the time he continued it in the firm; then say, the sum of the products is to each product, so is the whole gain or loss to each man's share.*

Examples.

1. A put in trade $500 for 4 months, and B $600 for 5 months. They gained $240: what was the share of each?

OPERATION.

A's cap. $500 \times 4 = 2000$

B's cap. $600 \times 5 = \underline{3000}$

Sum of products $= 5000 : 2000 :: 240 : x =$ $96, A's.

$5000 : 3000 :: 240 : x =$ $144, B's.

2. *Three men hire a pasture for $70.20: A put in 7 horses*

for 3 months; B, 9 horses for 5 months; and C, 4 horses for 6 months: what part of the rent should each pay?

3. A commenced business with a capital of $10000. Four months afterwards B entered into partnership with him, and put in 1500 barrels of flour. At the close of the year their profits were $5100, of which B was entitled to $2100: what was the value of the flour per barrel?

4 On the 1st of January, 1864, A commenced business with a capital of $23000; two months afterwards he drew out $1800; on the 1st of April, B entered into partnership with him, and put in $13500; four months afterwards he drew out $10000; at the end of the year their profits were $8400: how much should each receive?

5. Three persons divided their profits to the amount of $798. A put out $4000 for 12 months; B, $3000 for 15 months; and C, $5000 for 8 months: to what part of the profits was each entitled?

6. Three persons, C, D, and E, form a copartnership; C's stock is in trade 3 months, and he claims $\frac{1}{12}$ of the gain; D's stock is in 9 months; and E put in $756 for 4 months, and claims $\frac{1}{2}$ of the profits: how much did C and D put in?

7. Two persons form a partnership for one year and six months. A, at first, put in $3000 for 9 months, and then $1000 more. B, at first, put in $4000, and at the end of the first year, $500 more; but at the end of 15 months, he drew out $2000. At the end of 12 months, C was admitted as a partner with 7333\frac{1}{3}$. The gain was $7400: how much should each man receive?

8. Three men take an interest in a mining company. A put in $480 for 6 months; B, a sum not named for 12 months; and C, $320 for a time not named: when the accounts were settled, A received $600 for his stock and profits; B, $1200 for his; and C, $520 for his what was B's stock, and C's time?

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254. *What is the rule* when the capital is employed for unequal times

PERCENTAGE.

255. Per cent. means by the hundred. Thus, 1 per cent of a number is one-hundredth of it; 2 per cent. is two-hundredths of it; 3 per cent. three-hundredths, &c.

256. The Rate per cent. is the number of hundredths taken. Thus, if 1 hundredth is taken, the rate is 1 per cent.; if 2 hundredths are taken, the rate is 2 per cent.; if 3 hundredths, the rate is 3 per cent., &c.

257. The Base is the number whose part is taken.

258. The Percentage is the result of the operation, and the part of the base taken.

The rate per cent. is generally expressed decimally; thus,

1 per cent. of a number, is $\frac{1}{100}$ of it = .01 of it.
2 per cent. of a number, is $\frac{2}{100}$ of it = .02 of it.
25 per cent. of a number, is $\frac{25}{100}$ of it = .25 of it.
50 per cent. of a number, is $\frac{50}{100}$ of it = .50 of it.
100 per cent. of a number, is $\frac{100}{100}$ of it = 1 time it.
200 per cent. of a number, is $\frac{200}{100}$ of it = 2 times it.
½ per cent. of a number, is $\frac{\frac{1}{2}}{100}$ of it = .005 of it.
¾ per cent. of a number, is $\frac{\frac{3}{4}}{100}$ of it = .0075 of it.
75 per cent. of a number, is $\frac{.75}{100}$ of it = .0075 of it.
.8½ per cent. of a number, is $\frac{.8\frac{1}{2}}{100}$ of it = .0085 of it.

Note.—Per cent. is often expressed by the character %. Thus 5 per cent. is written 5%; 8 per cent., 8%.

Write, decimally, 5%; 8%; 15½%; 100%; 204%; 327½%, 672.3%; 49%; and 507.5%.

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255. What is the meaning of per cent.? What is 5 hundredths of a number?—256. What is the rate per cent.? If four hundredths of a number is taken, what is the rate?—257. What is the base?—258. *What is the percentage?* How is the rate per cent. generally expressed?

259. To find the percentage, when the base and rate are known.

1. What is the percentage of $450, the rate being 6 per cent.?

ANALYSIS.—The rate, expressed decimally, is .06. The percentage is, therefore, six hundredths of the base, or the *product of the base and rate*. Hence, to find the percentage of any number:

OPERATION

$$\begin{array}{r} 450 \\ .06 \\ \hline \$27.00 \end{array} \; Ans$$

Rule.—*Multiply the base by the rate, and the product will be the percentage.*

Examples.

Find the percentage of the following numbers:

1. 4 per cent. of $1256.
2. 12% of $956.50.
3. ¼ per cent. of 475 yards.
4. ⅞% of 324.5 cwt.
5. ⅘% of 125.25 lbs.
6. 1⅗ per cent. of 750 bush.
7. 4½% of $2000.
8. 9 per cent. of 186 miles.
9. 10⅜ per cent. of 460 sheep.
10. 5⅒ per cent. of 540 tons.
11. 8⅔ per cent. of $3465.75.
12. 12½% of 126 cows.
13. 50 per cent. of 320 bales.
14. 37½ per cent. of 1275 yds.
15. 95% of $4573.
16. 105 per cent. of 2500 bar.
17. 112½% of $4573.
18. 250 per cent. of $5000.
19. 305% of $1267.87½.
20. 500 per cent. of $3000.
21. What is 3% of $765?
22. What is 4½% of 960 bush?

23. What is the difference between 4¾% of $1000 and 7½ per cent. of $1500?

24. If I buy 895 gallons of molasses, and lose 17 per cent by leakage, how much have I left?

25. A grocer purchased 250 boxes of oranges, and found that he had lost in bad ones 18 per cent.: how many full boxes of good ones had he left?

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259. How do you find the percentage from the base and rate?

260. Parts of percentage.

There are three parts in percentage: 1st. The Base; 2d The Rate; and 3d. The Percentage.

261. To find the rate, when the base and percentage are known

1. What per cent. of $64 is $16? or, $16 is what part of 64?

OPERATION.

$\frac{16}{64} = \frac{1}{4} = .25$, or 25 per cent.

ANALYSIS.—In this example, 16 is the percentage, and 64 the base, and the rate is required. Since the percentage is equal to the base multiplied by the rate, the rate is equal to the quotient of the percentage divided by the base.

Rule.—*Divide the percentage by the base, and the first two decimal places will express the rate.*

Examples.

1. What per cent. of 10 dollars is 2 dollars?
2. What per cent. of 32 dollars is 4 dollars?
3. What per cent. of 40 pounds is 3 pounds?
4. Seventeen bushels is what per cent. of 125 bushels?
5. Thirty-six tons is what per cent. of 144 tons?
6. What per cent. is $84 of $96?
7. What per cent. is $\frac{1}{2}$ of $\frac{7}{8}$?
8. What per cent. is 3 miles of 400 miles?
9. Four and one-third is what per cent. of $9\frac{1}{3}$?
10. One hundred and four sheep is what per cent. of a drove of 312 sheep?
11. A grocer has $325, and purchases sugar to the amount of $121.87½: what per cent. of his money does he expend?
12. Out of a bin containing 450 bushels of oats, 56¼ bushels were sold: what per cent. is this of the whole?

NOTE.—If the base be regarded as a single thing, and denoted by 1, a fractional percentage expressed decimally will denote the rate.

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260. *How many* parts are there in percentage? What are they?
261. *How do you* find the rate, from the base and percentage?

13. $\frac{7}{8}$ of a number is what per cent. of the number?

14. $\frac{4}{5}$ of a ship is what per cent. of the ship?

15. $\frac{7}{10}$ of 50 is what per cent. of 50?

16. $\frac{2}{3}$ of a cargo is what per cent. of it?

17. $1\frac{3}{5}$ times a number is what per cent. of the number?

262. To find the base, when the rate and percentage are known.

1. Of what number is $960, 16 per cent.?

ANALYSIS.—By Art. 259, the percentage is equal to the base multiplied by the rate; hence, to find the base,

OPERATION.

$960 \div .16 = 6000$

Rule.—*Divide the percentage by the rate, expressed decimally.*

Examples.

2. The number 475 is 25% of what number?

3. The number $87\frac{1}{2}$ is $12\frac{1}{2}$% of what number?

4. Five hundred and sixty dollars is 140% of what number?

5. The number 75 is $\frac{1}{3}$% of what number?

6. One dollar and twenty-five cents is $\frac{7}{8}$% of what number?

7. The fraction $\frac{7}{8}$ is 45% of what number?

8. The fraction $\frac{3}{4}$ is $\frac{5}{6}$% of what number?

9 If a person receives $5850, and that sum is 75% of what is due him, what is the debt?

10. A bankrupt can pay only $37\frac{1}{2}$ per cent. of his debts: what did he owe to that merchant to whom he paid $1647?

11. In an army, 15600 men are mustered after a battle, in which 25% were killed and wounded: what was the original number of men?

263. AMOUNT is the base plus the percentage.

264. DIFFERENCE is the base minus the percentage.

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262. How do *you find the* base, when the rate and percentage are *known?—263. What is the* amount?—264. What is the difference?

265. Knowing the amount and rate, to find the base.

1. Mr. Jones buys 8 hogsheads of sugar, sells them at an advance of 15 per cent., and receives $470 : what did he pay for the sugar ?

ANALYSIS.—The amount received, $470, arises from *adding* the percentage to the base; that is, *it arises from multiplying the base by* 1 – *plus the rate per cent.;* hence, to find the base,

OPERATION.
1.15)470(400
460

Rule.—*Divide the amount by* 1 *plus the rate, expressed decimally.*

Examples.

1. What number is that, to which if 1 per cent. of it be added, the sum will be 392 ?

2. What was the population of a city in 1850, which increased 15 per cent., and in 1860 contained a population of 10350?

3. A farmer bought a flock of sheep, which in three years increased 96 per cent., and then numbered 1078 : how many did he buy ?

4. Mr. Jones sold his farm for $1008, which was thirty-three and one-third per cent. more than he gave for it : what did it cost him ?

5. A man bought a horse and cow ; he paid for the horse one hundred and twenty-two dollars, which was three hundred and five per cent. more than he paid for the cow : how much did the cow cost him ?

NOTE.—If the rate per cent. exceeds one hundred, the whole number, to which the rate is added, must be increased 1 for each hundred.

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265. How do you find the base, when the amount and rate are *known ?*

6. A crop of wheat, worth $1036, exceeds the value of the corn crop by 40 per cent.: what was the value of the corn crop?

7. A merchant, in closing up his business, had $85000; he had doubled his capital, and in addition, he had made twenty-five per cent. of it: what was the amount of his capital?

8. A man's income is $9180, arising from labor and investments: his investments yield 70 per cent. of his labor: what is the value of his labor?

9. A farmer bought 40 sheep, and after keeping them for one year, sold them at an advance of 55 per cent., and received $248: what did he pay for the sheep per head?

10. A merchant bought a lot of goods and marked them at an advance of 26 per cent.: when sold, he found that they brought him $6835.50: what did the goods cost him?

266. Knowing the difference and rate, to find the base.

1. A cask of wine, out of which 37 per cent. had leaked, was found to contain 33.39 gallons: how many gallons did the cask contain?

ANALYSIS.—Thirty-seven per cent. denotes .37 of the capacity of the cask; and hence, the part of the cask that is filled is denoted by 1 — .37 = .63. But .63 of the cask contains 33.39 gallons; therefore, the entire cask will contain as many gallons as .63 is contained times in 33.39, viz., 53; hence, to find the base,

OPERATION.

$$1 - .37 \text{ per cent.} = .63 \text{ per cent.}$$

$$.63 : 1 :: 33.39 : x \text{ gal.}$$

$$x = \frac{33.39}{.63} = 53 \text{ gal.}$$

Rule.—*Divide the difference by the difference between* 1 *and the rate, expressed decimally.*

Examples.

1. James, after giving John 15% of his marbles, has 136 left: how *many had he at first?*

2. Mr. Wilson lost 18 per cent. of his sheep by disease, and had a flock of 615 left: how many had he at first?

3. Mr. Jones invests 46% of his capital in land, and has $513 left: what is his capital?

4. An army fought two battles; in the first it lost 15 per cent., and in the second, 20 per cent. of the original number; after which it mustered 19500 men: what was its original strength?

5. A grocer bought a quantity of provisions, but finding them damaged, sold them at a loss of 19 per cent., and received $10935: what did they cost him?

6. A son, who inherited a fortune, spent 37½ per cent. of it, when he found that he had only $31250 remaining: what was the amount of his fortune?

7. A grocer purchased a lot of teas and sugar, on which he lost 16 per cent. by selling them for $4200: what did he pay for the goods?

8. A speculator invested in stocks, which, falling rapidly in price, he sold out at a loss of 13 per cent., and received $2262: what was the amount of his purchase?

267. Formulas of percentage.

Nearly every practical question, in Arithmetic, is a particular case of one or other of the five operations of Percentage: hence, we write the formulas:

1. $\text{Percentage} = \text{Base} \times \text{Rate}$ - - - - - - Art. 259.

2. $\text{Rate} = \dfrac{\text{Percentage}}{\text{Base}}$ - - - - - - Art. 261.

3. $\text{Base} = \dfrac{\text{Percentage}}{\text{Rate}}$ - - - - Art. 262.

4. $\text{Base} = \dfrac{\text{Amount}}{1 + \text{Rate}}$ - - - - - - Art. 265.

5. $\text{Base} = \dfrac{\text{Difference}}{1 - \text{Rate}}$ - - - - - Art. 266.

PROFIT AND LOSS.

268. **Profit and Loss** are commercial terms, indicating gain or loss in business transactions. The gain or loss is always estimated on the cost price.

The cost of an article is the amount paid for it.

The selling price of an article is the amount received for it.

The *cost* is the *base;* the gain or loss is the *percentage;* the rate per cent. of gain or loss is the *rate;* the selling price is the sum of the base and percentage, when there is a *gain*, and their *difference* when there is a *loss*.

The following examples may all be wrought by the five formulas and rules of Percentage:

Examples.

1. Bought 9 barrels of sugar, each weighing 250 pounds, at 7 cents a pound: how much profit would be made if it were sold at 8½ cents per pound?

2. If 15 pieces of muslin, each containing 43 yards, cost 27 cents per yard, what would be the gain if sold at 31¼ cents per yard?

3. A farmer bought a flock of 360 sheep; their keeping for 1 year cost $0.75 a head; their wool was worth 1 dollar and 25 cents a head, and one-fourth of them had lambs, each of which was worth one-half as much as a fleece: what was the profit of the purchase at the end of the year?

4. A merchant bought 65 barrels of flour, at $5¼ per barrel, and sold them so as to gain $42.50: what was the price per barrel?

5. A person bought 500 bushels of potatoes, at 62½ cents

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268 What do you understand by the terms profit and loss? What is cost? What is the selling price? What is the base? What is the *gain or loss?*

per bushel, and sold them so as to gain $35: at what price were they sold?

6. A house and lot were bought for $6450. The house was repaired at an expense of $575, painted for $796, and was then sold so as to gain $945: what was the price of the house and lot?

7. If in 3 hogsheads of molasses, which cost $68.04, one third leaked out, what must the remainder be sold for per gallon to realize a profit of $2.52 on the whole?

8. If a merchant's profits are 22 per cent. on the cost of the goods sold, what is his profit on $4162.50?

9. A quantity of goods were bought for $3612; the charges on them were $54: they were sold at an advance of 20%: what was the profit?

10. A merchant, on taking an inventory of stock, finds it worth $37649: what would be his profit on this stock, if sold at an advance of 31¼%?

11. A merchant bought goods to the amount of $2965; but being damaged, he sold them at a loss of 15%: what was the amount of his loss?

12. A quantity of flour was bought for $8550; $\frac{1}{10}$ of the flour was so damaged as to be sold at a loss of 12%; ¼ of it was sold at a profit of 19%; and the remainder at a profit of 30%: what was the net profit on the flour?

13. If sugar costs 16 cents per pound, and is sold at an advance of 12½%, what is the profit per lb.?

14. A bank, whose capital is $200000, after reserving $2860 for a surplus fund, declared a dividend of 8% on the capital: what were the entire profits?

15. The profits of a merchant averaged 25% of his capital, and his expenses are 5% of his profits: what part of the capital were the expenses?

16. A farmer sells 375 bushels of corn for 75 cents a bushel;

the purchaser sells it at an advance of 20 per cent.: how much a bushel did he receive for the corn?

17. A merchant buys a pipe of wine, for which he pays $322.56, and he wishes to sell it at an advance of 25 per cent.: what must he sell it for per gallon?

18. A man bought 3275 bushels of wheat, for which he paid $3493.33⅓, but finding it damaged, is willing to lose 10 per cent.: what must he sell it for per bushel?

19. If I purchase two lots of land for $150.25 each, and sell one for 40 per cent. more than it cost, and the other for 28 per cent. less, what is the gain on the two lots?

20. Bought a cask of molasses containing 144 gallons, at 45 cents a gallon, 36 gallons of which leaked out: at what price per gallon must I sell the remainder to gain 10 per cent. on the cost?

21. A person in Chicago bought 3500 bushels of wheat, at $1.20 a bushel: allowing 5 per cent. on the cost for risk in transportation, 3 per cent. for freight, and 2 per cent. commission for selling, what must it be sold for per bushel in New York to realize 40 per cent. net profit on the purchase?

22. Bought a quantity of goods for $348.50, and sold the same for $425: what per cent. did I make on the amount received?

23. Bought a piece of cotton goods for 6 cents a yard, and sold it for 7½ cents a yard: what was my gain per cent.?

24. If I buy rye for 90 cents a bushel, and sell it for $1.20, and wheat for $1.12½ a bushel, and sell it for $1.50 a bushel, upon which do I make the most per cent.?

25. If paper that cost $2 a ream, be sold for 18 cents a quire what is gained per cent.?

26. How much per cent. would be made upon a hogshead of sugar weighing 13 cwt. 3 qr. 14 lb., that cost $8 per cwt., if sold at 10 cents per pound?

27. A hardware merchant bought 45 T. 16 cwt. 25 lb. of iron,

at $75 per ton, and sold it for $78.50 per ton: what was his whole gain, and how much per cent. did he make?

28. A merchant buys 67560 feet of lumber for $7000: the expense of cartage and piling was $425, and the loss of material amounted to $216. If the lumber be sold at $97.50 per 1000 feet, what will be the entire loss?

29. A gentleman, having gold coin to the amount of $475 sold it for bank bills and obtained $593.75: what was the rate per cent. of premium on gold, and what the rate per cent. of depreciation on the bills?

30. In selling a quantity of wheat, a merchant gained $500 when his rate of profit was 31%: what was the cost?

31. In the course of 6 months a merchant gained $3745. what amount of goods must he have sold, allowing a gain of 25%?

32. The net profits of a shoe-dealer were $2965, and his expenses were $1260. If the rate of profit were 40%, what amount of goods were sold?

33. What must be the annual sales of a merchant, that he may realize $4500, after paying $2500 expenses, when his rate of profit is 35%?

34. The surplus fund of an insurance company, amounting to $32500, will pay 12½% on its capital: what is the capital?

35. The profits of a bank are 12% of its capital; the expenses are 10% of the profits: what % of the capital are the expenses?

36. A grocer sold a lot of sugars for $477.12, which was an advance of 12% on the cost: what was the cost?

37. Mr. A. bought a lot of sugars, but finding them of an inferior qaulity, sold them at a loss of 15%, and found that they brought $340: what did they cost him?

38. I sold a parcel of goods for $195.50, on which I made 15%: what did they cost me?

39. Sold 78 cwt. 3 qr. 14 lb. of sugar, at 8 cents a pound, and gained 15%: how much did the whole cost?

40. A dealer sold two horses for $472.50 each, and gained on one 35%, but lost 10% on the other: what was the cost of each, and what was his net gain?

41 A merchant having a lot of flour, asked $33\frac{1}{3}$% more than it cost him, but was obliged to sell it $12\frac{1}{2}$% less than his asking price: he received $7 per bbl.: what was the cost per bbl.?

42. If a merchant in selling a quantity of merchandise for $3850, loses 12% of the cost, what was the cost?

43. If 25 per cent. be gained on flour when sold at $10 a barrel, what per cent. would be gained when sold at $11.60 a barrel?

NOTE.—In this class of examples, first find the cost, as in Art. 267; then find the gain, or loss; and then divide by the number on which the per cent. is reckoned.

44. A lumber-dealer sold 25650 feet of lumber at $19.20 a thousand, and gained 20 per cent.: how much would he have gained or lost had he sold it at $15 a thousand?

45. A man sold his farm for $3881.25, by which he gained $12\frac{1}{2}$ per cent. on its cost: what was its cost, and what would he have gained or lost per cent. if he had sold it for $3277.50?

46. If a merchant sells tea at 66 cents a pound, and gains 20 per cent., how much would he gain per cent. if he sold it at 77 cents a pound?

47. Sold 5520 bushels of corn at 50 cents a bushel, and lost 8 per cent.: how much per cent. would have been gained had it been sold at 60 cents a bushel?

48. A grocer bought 3 hogsheads of sugar, each weighing $1112\frac{1}{2}$ pounds; he sold it at 11 cents a pound, and gained $37\frac{1}{2}$ per cent.: what was its cost, and for how much should he have sold it to gain 50 per cent. on the cost?

COMMISSION.

269. Commission is an allowance made to an agent for a transaction in business, and is reckoned at a certain rate per cent. on the amount of money used.

270. A Commission Merchant is one who sells or buys goods for another.

271. A consignment is a quantity of goods sent to a merchant for sale.

A consignor is the one who sends the goods.

A consignee is the one to whom the goods are sent.

Note.—The commission for the purchase or sale of goods, in the city of New York, varies from 2½ to 12½ per cent.; and, under some circumstances, even higher rates are paid. For the sale of real estate the rates are lower, varying from one-quarter to 2 per cent.

All the cases of Commission come under the rules and formulas of Percentage.

Examples.

1. A commission merchant sold a lot of goods, for which he received $7540; he charged 2½ per cent. commission: what was his commission, and how much must he pay over?

2. A commission merchant received $1399.77 to be invested in goods; he is to have 3 per cent. on the amount purchased: what is the amount of the purchase, and what the commission?

3. An auctioneer sold a house for $3125, and the furniture for $1520: what was his commission at ¾ per cent.?

4. A flour merchant sold 750 barrels of flour, at $9.75 a barrel: what was his commission at 2¼ per cent.?

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269. What is commission? How is it reckoned?—270. What is a commission merchant?—271. What is a consignment? What is a consignor? What is a consignee?

5. I sold at auction 96 hogsheads of sugar, each weighing 9 cwt. and 50 lb., at $6.50 per hundred: what was the auctioneer's commission at 1⅝%, and to how much was I entitled?

6. An agent purchased 2340 bushels of wheat at $1.75 a bushel, and charged 2⅗ per cent. for buying, 1⅖ per cent. for shipping, and the freight cost 2 per cent.: what was his commission, and what did the wheat cost the owner?

7. A town collector received 4½ per cent. for collecting a tax of $2564.25: what was the amount of his percentage?

8. I paid an attorney 6⅝ per cent. for collecting a debt of $7320.25: how much did I receive?

9. My commission merchant sold goods to the amount of $1000, on which I allowed him 5 per cent.; but as he paid over the money before it became due, I allowed him 1½ per cent. more: how much am I to receive?

10. A dairyman sent an agent 3476 pounds of cheese, and allowed him 3⅛ per cent. for selling it: how much would he receive after deducting the commission, if it were sold for 12½ cents per pound?

11. A person has $1500 in bills of the State Bank of Indiana, upon which there is a discount of 2½ per cent., and $1000 of the bank of Maryland, upon which there is a discount of 3¼ per cent.: what will be the loss in changing the amount into current money?

12. I am obliged to sell $2640 in bills on the bank of Delaware, upon which there is a discount of 2¾ per cent.: how much bankable money should I receive?

13. A merchant in New York received a consignment of 275 bbl. of flour, which he sold at $4.75 per bbl. He charged a commission of 2%, ¼% for storage, and ¾% for guarantee What were the charges, and what amount was transmitted to the consignor?

NOTE.—2 *per cent.* + ¼ per cent. + ¾ per cent. = 3 per cent

14. A commission merchant in New York receives $12000 for the purchase of sugar. He charges 2% commission. What amount is laid out in purchasing sugar, and what is the commission?

15. A factor receives $708.75, and is directed to purchase iron at $45 a ton; he is to receive 5 per cent. on the money paid: how much iron can he purchase?

16. I forward $2608.625 to a commission merchant in Chicago, requesting him to purchase a quantity of corn; he is to receive 2½ per cent. on the purchase: what does his commission amount to, and how much corn can he buy with the remainder, at 56 cents a bushel?

17. My agent at Havana purchased for me a quantity of sugar at 6¼ cents a pound, for which I allow him a commission of 1⅘ per cent. His commission amounts to $42.66: how many barrels of sugar of 240 pounds each did he purchase, and how much money must I send him to pay for it, including his commission?

18. A merchant in New Orleans received $187.50, to be laid out in the purchase of cotton. After allowing for commission at 2%, freight at ⅛%, insurance at ⅛%, and incidental expenses $\frac{1}{10}$%, what amount was expended in the purchase of cotton, and what was the commission?

19. A commission merchant, in selling a quantity of merchandise for $2785, received a commission of $60: what was the rate of commission?

Note.—In this example, the base and percentage are given, and the rate is required (Art. 267).

20. A land agent received $175 for selling a house for $6795: what was his rate of commission?

21. A collecting agent received $15 for collecting a debt of $175: what was his rate of commission?

22. A miller received for his toll 5 bushels on every 45 bushels of grain that he ground: what was the rate?

INTEREST.

272. **Interest** is a percentage paid for the use of money.

Principal, or **base**, is the money on which interest is paid.

Rate of interest is the per cent. paid per year.

Amount is the sum of the principal and interest.

Per annum means by the year.

273. In interest, by general custom, a year is reckoned at 12 months, each having 30 days. The Rate of Interest is generally fixed by law, and is called *Legal Interest.* Any rate above the legal rate is *usury,* and is generally forbidden by law.

274. In most of the States, the legal rate is 6 per cent.; in New York, South Carolina and Georgia, it is 7%; and in some of the other States the rate is fixed as high as 10 per cent.

275. There are five parts in interest: 1st, principal; 2d, rate; 3d, time; 4th, interest; 5th, amount.

CASE I.

276. To find the interest of any principal for one or more years

1. What is the interest of $3920 for 2 years, at 7 per cent.?

Analysis.—In this example, the base is $3920, the rate 7%, and the interest for 1 year is the percentage: this product multiplied by 2, the number of years, gives the interest for 2 years; hence,

OPERATION.

$3920	
.07	rate.
$274.40	int. for 1 year.
2	No. of years.
$548.80	interest.

Rule.—*Multiply the principal by the rate, expressed decimally, and the product by the number of years.*

Examples.

1. What is the interest of $675 for 1 year, at 6½ per cent.?
2. What is the interest of $871.25, for 1 year, at 7%?
3 What is the interest of $535.50, for 7 years, at 6%?
4 What is the interest of $1125.885, for 4 years, at 8%
5 What is the interest of $789.74, for 12 years, at 5%?
6 What is the interest of $2500, for 7 years, at 7½%?
7. What is the interest of $3153.82, for 2 years, at 4½%?
8. What is the amount of $199.48, for 16 years, at 7%?
9. What is the amount of $897.50, for 3 years, at 8%?
10. What is the interest of $982.35, for 4 years, at 6¾%?
11. What is the amount of $1500, for 5 years, at 5¼%?
12. What is the interest of $1914.10, for 6 years, at 3¼%?
13. What is the interest of $350, for 21 years, at 10%?
14. What is the amount of $628.50, for 5 years, at 12⅓%?
15. What is the amount of $75.50, for 10 years, at 6%?
16. What is the amount of $5040, for 2 years, at 7½%?

Note.—When there are years and months, and the months are an aliquot part of a year, *multiply the interest for 1 year by the years and the months, reduced to the fraction of a year.*

17. What is the interest of $119.48, for 2 yrs. 6 mo., at 7%?
18. What is the interest of $250.60, for 1 yr. 9 mo., at 6%?
19. What is the interest of $956, for 5 yrs. 4 mo., at 9%?
20. What is the amount of $1575.20, for 3 yrs. 8 mo., at 7%?
21. What is the amount of $5000, for 2 yrs. 3 mo., at 5⅓%?
22. What is the interest of $1508.20, for 4 yrs. 2 mo., at 10%?
23. What is the interest of $75, for 6 yrs. 10 mo., at 12½%?
24. What is the amount of $125, for 5 yrs. 6 mo., at 4¾%.

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272. What is interest? What is the principal or base? What is rate? What is amount? What is the meaning of per annum?—273. How is a year reckoned, in computing interest? How many days are *reckoned* in a month? What is legal interest? What is usury?

CASE II.

277. To find the interest on a given principal for any rate and time.

1. What is the interest of $1752.96, at 6 per cent., for 2 yrs. 4 mo. and 29 da.?

ANALYSIS.—The interest for 1 year is the product of the principal and rate. If the interest for 1 year be divided by 12, the quotient will be the interest for 1 month; if the interest for 1 month be divided by 30, the quotient will be the interest for 1 day.

The interest for 2 years is two times the interest for 1 year; the interest for 4 months, 4 times the interest for 1 month; and the interest for 29 days, 29 times the interest for 1 day.

OPERATION.

$1752.96					
.06					
2)105,1776	int. for 1 *yr.*	$105.1776 × 2	=	$210.3552	2*yr.*
30)8,7648	int. for 1 *mo.*	8.7648 × 4	=	35.0592	4*mo.*
.29216	int. for 1 *da.*	0.29216 × 29	=	8.47264	29*da.*
		Total interest,		$253.88704	

Hence, we have the following,

Rule.

I. *Find the interest for* 1 *year:*

II. *Divide this interest by* 12, *and the quotient will be the interest for* 1 *month:*

III. *Divide the interest for* 1 *month by* 30, *and the quotient will be the interest for* 1 *day:*

IV. *Multiply the interest for* 1 *year by the number of years, the interest for* 1 *month by the number of months, and the interest for* 1 *day by the number of days, and the sum of the products will be the required interest.*

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274. What are the general rates of legal interest?—275. How many parts are there in interest? What are they?—276. How do you find the interest of any principal for one or more years?—277. How do you find the *interest for any rate* and time?

NOTE.—This method of computing interest for days, is the one in general use. It supposes the month to contain 30 days, or the year 360 days; whereas, it actually contains 365 days.

To find the *exact* interest for 1 day, we must regard the month as containing $\frac{365}{12}$ days $= 30\frac{5}{12}$ days; and this is the number by which the interest for one month should be divided, in order to find the *exact* interest for one day. As the divisor, commonly used, is *too small* the interest found for 1 day, is a trifle too large. If entire accuracy is required, the interest for the days must be diminished by its $\frac{5}{365}$ part $= \frac{1}{73}$ part.

2D METHOD.

278. There is another rule resulting from the last analysis which is regarded as the best general method of computing interest.

Rule.

I. *Find the interest for* 1 *year, and divide it by* 12; *the quotient will be the interest for* 1 *month:*

II. *Multiply the interest for* 1 *month by the time expressed in months and decimal parts of a month, and the product will be the required interest.*

NOTE.—Since a month is reckoned at 30 days, *any number of days is reduced to decimals of a month by dividing the number of days by* 3.

Examples.

1. What is the interest of $655, for 3 years 7 months and 13 days, at 7%?

OPERATION.

3 *yrs.* = 36 *mos.*
7 *mos.*
13 *da.* = $.4\frac{1}{3}$ *mos.*
Time = $43.4\frac{1}{3}$ *mos.*

$655	
.07	
12)45.85	int. for 1 year.
3.82083+	int. for 1 month
$43.4\frac{1}{3}$	time in months.
127361	
1528332	
1146249	
1528332	
165.951383	*Ans.*

2. What is the interest of $358.50, for 1 yr. 8 mo. 6 da., at 7 %?

3. What is the interest of $1461.75, for 4 yrs. 9 mo. 15 da., at 6 per cent.?

4. What is the interest of $1200, for 2 years 4 months and 12 days, at 7½ %?

5 What is the interest of $4500, for 9 mos. 20 da., at 5 %?

6 What is the interest of $156.25, for 10 mo. 18 da., at 8 %?

7. What is the interest of $640, for 3 yrs. 2 no. 9 da., at 6½%?

8. What is the interest of $276.50, for 11 mo. 21 da., at 10%?

9. What is the amount of $378.42, for 1 yr. 5 mo. 3 da., at 7%?

10. What is the amount of $1250, for 7 mo. 21 da., at 10½ %?

11. What is the interest of $6500, for 2 mo. 10 da., at 9½ %?

12. What is the interest of $70.50, for 10 years and 10 months, at 5¼ per cent.?

13. What is the amount of $45, for 12 years and 27 days, at 6¾ per cent.?

14. What will $100 amount to in 15 years and 6 months, if put at interest at 4 per cent.?

15. How much will $475.50 gain in 5 years 9 months and 24 days, at 8 per cent.?

16. What will be the interest of $4560, for 14 months and 19 days, at 7 per cent.?

17. What will $128.37½ amount to in 10 months and 27 days, at 6 per cent.?

18. What is the interest of $264.52, for 2 years 8 months and 14 days, at 6 per cent.?

19. What is the amount of $76.50, for 1 year 9 months and 12 days, at 6 per cent.?

20. What will be the interest for 3 years 3 months and 15 days, of $241.60, at 7 per cent.?

21. What is the interest of $5600, for 30 days, at 7 per cent.?

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278. What is the rule for the second method?

22. What will $8450 amount to in 60 days, at 10 per cent.?

23. What is the interest of $4000, for 1 month and 6 days, at 9 per cent.?

24. What will be the amount of $87.60, from September 9th, 1852, to October 10th, 1853, at 6¼ per cent.?

25. What will be due on a note of $126.75, given July 8th. 1854, and payable April 25th, 1858, at 7 per cent.?

26. What is the interest of $350, from January 1st, 1856, to 15th of September next following, at 5¼ per cent.?

27. Gave a note of $560.40, March 14th, 1855, on interest, after 90 days: what interest was due December 1st, 1856, at 10 per cent.?

28. What is the interest of $1256, for 11 months and 9 days, at 6 per cent.?

29. What is the amount of $745.40, at 5 per cent. interest being reckoned from the 5th day of the 10th month of 1850, to the 10th day of the 5th month of 1854?

30. September 10th, 1852, James Trusty borrowed of Peter Credit $250, and March 4th, 1853, $500 more, agreeing to pay 7 per cent. interest on the whole: what was the amount of his indebtedness January 1st, 1854?

31. Ordered drygoods of A. T. Stewart & Co., at different times, to the following amounts: viz., Jan. 1st, 1854, $254; March 15th, 1854, $154.60; April 20th, 1854, $424.25; and June 3d, 1854, $75.50. I bought on time, at 6 per cent. interest: what was the whole amount of my indebtedness on the first day of September following?

32. If I borrow $475.75 of a friend at 7 per cent., what will I owe him at the end of 8 months and a half?

33. In settling with a merchant, I gave my note for $127.28, due in 1 year 9 months, at 6 per cent.: what must be paid when the note falls due?

34. A person buying a piece of property for $4500, agreed

to pay for it in three equal annual instalments, with interest at 6½ per cent.: what was the entire amount of money to be paid?

35. A mechanic hired a journeyman for 9 months, at $40 a month, to be paid monthly; at the end of the time he had paid nothing; he then settled, allowed interest at 7 per cent., and gave his note, on interest, due in 1 year 4 months and 15 days: what will he pay when his note falls due?

36. A person owning a part of a woolen factory, sold his share for $9000. The terms were, one-third cash, on delivery of the property, one-half of the remainder in 6 months, and the rest in 12 months, with 7½ per cent. interest: what was the whole amount paid?

INTEREST ON NOTES.

$382.50 CHICAGO, January 1st, 1864.

1. For value received, I promise to pay on the 10th day of June next, to C. Hanford or order, the sum of three hundred and eighty-two dollars and fifty cents, with interest from date, at 7 per cent.?

JAMES JONES.

$612 BALTIMORE, January 1st, 1862.

2. For value received, I promise to pay on the 4th of July, 1864, to Wm. Johnson or order, six hundred and twelve dollars, with interest at 6 per cent. from the 1st of March, 1862.

JOHN LIBERAL.

$3120 CHARLESTON, July 3d, 1860.

3. Six months after date, I promise to pay to C. Jones or order, three thousand one hundred and twenty dollars, with interest from the 1st of January last, at 7 per cent.

JOSEPH SPRINGS.

$786.50 **New York, July 7th, 1861.**

4. Twelve months after date, I promise to pay to Smith & Baker or order, seven hundred and eighty-six $\frac{50}{100}$ dollars, for value received, with interest from December 3d, 1861, at 8 per cent. Silas Day.

$4560.72 **Cincinnati, March 10th, 1863.**

5. Nine months after date, for value received, I promise to pay to Redfield, Wright & Co. or order, four thousand five hundred and sixty $\frac{72}{100}$ dollars, with interest, after 6 months, at 7 per cent. Frederick Stillman.

$1854.83 **Boston, July 17, 1863.**

6. Eleven months after date, for value received, we promise to pay to the order of Fondy, Burnap & Co. one thousand eight hundred and fifty-four $\frac{83}{100}$ dollars, with interest from May 13th, 1863, at 6 per cent. Palmer & Blake.

POUNDS, SHILLINGS, AND PENCE.

279. To find the interest of pounds, shillings, and pence.

Rule.—I. *Reduce the shillings and pence to the decimal of a pound* (Art. 214):

II. *Then find the interest as though the sum were dollars and cents; after which, reduce the decimal part of the answer to shillings and pence* (Art. 215).

Examples.

1. What is the interest, at 6 per cent., of £27 15s. 9d for 2 years?

£27 15s. 9d. = £27.7875.

£27.7875 × .06 × 2 = £3.3345 interest.

£3.3345 = £3 6s. 8$\frac{1}{4}$d. *Ans.*

2. What is the interest on £203 18s. 6d., at 6 per cent. for 3 years 8 months 16 days?

3. What is the interest of £215 13s. 8d., at 6 per cent., for 3 years 6 months and 9 days?

4. What is the interest of £1543 10s. 6d., for 2 years and a half, at 4 per cent.?

5. What is the amount of £1047 3s., for 1 yr. 4 mo. 15 da., at 6 per cent.?

6. What is the interest on £511 1s. 4d., at 6 per cent. per annum, for 6 yr. 6 mo.?

7. What is the interest on £161 15s. 3d., at 6 per cent., for 8 mo. 13 da.?

PARTIAL PAYMENTS.

280. A PARTIAL PAYMENT is the payment of a part of the amount due on a note or bond. When partial payments are made, they are indorsed on the note or bond.

281. We shall now give the rule established in New York (see Johnson's Chancery Reports, vol. i., page 17,) for computing the interest on a bond or note, when partial payments have been made. The same rule is also adopted in Massachusetts, and in most of the other States.

Rule.

I. *Compute the interest on the principal to the time of the first payment; if the payment equals or exceeds this interest, add the interest to the principal, and from the sum subtract the payment; the remainder forms a new principal:*

II. *But if the payment is less than the interest, take no notice of it, except to indorse it on the note or bond, until other payments shall have been made, which in all, shall exceed the interest computed to the time of the last payment, then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments; the remainder will form a new principal, on which interest is to be computed as before.*

Examples.

$349.998

RICHMOND, Va., May 1st, 1846

1. For value received, I promise to pay James Wilson, or order, three hundred and forty-nine dollars ninety-nine cents and eight mills, with interest, at 6 per cent

JAMES PAYWELL.

On this note were indorsed the following payments:

Dec. 25th, 1846,	received - - - - - -	$49.998
July 10th, 1847,	" - - - - - -	4.998
Sept. 1st, 1848,	" - - - - - -	15.008
June 14th, 1849,	" - - - - - -	99.999

What was due April 15th, 1850?

Principal on interest from May 1st, 1846 - - - -		$349.998
Interest to Dec. 25th, 1846, time of first payment, 7 months 24 days - - - - - - - -		13.649+
Amount - - - - - -		$363.647+
Payment Dec. 25th, exceeding interest then due -		49.998
Remainder for a new principal - - - - - - - -		$313.649
Interest of $313.649 from Dec. 25th, 1846, to June 14th, 1849, 2 years 5 months, 19 days - - -		46.472+
Amount - - - - - -		$360.121
Payment, July 10th, 1847, less than interest then due - - - - - - -	$4.998	
Payment, Sept. 1st, 1848 - - - - - -	15.008	
Their sum, less than interest then due -	$20.006	
Payment, June 14th, 1849 - - - - - -	99.999	
Their sum exceeds the interest then due - - - -		$120.005
Remainder for a new principal, June 14th, 1849 -		240.116
Interest of $240.116 from June 14th, 1849, to April 15th, 1850, 10 months 1 day - - -		12.045
Total due, April 15th, 1850 - - - -		$252.161+

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279. *How do you find the interest when the principal is in pounds shillings, and pence?*

$6478.84

New Haven, Feb. 6th, 1850.

2. For value received, I promise to pay William Jenks, or order, six thousand four hundred and seventy-eight dollars and eighty-four cents, with interest from date, at 6 per cent.

John Stewart.

On this note were indorsed the following payments:

May 16th, 1853, received - - - - - -	$ 545.76
May 16th, 1855, " - - - - - -	1276
Feb. 1st, 1856, " - - - - - -	2074.72.

What remained due, August 11th, 1857?

3. A's note of $7851.04 was dated Sept. 5th, 1851, on which were indorsed the following payments: viz., Nov. 13th, 1853, $416.98; May 10th, 1854, $152: what was due March 1st, 1855, the interest being 6 per cent.?

$8974.56

New York, Jan. 3d, 1854.

4. For value received, I promise to pay to James Knowles, or order, eight thousand nine hundred and seventy-four dollars and fifty-six cents, with interest from date at the rate of 7 per cent.

Stephen Jones.

On this note are indorsed the following payments:

Feb. 16th, 1855, received - - - - - -	$1875.40
Sept. 15th, 1856, " - - - - - -	3841.26
Nov. 11th, 1857, " - - - - - -	1809.10
June 9th, 1858, " - - - - - -	2421.04.

What will be due, July 1st, 1858?

$345.50

Buffalo, Nov. 1st, 1852.

5. For value received, I promise to pay C. B. Morse, or order, three hundred and forty-five dollars and fifty cents, with interest from date, at 7 per cent.

John Doe.

On this note are the following indorsements:

June 20th, 1853,	received	$75
Jan. 12th, 1854,	"	10
March 3d, 1855,	"	15.50
Dec. 13th, 1856,	"	52.75
Oct. 14th, 1857,	"	106 75

What will there be due, Feb. 4th, 1858?

$450

MOBILE, Oct 19th, 1850.

6. For value received, we jointly and severally promise to pay Jones, Mead & Co., or order, four hundred and fifty dollars on demand, with interest, at 8 per cent.

MANNING & BROS.

The following indorsements were made on this note:

Sept. 25, 1851, received $85.60; July 10, 1852, received $20; June 6, 1853, received $150.45; Dec. 28, 1854, received $25.12½; May 5, 1855, received $169: what was due, Oct. 18, 1857?

PROBLEMS IN SIMPLE INTEREST.

282. In every question of Interest, there are four parts: 1st, Principal; 2d, Rate; 3d, Time; and 4th, Interest.

If any three of these parts are known, the fourth can be found. The interest is found by multiplying the principal by the rate and time in years (Art. 276); therefore, the interest is the product of the three factors, principal, rate, and time. Any one of these factors is found by dividing their product by the other two: Hence, we have the following principles;

1st, *The interest is equal to the product of the principal, rate, and time;* 2d, *The principal is equal to the interest divided by the product of rate and time;* 3d, *The rate is equal to the interest divided by the product of the principal and time;* 4th, *The time is equal to the interest divided by the product of the principal and rate.*

283. Formulas.

1. $$\text{Interest} = I = P \times R \times T.$$

2. $P = \dfrac{I}{R \times T}$; 3. $R = \dfrac{I}{P \times T}$; 4. $T = \dfrac{I}{P \times R}$.

Examples.

1. At what rate per cent. must $325 be put at interest for 1 year and 6 months, to produce an interest of $34.125?

ANALYSIS.—The product of principal by the time is $325 \times 1\frac{1}{2} = 487\frac{1}{2}$. By principle 3d, the rate equals $34.125 ÷ 487.5 = .07, or 7 per cent.

2. What principal, at 6 per cent., will in 9 months give an interest of $178.9552?

3. The interest for 2 years and 6 months, at 7 per cent., is $76.965: what is the principal?

4. What sum must be invested, at 6 per cent., for 10 months and 15 days, to produce an interest of $327.3249?

5. If my salary is $1500 a year, what sum invested at 5 per cent. will pay it?

6. What sum put at interest for 4 years and 3 months, at 7 per cent., will gain $283.3914?

7. The interest of $2100 for 3 years 1 month and 18 days is $460.60: what is the rate per cent.?

8. A person owning property valued at $2470.80, rents it for 1 year and 10 months for $452.98: what per cent. does it pay?

9. At what rate per cent. must $3456 be loaned for 2 years 7 months and 24 days, to gain $503.712?

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280. What is a partial payment?—281. What is the rule for partial payments?—282. How many parts are there in a problem of simple interest? What are they?—283. Write on the blackboard the formulas *for the problems of simple* interest.

COMPOUND INTEREST.

281. COMPOUND INTEREST is the interest computed on principal and interest. It may be computed annually, semi-annually, quarterly, monthly, weekly, or daily. In Savings Banks, the interest is generally computed semi-annually.

NOTE.—The laws do not favor the payment of compound interest. In most of the States, an agreement to pay compound interest could not be enforced.

Rule.—*Compute the interest to the time at which it becomes due; then add it to the principal and compute the interest on the amount as on a new principal: add the interest again to the last principal and compute the interest as before; do the same for all the times at which payments of interest become due; from the last result subtract the first principal, and the remainder will be the compound interest.*

NOTE.—When there are months and days in the time, find the amount for the *years*, and on this amount cast the interest for the months and days: this, added to the last amount, will be the required *amount* for the whole time.

Examples.

1. What will be the compound interest of $3750 for 2 years, at 7 per cent.?

	$3750.000	principal for 1st year.
$3750 × .07 =	262.500	interest for 1st year.
amount	4012.500	principal for 2d "
$4012.50 × .07 =	280.875	interest for 2d "
	4293.375	amount.
	3750.000	
	543.373	interest. *Ans.*

2. What will be the compound interest of $175 for 2 years, at 7 per cent., the interest being computed annually?

3. What will be the *amount* of $240, compound interest, for 4 years, at 5 per cent., the interest being computed annually?

4. What is the compound annual interest of $540.50 at 6 per cent., for 3 years 6 months and 15 days? (See Note.)

TABLE.—The following table shows the *amount* of $1 or £1, compound interest, from 1 year to 20, at the rates of 3, 4, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	1.03000	1.04000	1.05000	1.06000	1.07000	1
2	1.06090	1.18160	1.10250	1.12360	1.14490	2
3	1.09272	1.12486	1.15762	1.19101	1.22504	3
4	1.12550	1.16985	1.21550	1.26247	1.31079	4
5	1.15927	1.21665	1.27628	1.33822	1.40255	5
6	1.19405	1.26531	1.34009	1.41851	1.50073	6
7	1.22987	1.31593	1.40710	1.50363	1.60578	7
8	1.26677	1.36856	1.47745	1.59384	1.71818	8
9	1.30477	1.42331	1.55132	1.68947	1.83845	9
10	1.34391	1.48028	1.62889	1.79084	1.96715	10
11	1.38423	1.53945	1.71033	1.89829	2.10485	11
12	1.42576	1.60103	1.79585	2.01219	2.25219	12
13	1.46853	1.66507	1.88564	2.13292	2.40984	13
14	1.51258	1.73167	1.97993	2.26090	2.57853	14
15	1.55796	1.80094	2.07892	2.39655	2.75903	15
16	1.60470	1.87298	2.18287	2.54035	2.95216	16
17	1.65284	1.94790	2.29201	2.69277	3.15881	17
18	1.70243	2.02581	2.40661	2.85433	3.37993	18
19	1.75350	2.10684	2.52695	3.02559	3.61652	19
20	1.80611	2.19112	2.65329	3.20713	3.86968	20

Rule.—*Take from the table the amount of* $1 *for the given rate and time. Multiply it by the given principal, and the product will be the amount; from this, subtract the given principal and the remainder will be the required interest.*

5. What will be the amount of $75, at annual compound interest, 7 per cent., for 10 years, 4 months, 21 days?

6. What will $650 amount to, at annual compound interest, at 3 per cent., for 12 years?

7. What will be the compound interest of $3475, computed annually for 5 years, 5 months, and 9 days, at 6 per cent.?

8. What will be the compound annual interest of $95637.50, for 7 years, at 6 per cent.?

9. What will be the compound interest of $75439.75, for 4 years, at 4½ per cent.?

DISCOUNT.

285. Discount is an allowance made for the payment of money before it is due.

The Face of a note is the amount named in the note.

The present value of a note, is such a sum as, being put at interest until the note becomes due, would increase to an amount equal to its face.

The discount, on a note, is the difference between the face of the note and its present value.

286. Knowing the face of a note, due at a future time, and the rate of interest, to find its present value.

1. I give Mr. Wilson my note for $106, payable in 1 year: what is the present value of the note, if the interest is 6 per cent.? What is the discount?

Analysis.—The present value is the base, the rate is 6 per cent., and the face of the note is the amount (Art. 267).

OPERATION.

$$\text{Pres. value} = \frac{106}{1+.06} = \$100$$

Rule.—*Divide the face of the note by* 1 *dollar plus the interest of* 1 *dollar for the given time.*

Note.—When payments are to be made at different times, *find the present value of the sums separately, and their sum will be the present value of the note.*

Examples.

1. What is the present value of a note of $615, due 1 year 4 months hence, at 7 per cent.?

2. What is the present value of $202.58, due in 1 year 7 months and 18 days, at 6 per cent.?

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285. What is discount? What is the face of a note? What is present value? What is the discount on a note?—**286.** Knowing the face of a note and rate, how do you find the present value?

3. How much should I deduct for the present payment of a note of $721, due in 7 months and 6 days, at 5 per cent.?

4. If a note for $5160 is payable Feb. 4th, 1864, what is its value Sept. 10th, 1863, interest being reckoned at 8 per cent.?

5. What sum of money will amount to $2500, in 2 years 7 months and 12 days, at 12 per cent.?

6. What is the present value and discount of $3000, payable in 1 year 2 months and 20 days, at 7 per cent.?

7. A held a note of $1400 against B, payable Aug. 1st, 1856; B paid it May 15th, 1856: what sum did he pay, the interest being 7 per cent.?

8. A flour merchant bought for cash 300 barrels of flour, for $10.50 per barrel; he sold it the same day for $12 a barrel, and took a note at 3 months: what was the cash value of the sale, and what his gain, if the interest is reckoned at 7 per cent.?

9. A man purchased a house and lot for $10000, on the following terms: 5000 in cash, 2500 in 3 months, and the balance in six months: what was the cash value of the property, interest being reckoned at 6 per cent.?

10. Which is the more advantageous, to buy sugar at 7½ cents a pound, on 4 months, or at 8 cents a pound on 6 months, at 6 per cent. interest?

11. Bought land at $10 an acre: what must I ask per acre if I abate 10 per cent., and still make 20 per cent. on the purchase money?

12. A merchant owed three notes, viz., $1000, payable Aug. 1st, 1855; $500, payable Oct. 10th, 1855, and $900, payable Nov. 1st, 1855: what was the cash value of the three notes, July 1st, 1855, reckoning interest at 6 per cent.; and what was the difference between that value and their amounts at the times when they fell due, if interest were reckoned from July 1st.

BANKING.

287. A CORPORATION is a collection of persons authorized law to do business together. The instrument which defin their rights and powers is called a *Charter*.

288. BANKS are Corporations for the purpose of receiving d posits, loaning money, and furnishing a paper circulation rep sented by specie.

BANK NOTES are the notes made by a bank to circulate money, and should be payable in specie, on presentation at t bank.

A PROMISSORY NOTE is the note of an individual, and is positive engagement, in writing, to pay a given sum, either demand or at a specified time.

FORMS OF NOTES.

No. 1. *Negotiable Note.*

$25.50

PROVIDENCE, May 1, 1856.

For value received, I promise to pay on demand, to Al Bond, or order, twenty-five dollars and fifty cents.

REUBEN HOLMES.

No. 2. *Note Payable to Bearer.*

$875.39

ST. LOUIS, May 1, 1856

For value received, I promise to pay, six months after da to John Johns, or bearer, eight hundred and seventy-five d lars and thirty-nine cents.

PIERCE PENNY.

No. 3. *Note by two Persons.*

659.27

BUFFALO, June 2, 1856

For value received, we jointly and severally promise to p to Richard Ricks, or order, on demand, six hundred and fif nine dollars and twenty-seven cents.

ENOS ALLAN.
JOHN ALLAN.

No. 4. *Note Payable at a Bank.*

$20.25

CHICAGO, May 7, 1856.

Sixty days after date, I promise to pay John Anderson, or order, at the Bank of Commerce, in the city of New York, twenty dollars and twenty-five cents, for value received.

JESSE STOKES.

Remarks Relating to Notes.

1. The person who signs a note is called the *drawer* or *maker* of the note; thus, Reuben Holmes is the drawer of note No. 1.

2. The person who has the rightful possession of a note is called the *holder* of the note.

3. A note is said to be *negotiable* when it is made payable to A. B., or order, who is called the payee (see No. 1). Now, if Abel Bond, to whom this note is made payable, writes his name on the back of it, he is said to *indorse* the note, and he is called the indorser; and when the note becomes due, the holder must first demand payment of the maker, Reuben Holmes; and if he declines paying it, the holder may then require payment of Abel Bond, the indorser.

4. When a note is not paid at the time it becomes due, the indorser must be notified of the fact, and of the time it was due. This notice is generally given by an officer called a notary public, and is called a *protest*.

5. If the note is made payable to A. B., or bearer, then the drawer alone is responsible, and he must pay to any person who holds the note.

6. The time at which a note is to be paid should always be named; but if no time is specified, the drawer must pay when required to do so, and the note will draw interest after the payment is demanded.

7. When a note, payable at a future day, becomes due, it will draw interest, though no mention is made of interest.

8. In each of the States there is a *rate* of interest established by law, which is called the legal interest; and when no rate is specified, the note will always draw legal interest. If a rate *higher* than legal interest is named in the note, or agreed upon, the drawer, in most of the States, is not bound to pay the note.

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287. What are corporations? What is a charter?—288. What are banks? *What are bank-notes?* What is a promissory note?

9. If two persons jointly and severally give their note (see No. 8), it may be collected of either of them.

10. The words, "For value received," should be expressed in every note.

11. When a note is given, payable on a fixed day, and in a specific article, as in wheat or rye, payment must be offered at the specified time; and if it is not, the holder can demand the value in money.

12. Days of grace are days allowed for the payment of a note after the expiration of the time named on its face. By mercantile usage, a note does not *legally* fall due until 3 days after the expiration of the time named on its face, unless the note specifies "*without grace.*" For example, No. 2 would be due on the 4th of November, and the three additional days are called *days of grace.*

When the last day of grace happens to be a Sunday, or a holiday, such as New Year's day, or the 4th of July, the note must be paid the day before; that is, on the second day of grace.

13. There are two kinds of notes discounted at banks: 1st. Notes given by one individual to another for property actually sold; these are called *business notes,* or *business paper.* 2d. Notes made for the purpose of borrowing money, which are called *accommodation notes,* or *accommodation paper.* The first class of paper is much preferred by the banks, as more likely to be paid when it falls due, or, in mercantile phrase, "when it comes to maturity."

BANK DISCOUNT.

289. Bank Discount is the deduction made by a bank from the face of a note due at a future time.

Bank discount, by custom, is the interest of the face of the note, calculated from the time when it is discounted to the time when it falls due; in which time three days of grace are always included (see remark 12). The interest on notes discounted at bank is always *paid in advance.*

The proceeds of a note is the difference between its face and the discount.

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289. What is bank discount? How is interest calculated? When *is it paid?* What are the proceeds of a note?

290. To find the bank discount.

Rule.—*Add 3 days to the time which the note has to run, and then calculate the interest for that time at the given rate.*

Examples.

1. What is the bank discount on a note of $300, for 4 months, at 6 per cent. per annum.

2. What is the bank discount on a note of $200, payable in five months, at 9 per cent.?

3. What is the bank discount and proceeds of a note of $500, at 6½ per cent., payable in 8½ months?

4. What is the cash value of a note, payable at bank, of $1255.38, and due in 4 months, at 7 per cent.?

5. What was the bank discount on a note of $500, due August 13th, 1855, and discounted July 1st, 1855, reckoning interest at 7 per cent.?

6. I bought 4368 bushels of wheat, at $1.25 a bushel, and sold it the same day for $1.30 a bushel on a note of 4 months. If I get this note discounted, at bank, at 7 per cent., what do I gain or lose?

7. What is the difference between the true and bank discount, of $7000, payable in 7 months, at 6 per cent.?

8. What is the difference between the true and bank discount, of $10000, payable in 4½ months, at 8 per cent.?

9. January 1st, 1855, a note was given for $1000, at 5½ per cent., to be paid May 1st, next following: what was its cash value at bank?

10. A holds a note against B for $1500, to run 6 months from Aug. 1st, without interest. Oct. 1st, he wishes to pay a debt at the bank of $1000, and turns in the note at a discount of 5 per cent. in payment: how much should he receive back from the bank?

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290. How do you find the bank discount?

291. To draw a note due at a future time, whose proceeds shall be a given amount.

1. For what sum must a note be drawn at 4 months and 12 days, so that, when discounted at bank, at 6 per cent., the proceeds shall be $400.

ANALYSIS.—The face of the note must be such, that the interest for the given time, subtracted from the face, shall leave the required proceeds. Hence, the proceeds correspond to the *difference*; the rate of interest of $1 for the given time, to the rate; and the face of the note to the base (Art. 267).

Rule.—*Divide the given proceeds by* 1 *minus the rate of* $1 *for the given time, and the quotient will be the face of the note.*

OPERATION.

$$\text{Face} = \frac{\text{Proceeds}}{1 - \text{R}}$$

$$\text{Face} = \frac{400}{.9775} = \$409.207+$$

$$\text{Proceeds} = \$400.$$

$$\text{R. } \$1 \text{ for } 4\,mo.\ 15\,da. = 0.0225$$

$$1 - .0225 = 0.9775$$

2. For what sum must a note be drawn at 7 per cent., payable in 6 months, so that when discounted at a bank it shall produce $285.95.

3. How large a note must I make at a bank, at 6 per cent., payable in 6 months and 9 days, to produce $674.89?

4. For what sum must a note be drawn, at 5 per cent., payable in 9 months and 15 days after date, so that when discounted at bank, it shall produce $1000.

5. Marsh, Dean & Co. purchase of John Jones 380 barrels of flour, at $9.12½ a barrel, for which they give him a note at 90 days, for such sum, that if discounted at 6 per cent., he shall receive the above price for his flour: what was the face of the note?

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291. How do you draw a note whose proceeds shall be a given *amount?*

STOCKS.

292. **Capital** or **Stock** is the amount of money paid in to carry on the business of a corporation.

Stockholders are the owners of the stock.

Certificates are the written evidences of ownership.

293. **United States** or **State Stocks** are the bonds of the United States, or of a State, bearing a fixed interest.

A **Coupon** is a due-bill for interest, attached to bonds or certificates of stock, and payable at specified times.

294. **Par Value** of stock is the number of dollars named in each share, generally 100; sometimes 50, and sometimes 25.

Market Value of a stock, is what it brings per share, when sold for cash.

295. **Premium** is the rate per cent. which a stock sells for above its par value.

Discount is the rate per cent. which a stock sells for below its par value.

296. **Dividend** is a profit divided among the stockholders, and is generally estimated at a certain rate per cent. on the par value of the stock.

297. **Brokerage** is a commission made to an agent for buying and selling stock, uncurrent money, or bills of exchange.

Notes.—1. The brokerage in the city of New York is generally one-fourth of one per cent. on the par value.

2. In questions of stocks, the *par value* is always the *base.*

3. In the examples, the shares are $100 each, unless another amount is named.

298. **To find the dividend on a given amount of stock.**

1. What is the percentage on 25 shares, $100 each, of Kings County Insurance Company, the dividend being 25 per cent.?

Analysis.—Here, the base and rate are given to find the percentage *(Art. 259).*

2. The Atlantic Fire Insurance Co. declares a semi-annual dividend of 4½% on the capital stock: what is the annual dividend of 43 shares at that rate?

3. The Atlantic Bank of Brooklyn has declared a semi-annual dividend of 5%: what is the dividend on 18 shares?

4. A bankrupt is indebted to A, $5416, and to B, $6795: what does each receive, when the dividend to the creditors is 47¼ per cent.?

5. A mining company, shares $25 each, declared a dividend of 17%: what was the dividend on 36 shares?

299. To find the value of stock which is above or below par.

1. What is the value of $5600 of stock, reckoned at par, when the stock is at a premium of 9 per cent.?

ANALYSIS.—In this class of examples, the base and rate are given. When the stock is above par, the amount is required (Art. 265); when it is below par, the difference (Art. 266).

2. What is the cost of 56 shares of New York Central Railroad stock, at 5½ per cent. below par, and the brokerage ½ per cent.?

3. I bought 36 shares in the Pennsylvania Coal Company, at a discount of 12½ per cent., and sold them at a premium of 7 per cent., paying ½ per cent. brokerage in each case: how much did I make by the operation?

4. What is the market value of 216 shares of bank-stock, each share $75, and the premium 7¾%?

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292. What is capital or stock? Who are stockholders? What are certificates?—293. What are United States stocks? What is a coupon?—294. What is the par value of a stock? What is market value?—295. What is premium? What is discount?—296. What is dividend?—297. What is brokerage? What is the general rate in the city of New York? What is the base, in stocks?—298. How do you find the dividend on stock?—299. How do you find the value of stock *when it is* above or below par?

5. The par value of 257 shares of bank-stock is $200 a share: what is the present value of all the shares, the stock being at a premium of 15 per cent.?

6. What is the value of 120 shares of Exchange Bank stock, it being at a premium of 18¾ per cent., and the par value being $150 a share?

7. What will be the cost of 69 shares of Panama Railroad stock, at a discount of 8%, the par value being $125, and brokerage ¾ per cent.?

8. Gilbert & Co. buy for Mr. A, 200 shares of United States stock, at a premium of 6½ per cent., and charge ¼ per cent. brokerage: if the shares are $1000 each, how much money does A pay for the stock?

9. Mr. B. bought 125 shares of stock in the American Guano Company, at par, the shares being $20 each. At the end of 4 months, he received a dividend of 5 per cent., and at the end of 10 months, a second dividend of 4 per cent. At the end of the year, he sold his stock at a premium of 10 per cent.: how much did he make by the operation, reckoning the interest of money at 7 per cent.?

300. To find how much stock, at par value, a given sum of money will purchase, when the stock is at a premium or discount.

1. What value of stock, at par, can be purchased by $3045.38, if the stock is at a premium of 10 per cent., and ½ per cent. is charged for brokerage?

Analysis.—When the stock is above par, the amount and the rate are given to find the base (Art. 267); when below par, the difference and rate are given to find the base (Art. 267).

2. A person wishes to invest $3000 in bank-stock, which is at a discount of 15 per cent.: what amount at par can he purchase?

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300. How do you find how much stock, at par, a given sum of money will buy *when the stock is at* a premium? How when it is at a discount?

3. How many shares of Galena and Chicago Railroad stock can be bought for $6384, at 14 % premium?

4. When bank-stock sells at a discount of 7½ %, what amount of stock, at par value, will $3700 buy?

5. A person has $7000, which he wishes to invest; what will it purchase in 5 per cent. stocks, at a discount of 3½ per cent., if he pays ¼ per cent. brokerage?

6 How much 6 per cent. stock, at par, can be purchased for $8700, at 8½ per cent. premium, ¼ per cent being paid for brokerage?

7. A person owning $12000 in government funds, desires to purchase stock in the American Exchange Bank. The funds are at a discount of 3½ per cent., while the bank-stock is at a premium of 10¼ per cent.: what amount of stock, at par value, can he purchase, allowing the broker's charges for the purchase to be ¾ per cent.?

301. To find the rate of interest on an investment in stock, when the stock is above or below par.

1. What is the rate of interest on an investment in 6 per cent. stocks, when they are at a discount of 25 per cent.?

ANALYSIS.—The interest on the stock is computed on its *par value;* the interest on the investment is computed on the market value, and the percentage in each case is the same. Hence, 1 dollar of the stock multiplied by its rate of interest, will be equal to the market value of $1 of the stock multiplied by its rate of interest.

OPERATION.

$$\left.\begin{matrix}.75\\ x\end{matrix}\right\} = \$1 \times .06 = \$.06$$

$$x = \frac{.06}{.75} = .08$$

Ans. 8 per cent.

Rule.—*Multiply* $1 *of the stock by its rate of interest, and divide the product by the market value of* $1 *of the stock. the quotient will be the rate of interest on the investment.*

2. If I buy 7 per cent. stock at 12½ per cent discount, *what is the rate* per cent. on the investment?

3. If the stock of the Erie Railroad sells at 62½ per cent., and pays semi-annual dividends of 2½ per cent., what would be the rate of interest on an investment?

4. The bonds of the Illinois Central Railroad Company, which bear interest of 7 per cent., are worth 87 per cent., and the charge for brokerage is ½ per cent.: what would be the interest on an investment in these funds?

5. The stock of the Hartford and New Haven Railroad is at a premium of 20 per cent.: reckoning the interest on money at 6 per cent., what will be the interest on an investment?

302. To find how much a stock must be above or below par, to produce a given rate of interest.

1. At what rate must a 6 per cent. stock be bought, so that the investment shall yield 9% interest?

ANALYSIS.—Since the percentage is the same in both cases, $1 of the stock multiplied by its rate of interest, is equal to the market value of $1 of the investment multiplied by its rate.

OPERATION.

$$\left.\begin{array}{c} x \\ .09 \end{array}\right\} = \$1 \times .06 = \$.06$$

$$x = \frac{.06}{.09} = .66\tfrac{2}{3}$$

$$1 - .66\tfrac{2}{3} = .33\tfrac{1}{3} \text{ dis.}$$

Rule.—I. *Multiply $1 of the stock by its rate of interest, and divide the product by the rate of interest on the investment: the quotient will be the per cent. of the market value of $1 of the stock:*

II. *If the market value is greater than* 1, *subtract* 1 *from it, and the remainder will be the per cent. of premium; if less than* 1, *subtract it from* 1, *and the remainder will be the per cent. of discount.*

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301. How do you find the rate of interest on an investment when the stock is above or below par?—302. How do you find how much a stock must be above or below par, to produce a given rate of interest?

2. At what rate of discount must I invest in 8 per cent. stock, in order to yield me 10 per cent.?

3. If the par value of a stock is $100, and the interest 7 per cent., what is the discount when an investment yields 12 per cent.?

4. At what rate must I invest in a 9% stock, that I may receive 8 per cent. on my investment?

303. Which is the best investment?

1. I invest $1250 in State stocks bearing an interest of 6 per cent., and a premium of 15 per cent. I invest an equal amount in State fives at 12 per cent. discount. Which will yield the larger interest?

Analysis.—Find the rate of interest of each investment, and then compare the two rates. That investment which produces the greater rate is the more advantageous.

OPERATION.

1st.	2d.
$\frac{\$1 \times .06}{1.15} = .0521 = 5\frac{21}{100}\%$	$\frac{\$1 \times .05}{.88} = .0568 = 5\frac{68}{100}\%$

The second investment is the more advantageous.

2. Which is the better investment, to buy sixes at par, or sevens at 107?

3. Which will yield the larger profit, 8 per cent. stock at a premium of 20 per cent., or 5 per cent. stock at 80 per cent.?

4. If I invest $2000 in State stocks at 5 per cent., at par, and an equal amount at 6 per cent., at 90, what will be the difference of the proceeds of the investments at the end of 5 years?

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303. How do you determine which is the best investment?

INSURANCE.

304. **An Insurance Company** is a company chartered to insure against risks.

Insurance is an indemnity for loss or injury. It is made by companies or individuals, in consideration of a certain sum paid

Underwriters or **Insurers** are the companies or persons who insure.

305. **Insurance** is now limited, chiefly, to three classes of cases:

1. **Fire insurance**, or insurance against loss by fire.
2. **Marine insurance**, or insurance against loss by water.
3. **Life insurance**, or insurance against loss by death.

306. A **Mutual insurance** is one in which the insured share in the profits.

307. A **policy** is the mutual agreement of the parties.

308. **Premium** is the percentage paid by him who owns the property to him who insures it, as a compensation for risk.

309. All the cases of insurance are simple applications of the principles of percentage. There are four:

1. To find the premium, when the base and rate are known (Art. 259).

2. To find the rate, when the base and premium are known (Art. 261).

3. To find the base, when the rate and premium are known (Art. 262).

4. To find the percentage, when the premium is insured as well as the base. The base insured is then the premium plus the first base.

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304. *What is an insurance company? What is insurance? Who are underwriters?*

Examples.

1. What would be the premium for insuring a ship and cargo, valued at $147674, at 3½ per cent.?

2. What would be the insurance on a ship, valued at $47520, at ½ of 1 per cent.? At ⅓ of 1 per cent.?

3. What would be the insurance on a house, valued at $16800, at 1½ per cent.? At ¾ of 1 per cent.?

4. A merchant owns ⅔ of ¾ of a ship, valued at $24000, and insures his interest at 2½ per cent.: what does he pay for his policy?

5. What will it cost to insure a store, worth $5640, at ¾ per cent., and the stock, worth $7560, at ⅝ per cent.?

6. A carriage-maker shipped 15 carriages, worth $425 each: what must he pay to obtain an insurance upon them at 75 cents on a hundred dollars?

7. A merchant imported 150 hhd. of molasses, at 35 cents a gallon; he gets it insured for 3½ per cent. on the selling price of 50 cents a gallon: if the whole should be destroyed, and he get the amount of insurance, how much would he gain?

8. If I get my house and furniture, valued at $3640, insured at 4½ per cent., what would be my actual loss if they were destroyed?

9. The ship Astoria was valued at $20450, and her cargo at $25600, and was bound on a voyage from New York to Canton. The vessel was insured at the St. Nicholas office for $12000, at 2¾ per cent., and the cargo for $18500, at 3¼ per cent. The vessel foundered at sea: what was the entire loss of the owner?

10. Shipped from New York to the Crimea, 5000 barrels of

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305. To how many classes is insurance limited? What are they? —**306.** What is mutual insurance?—**307.** What is the policy?—**308.** What is premium?—**309.** How many cases are there of insurance? *What are they?*

flour, worth $10.50 a barrel. The premium paid was $2887.50: what was the rate per cent. of the insurance?

11. Paid $120 for insurance on my dwelling, valued at $7500: what was the rate per cent.?

12. A merchant imported 225 pieces of broadcloth, each piece containing 40 yards, at $3.50 a yard: he paid $1323 for insurance: what was the rate per cent.?

13. A merchant paid $1320 insurance on his vessel and cargo, which was $5\frac{1}{2}$ per cent. on the amount insured; how much did he insure?

14. A man pays $51 a year for insurance on his storehouse, at $1\frac{1}{2}$ per cent., and $126.45 on the contents, at $2\frac{1}{4}$ per cent: what amount of property does he get insured?

15. A person shipped 15 pianos, valued at $275 each. He insures them at 3 per cent., and also insures the premium at the same rate: what insurance must he pay?

16. A store and its contents are valued at $16750. The owner insures them at $1\frac{3}{4}$ per cent., and then insures the premium at the same rate: what insurance must he pay?

LIFE INSURANCE.

310. LIFE INSURANCE is an agreement to pay, in consideration of a premium, a specified amount to parties named in the agreement, in case of death, or other specified contingency.

311. To enable the company to fix their premiums at such rates as shall be both fair to the insured and safe to the association, they must know the *average* duration of life from any given time to its probable close. This average is called the "Expectation of Life," and is determined by collecting

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310. What is life insurance?—311. What is necessary to enable a company to fix *their premiums*? How is the expectation determined? *What do you understand* by the expectation of life?

from many sources the most authentic information in regard to the *average* duration of life from any period named.

If we take 100 infants, some will die in infancy, some in childhood, and some in old age. It has been found, from careful observation, that if the sum of their ages, after the last shall have died, be divided by 100, the quotient will be 38.72 very nearly: hence 38.72 is said to be the "Expectation of Life" at infancy.

The Carlisle Tables of England, which differ but little from those used in this country, show the "Expectation of Life" from 1 to 100 years. At 10 years old it is found to be 48.82; at 20, 41.46; at 30, it is 34.34; at 40, 27.61; at 50, it is 21.11; at 60, 14 years; at 70, 9.19; at 80, 5.51; at 90, 3.28; and at 100, it is 2.28 years.

If we wish the expectation of life, between the periods named in the table, we can readily find it by the rules of proportion. Thus, if we wished the expectation of life at 16 years, we should observe that, at 10 years, it has been found to be 48.82; at 20 years, it has been found to be 41.46; hence, for 10 years it varies 48.82 — 41.46 = 7.36 years:

Then, $10 : 6 :: 7.36 : 4.416$;

which number being subtracted from 48.82, leaves 44.40, the expectation of life at 16 years of age.

312. From the above facts, and the value of money (which is shown by the rate of interest), a company can calculate with great exactness the amount which they should receive annually for an insurance on a life for any number of years, or during its entire continuance.

313. The Mutual Life Insurance Company, of New York, has made a valuable collection of statistics, and its actuary, Shephard Homans, has constructed therefrom, a set of Tables, differing slightly from the Carlisle Tables. These changes have been adopted by several other Companies; but, as no settled standard has yet been fixed, the examples, as before, are wrought from the Carlisle

Tables. Tables are also published, showing the quarterly, semi-annual, and annual premiums that must be paid on each $100, or $1000 insured.

NOTE.—Experience has demonstrated that the risks are about equal on all ages between 14 and 25 years. Persons under the age of 25 years are charged for *whole life policies,* the rate at that age; though dividends are based on the true age. An extra charge, on the above rates, of one-half per cent. on the amount insured, is made for insuring the lives of women under the age of 48 years.

Examples.

1. A person, 20 years of age, finds that the premium, per annum, is $1.36 on $100: what must he pay to insure his life for 1 year for $8950?

2. A man, aged 40 years, wishes to insure his life for 5 years, and finds that the annual rate is $1.86 for $100: how much premium must he pay per annum on $12500?

3. A person, 38 years of age, obtains an insurance on his life for 5 years, at the rate of $1.75 per annum on $100: how much is the annual premium on $15000?

4. A person going to Europe, expecting to return in 2 years, effects an insurance on his life at $\frac{1}{2}$ of $\frac{3}{4}$ per cent premium on $100; he insures for $5000: what is the annual premium?

5. What will be the annual premium for insuring a person's life, who is 60 years of age, for $2000, at the rate of $4.91 on $100?

6. A person, at the age of 50 years, obtained an insurance at $4\frac{3}{4}$ per cent. per annum on each $100; he insured for $1500, and died at the age of 70. How much more was the insurance than the payments, without reckoning interest?

7. A gentleman, 47 years of age, going to China as ambassador, obtains an insurance on his life for $10000, by paying a premium of $2.71 per annum on every $100, and dies at the middle of the third year: reckoning simple interest on his *payments at 7 per cent.,* what is gained by the insurance?

ENDOWMENTS.

314. An Endowment is a certain sum to be paid at the expiration of a given time, in case the person on whose life it is taken shall live till the expiration of the time named.

The following table shows the value of an endowment purchased for $100, at the several periods mentioned in the column of ages, the endowment to be paid if the person attains the age of 21 years. The table is calculated under the hypothesis that money is worth 6 per cent. interest.

TABLE OF ENDOWMENTS,

Showing the sum to be paid at 21 years, if alive.

Age.		Age.		Age.	
Birth........	$376.84	5 years....	$210.53	13 years.....	$144.12
3 months....	344.28	6 "	198.83	14 "	137.86
6 "	331.46	7 "	188.83	15 "	131.83
9 "	318.90	8 "	179.97	16 "	125.97
1 year.......	306.58	9 "	171.91	17 "	120.31
2 "	271.03	10 "	164.46	18 "	114.89
3 "	243.69	11 "	157.43	19 "	109.70
4 "	225.42	12 "	150.64	20 "	104.74

This table shows that if $100 be paid at the birth of a child, he will be entitled to receive $376.84, if he lives to attain the age of 21 years. If $100 be paid when he is ten years old, he will be entitled to receive $164.46, if he lives to attain the age of 21 years. And similarly for other ages. We can easily find by proportion,

1st. How much must be paid, at any age under 21, to purchase a given endowment at 21; and,

2d. What endowment a sum paid at any age under 21, will purchase.

Examples.

1. What endowment, at 21, can be purchased for $250, paid at the age of 10 years?

2. What endowment, at 21, can be purchased for $360, paid *at the age* of 5 years?

3. If my child is 7 years old, and I purchase an endowment for $650, what will he receive if he attains the age of 21 years?

ANNUITIES.

315 An Annuity is a fixed sum of money to be paid at regular periods, generally, yearly, either for a limited time, or forever, in consideration of a given sum paid in hand.

The Present Value of an annuity is that sum which, being put at compound interest, would produce the sums necessary to pay the annuity.

The purchaser of an annuity should pay more than the compound interest; for the seller cannot afford to take the money of the purchaser, invest it, reinvest the interest, and pay over the entire proceeds.

Knowing the rate of interest on money, and the present value of an annuity, a close estimate may be made of the price it ought to sell for.

Table,

Showing the present value of an annuity of $1, *from* 1 *to* 30 *years, at different rates of interest.*

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0.952381	0.943396	16	10.837770	10.105895
2	1.859410	1.833393	17	11.274066	10.477260
3	2.723248	2.673012	18	11.689587	10.827603
4	3.545950	3.465106	19	12.085321	11.158116
5	4.329477	4.212364	20	12.462216	11.469921
6	5.075692	4.917324	21	12.821153	11.764077
7	5.786373	5.582381	22	13.163003	12.041582
8	6.463213	6.209794	23	13.488574	12.303379
9	7.107822	6.801692	24	13.798642	12.550358
10	7.721735	7.360087	25	14.093945	12.783356
11	8.306414	7.886875	26	14.375185	13.003166
12	8.863252	8.388844	27	14.643034	13.210534
13	9.393573	8.852683	28	14.898127	13.406164
14	9.898641	9.294984	29	15.141074	13.590721
15	10 379658	9.712249	30	15.372451	13.764831

To find the present value of an annuity for any rate, and for any time, we simply multiply the present value of an annuity of $1 for the same rate and time, by the annuity, and the product will be its present value.

Thus, the present value of an annuity of $600 for 8 years, at 6 per cent., is

$$\$6.209794 \times 600 = \$3725.8764\text{; that is,}$$

$$\text{pres. val. of } \$1 \times \text{annuity} = \text{pres. val.; hence,}$$

$$\text{annuity} = \frac{\text{pres. val.}}{\text{pres. val. of } \$1}\text{; therefore,}$$

316. To find what sum will produce a certain annuity at a given rate and for a given time.

Rule.—*Multiply the present value of an annuity of $1, at the given rate and for the given time, by the given annuity; the product will be that sum.*

317. To find what annuity a given sum will produce at a given rate, and for a given time.

Rule.—*Divide the given sum, or present value, by the present value of $1, for the given rate and time, and the quotient will be the annuity.*

Examples.

1. What is the present value of an annuity of $550, at 5 per cent., for 21 years?

2. What would be the value of an annuity that should yield eight hundred and thirty-five dollars a year for sixteen years, the interest being compound, and at the rate of 5 per cent per annum?

3. What is the present value of an annuity of $1500 a year, for 30 years, the compound interest being reckoned at 5 per cent.?

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314. What is an endowment? What does the table of endowments show? What may be found from the table?—**315.** What is an annuity? What is the present value of an annuity?—**316.** How do you find the present value of an annuity for a given rate and time?

4. What annuity, for twenty-four years, could be purchased for the sum of twenty-seven thousand five hundred and sixty dollars, the compound interest being reckoned at 6 per cent.?

5. Mr. Jones having a small fortune of $25000, and calculating that he would live about 20 years, purchased an annuity at 6 per cent., with an agreement that he would pay $20 a year to an invalid sister: what was his annual income from the investment after making that payment?

ASSESSING TAXES.

318. A Tax is a certain sum required to be paid by the inhabitants of a town, county, or State, for the support of government. It is generally collected from each individual, in proportion to the amount of his property.

In some States, however, every white male citizen, over the age of twenty-one years, is required to pay a certain tax. This tax is called a poll-tax; and each person so taxed is called a *poll.*

319. In assessing taxes, the first thing to be done is to make a complete inventory of all the property in the town, on which the tax is to be laid. If there is a poll-tax, make a full list of the polls, and multiply the number by the tax on each poll, and subtract the product from the whole tax to be raised by the town; the remainder will be the amount to be raised on the property. This remainder is the percentage or tax to be raised. The value of the property taxed is the base; hence this remainder, divided by the value of the property

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317. How do you find what annuity a given sum will produce, at a given rate and for a given time?—318. What is tax? How is it generally collected? What is a poll-tax?—319. What is the first thing to be done in assessing a tax? If there is a poll-tax, how do you find the amount? How, then, do you find the per cent. of tax to be levied on a dollar? How do you find the tax to be raised on each individual?

gives the rate. Each man's property, multiplied by the rate, gives his tax or percentage.

Examples.

A certain town is to be taxed $4280; the property on which the tax is to be levied is valued at $1000000. Now there are 200 polls, each taxed $1.40. The property of A is valued at $2800, and he pays 4 polls,

B's at $2400, pays 4 polls,
C's at $2530, pays 2 "
D's at $2250, pays 6 "

E's at $7242, pays 4 polls,
F's at $1651, pays 6 "
G's at $1600.80, " 4 "

What will be the tax on one dollar, and what will be A's tax; and, also, that of each on the list?

First, $1.40 × 200 = $280, amount of poll-tax.
$4280 − $280 = $4000, amount to be levied on property.
Then, $4000 ÷ $1000000 = .004 = $\frac{4}{10}$% = 4 mills on $1.

Now, to find the tax of each, as A's, for example:

A's inventory, - - - - - - - - - - -	$2800
	.004
	$11.20
4 polls, at $1.40 each, - - - - - - -	5.60
A's whole tax, - - - - - - - - - - -	$16.80

In the same manner, the tax of each person in the township may be found.

Examples.

1. In a county embracing 350 polls, the amount of property on the tax-list is $318200; the amount to be raised is as follows: for State purposes, $1465.50; for county purposes, $350.25; and for town purposes, $172.97. By a vote of the county, a tax is levied on each poll of $1.50: how much per cent. will be laid upon the property?

2. In a county embracing a population of 98415 persons, a

tax is levied for town, county, and State purposes, amounting to $100406. Of this sum, a part is raised by a tax of 25 cents on each inhabitant, and the remainder by a tax of two mills on the dollar: what is the amount of property taxed?

3. In a county, embracing a population of 56450 persons, a tax is levied for town, county, and State purposes, amounting to $87467; the personal and real estate is valued at $4890300. Each inhabitant is taxed 25 cents: what per cent. is the tax, and how much will a man's tax be, who pays for five persons, and whose property is valued at $5400?

What is B's tax, who is assessed for 2 persons, and whose property is valued at $3760.50?

4. A banking corporation, consisting of 40 persons, was taxed $957.50; their property was valued at $125000, and each person was assessed 50 cents: what per cent. was their tax, and what was a man's tax, who paid for 1 person, and whose share was assessed for $2000?

5. What sum must be assessed to raise a net amount of $5674.50, allowing 2½ per cent. commission on the money collected?

6. Allowing 4 per cent. for collection, what sum must be assessed to raise $21346.75 net?

7. In a certain township, it becomes necessary to levy a tax of $4423.2475, to build a public hall. The taxable property is valued at $916210, and the town contains 150 polls, each of which is assessed 50 cents. What amount of tax must be raised to build the hall, and pay 5 per cent. for collection and what is the tax on a dollar?

What is a person's tax who pays for 3 polls, and whose personal property is valued at $2100, and his real estate at $3000?

What is G's tax, who is assessed for 1 poll, and $1275.50?

What *is H's tax, who is* assessed for 1 poll, and $2456?

8. The people of a school district wish to build a new school-house, which shall cost $2850. The taxable property of the district is valued at $190000: what will be the tax on a dollar, and what will be a man's tax, whose property is valued at $7500?

How much is Mr. Merchant's tax, whose personal and real estate are assessed for $1200?

9. In a school district, a school is supported by a rate-bill. A teacher is employed for 6 months, at $60 a month; the fuel and other contingencies amount to $66. They drew $41.60 public money, and the whole number of days' attendance was 7688: what was D's tax, who sent 148 days?

What was F's tax, who sent 184½ days?

EQUATION OF PAYMENTS.

320. Equation of Payments is the process of finding the average time of payment of several sums due at different times, so that no interest shall be gained or lost.

The average or equated time, is the time that elapses from the time at which we begin to reckon interest to the time of payment of all the debts. The equated *date* is the date of payment of all the debts.

321. When the times of payment are reckoned from the same date.

1. B owes Mr. Jones $57: $15 is to be paid in 6 months; $18 in 7 months; and $24 in $10 months: what is the average time of payment, so that no interest shall be gained or lost?

Analysis.—The interest of $15 for 6 months, is the same as the interest of $1 for 90 months; the interest of $18 for 7 months, is the same as the interest of $1 for 126 months; and the interest of $24 for 10 months, is the same as the interest *of $1 for 240* months; hence, the sum of

OPERATION.

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$15 ×  6 =  90
$18 ×  7 = 126
$24 × 10 = 240
———       ————
 57     57)456(8
           456
           ———
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these products, 456, is the number of months it would take $1 to produce the required interests. Now, the sum of the payments, $57, will produce the same interest in one *fifty-seventh part of the time;* that is, in 8 months: hence, to find the average time of payment:

Rule.—*Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments: the quotient will be the average time.*

Examples.

1. A merchant owes $1200, of which $200 is to be paid in 4 months, $400 in 10 months, and the remainder in 16 months: if he pays the whole at once, at what time must he make the payment?

2. A owes B $2400; one-third is to be paid in 6 months, one-fourth in 8 months, and the remainder in 12 months: what is the mean time of payment?

3. A merchant has due him $4500; one-sixth is to be paid in 4 months, one-third in 6 months, and the rest in 12 months: what is the equated time for the payment of the whole?

4. A owes B $1200, of which $240 is to be paid in three months, $360 in five months, and the remainder in 10 months; what is the average time of payment?

5. Mr. Swain bought goods to the amount of $3840, to be paid for as follows, viz.: one-fourth in cash, one-fourth in 6 months, one-fourth in 7 months, and the remainder in one year: what is the average time of payment?

6. A man bought a farm for $5000, for which he agreed to pay $1000 down, $1200 in 3 months, $800 in 8 months, $1500 in 10 months, and the remainder in one year: if he pay the whole at once, what would be the average time of payment?

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320 What is equation of payments? What is the average or equated time?—*321. How do you find equated time?*

7. A person owes three notes: the first is for $200, payable July 1st; the second for $150, payable August 1st; and the third for $250, payable August 15th what is the average time, reckoned from July 1st?

322. When the times are reckoned from different dates.

1 E. BOND,

Bought of TRUST & Co.

1861	Aug. 1,	450 yds. muslin, at 10 cents - - -	$45	00
	Aug 16,	800 yds. calico, at 12½ cents - - -	100	00
	Sept. 5,	720 yds. bombazine, at 80 cents - -	576	00
	Oct. 1,	300 yds. cloth, at $3.50 - - - -	1050	00

On what day may the whole be supposed to have been purchased; or, what is the equated date of purchase?

ANALYSIS.—The owner parted with his goods, and therefore with their values, at the dates specified; and the question is, to find at what time he could have sold the whole at the same advantage. Reckoning from Aug. 1st, the earliest date, he had the use of $45, the amount of the first sale, for no time; of $100 for 15 days, viz., from Aug. 1st to Aug. 16th; of $576 for 35 days, viz., from Aug. 1st to Sept. 5th; of $1050 for 61 days, from Aug. 1st to Oct. 1st: then, by the preceding Article, we have the following operation:

OPERATION.

$45 \times 0 = 000$

$100 \times 15 = 1500$

$576 \times 35 = 20160$

$1050 \times 61 = 64050$

$1771 \qquad 1771)85710(48\tfrac{702}{1771}$ = equated time.

7084

14870 48 days from Aug. 1st.

14168 equated date, Sept. 18th.

702

1771

---

322. From what date may the equated time be reckoned? What is the multiplier of the date used as the point of departure? What do you do when the quotient contains a fraction? What is the rule when the times are reckoned for different dates?

Instead of reckoning from the earliest date, we might have reckoned backward from the latest date.

From Oct. 1st to Sept. 5th there are 26 days.
" " to Aug. 16th " 46 "
" " to Aug. 1st " 61 "

$$
\begin{array}{rcrcr}
45 & \times & 61 & = & 2745 \\
100 & \times & 46 & = & 4600 \\
576 & \times & 26 & = & 14976 \\
\underline{1050} & \times & 0 & = & \underline{0000} \\
1771 & & & & 22321 \div 1771 = 12\frac{1069}{1771} \text{ days.}
\end{array}
$$

13 days from Oct. 1st or Sept. 18th.

Rule.—I. *From the date assumed as the point of reckoning, find the intervening days to each date, and multiply each amount by its number of days:*

II. *Divide the sum of these products by the sum of the payments, and the quotient will be the equated time in days. This number, reckoned from the assumed date, will give the equated date.*

NOTE.—1. The equated time may be reckoned from the earliest or any previous date, or from the latest date or any date subsequent to it

2. The multiplier of the date used as the point of reckoning is 0, and the corresponding product is nothing. The payment must be added, in finding the sum of the payments.

3. When the fraction in the quotient is less than ½, it is rejected; when greater than ½, 1 is added to the days

2. Mr. Johnson sold, on a credit of 8 months, the following bills of goods:

April 1st, a bill of $4350,
May 7th, a bill of 3750,
June 5th, a bill of 2550

At what time will the whole become due?

NOTE.—Find the equated date of purchase, to which add the time of credit; if the times of credit vary, find the times of payment, and then equate.

3. A purchased of B the following bill of goods, on different times of credit:

May 1st,	1857,	a bill	amounting	to	$800	on	3	months
June 1st,	"	"	"	"	700	"	3	"
" 15th	"	"	"	"	900	"	4	"
July 25th,	"	"	"	"	1000	"	6	"

What is the equated time for the payment of the whole, and on what day, reckoned from Aug. 1st, is the bill due?

4. A person purchased the following bills of goods, on different times of credit:

Jan. 1st,	1855,	a bill	amounting	to	$367.20	on	4	months.
" 28th,	"	"	"	"	901.80	"	3	"
Feb. 24th,	"	"	"	"	826.38	"	5	"
March 30th,	"	"	"	"	854.88	"	6	"
May 1st,	"	"	"	"	396.50	"	4	"

What is the average time of payment from the time the first bill falls due? On what day is the payment made?

5. A flour merchant bought at one time 150 barrels of flour, at $8 a barrel; 15 days afterward he bought 176 barrels, at $8.50 a barrel; 25 days after that he bought 200 barrels, at $9 a barrel: how many days after the first purchase would be the equated time of payment?

323. To find how long a sum of money must be at interest to balance the interest on a given sum for a given time.

1 If A lends B $700 for 3 months, how long ought B to lend A $500 to balance the interest?

Analysis.—Since $700, in 3 months, will produce as much interest as $2100 in 1 month, it will require as many months for $500 to produce the same interest, as 500 is contained times in 2100.

OPERATION.

$700 \times 3 = 2100$;

$2100 \div 500 = 4\frac{1}{5}$

Ans. $4\frac{1}{5}$ months

2. A lends B his note for $900, payable in 5 months: for what length of time should B lend to A his note for $480, to *balance the favor?*

3. C buys of D 100 barrels of flour, at $7½ per barrel, and in payment gives his note for 3 months; D buys of C 500 bushels of wheat at 80 cents per bushel, and gives his note in payment: how long must this note run, that each may have an equal use of the other's money?

324. To find how long the balance may be kept, when payments are made before they are due.

1. A owes B $800, payable in 6 months; at the expiration of 4 months, he pays $500: how long beyond the 6 months should A retain the balance, so that neither shall make or lose interest?

ANALYSIS.—A has the right to retain the $800 for 6 months, or $4800 for 1 month. He retains $500 for 4 months, or $2000 for 1 month. Hence, he may still retain $2800 for 1 month, or the balance, $300, as many months as 300 is contained times in $2800; or, 9⅓ months from the date of the debt; or, 9⅓ − 6 = 3⅓ months beyond the time of six months.

OPERATION.

$$\begin{array}{rcr} 800 \times 6 & = & 4800 \\ \underline{500} \times 4 & = & \underline{2000} \\ 300 & & 2800 \end{array}$$

$$2800 \div 300 = 9\tfrac{1}{3}$$

2. C owes D $2500, payable in 4 months; but at the end of 3 months pays him $1600: how long after the payment of $1600 should the remainder be retained to balance the account?

3. One merchant owes another $1600, payable in 6 months, but at the end of 3 months pays $400; at the end of 4 months $400, and at the end of 5 months $300: how long, from the ~~last payment~~, may the balance be retained to square the account?

4. Mr. Jones owes his grocer $900, due 9 months from the 1st day of January; June 15, paid $520: on what day was the remainder due?

5. A note for $500, dated November 6th, 1856, payable in 8 months, was given by E to F. On December 3d, E paid $350: at what time was the balance due?

325. To find the cash balance of an account.

An account is said to be *balanced* when the sum of tl items on the debit side is equal to the sum of the items c the credit side. When these two sums are unequal, such a amount is added to the less as will make the sum equal t the greater. This is called the *balance*. There are three kin of balances:

1st The *merchandise balance*, in which interest on the iten is not considered;

2d The *interest balance*, which adjusts the interest on th two sides of an account; and

3d. The *cash balance*, which arises from combining the me chandise balance with the interest balance.

Accounts are settled either by cash or by note. In asce taining the cash balance of an account, interest is allowed o all the items of both sides; the balance of interest makes new item, and may belong to either side of the account.

Ascertain the cash balance of the following account on th 25th of April, 1850:

Dr.	S. SNODGRASS.			*Cr.*
1850.	April 1st, To goods,	$375.00	April 7th, By goods,	$675.0
	" 17th, " "	268.00	" 15th, " "	380.0
	" 25th, " "	175.00	" 25th, Bal. of Int.,	.9
	Cash balance,	237.93		
		$1055.93		$1055.9

ANALYSIS.—Reckoning backwards from April 25th, we find th days for which we charge interest, and these are used as multiplier The interest of $375 for 24 days is the same as the interest of $1 fo 9000 days; and so of the other items. The difference of the sum of these products, is the number of days which $1 must be at intere to produce the balance of interest, and the balance always goes wit the larger sum of the products.

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325. *What is* the rule for finding the cash balance of an account

OPERATION.

Debtor Items.	*Creditor Items.*
$375 \times 24 = 9000$	$675 \times 18 = 12150$
$268 \times 8 = 2144$	$380 \times 10 = 3800$
$175 \times 0 = 0000$	15950
11144	11144
	4806

Then, $4806 \times \frac{.07}{360} = 0.93 +$, balance of interest.

Rule.—I. *Take the latest date of the account, or any later date at which the balance is to be struck, as the point of reckoning, and find the days between this date and the date of each item; and consider these days as multipliers:*

II. *Multiply each item by its multiplier; then take the difference of the sums of these products, and multiply it by the interest for one day: the result will be the interest balance, which is to be added to the side having the greater sum:*

III. *Then find the cash balance.*

NOTES.—1. If the cash balance had been required on any day after the 25th of April, the mode of proceeding would have been *exactly* the same.

2. In the examples, the rate of interest will be taken at 7 per cent. and 360 days in the year.

3. After the balance of interest is found, the cash balance is obtained by adding the two sides of the account, taking the difference of the sums, and placing it on the smaller side of the account.

4. If the cash balance is settled by a note, interest should run on the note from the date of the cash balance to the time of payment.

5. Let the pupil find the interest and cash balance in each of the following examples:

2. What is the balance of interest, and what the cash balance, *on the following* account, on March 20th?

Dr. S. JOHNSON. *Cr.*

1856. Jan. 1,	To merch.,	$500	Jan. 5,	By cash,	$350
" 16,	" cash,	450	" 19,	" merch.,	780
Feb. 5,	" merch.,	680	" 25,	" "	250
" 24,	" "	300	Feb. 15,	" cash,	600
Mar. 1,	" cash,	150		Cash balance,	700.58
" 16,	" merch.,	600			
	Interest balance,	.58			
		$2680.58			$2680.58

NOTE.—1. When the items have the same or different times of credit allowed, find when the items are payable, and then proceed as before.

2. If the cash balance is required on a day *previous* to the latest date of the items, find the cash balance for this latest date; then find the present value for the given date: this will be the cash balance.

3. Allowing a credit of six months on each item, what is the interest and cash balance, Feb. 1st, 1856?

Dr. R. SHERMAN. *Cr.*

1855. July 1st,	To merch.,	$750	Feb. 6th,	By merch.,	$800
" 17th,	" "	600	Mar. 7th,	" "	900
" 25th,	" "	800		Interest balance,	46.20
				Cash balance,	403.80
		$2150			$2150.00

4. Allowing a credit of 3 months on each of the items of the following account, what would be the interest and cash balance on October 31st, 1856?

Dr. R. RIVERS. *Cr.*

1856. May 1,	To merch.,	$500	May 6th,	By cash,	$400
" 20,	" "	675	" 25th,	" mer.,	620
June 6th,	To cash.	350	June 16th,	" cash,	900
July 9th,	" merch.,	175	July 20th,	" mer.,	400
	Cash balance,	620.70		Interest balance,	.70
		$2320.70			$2320.70

326. To find the equated time of settling an account containing debtor and creditor items.

To equate an account, is to fix the time of payment of the merchandise balance in such manner that the interest of each side shall be equal. The object of equating accounts is two-fold: 1st. To find for what time interest must be charged on the balance; 2d. To find the date of a note, whose running time is fixed, and which is given in payment of the balance.

Properly, the face of the note should be the sum, whose present value is the balance.

If the note is given without interest, then its face is the balance; and if the note becomes payable before the latest date, then interest must be charged for the remaining time. The process of equating accounts is similar to that of finding the cash balance: hence, we have the following

Rule.—I. *Find the merchandise balance:*

II. *Find the number of days between the latest date of either side, and the date of each item, and consider these numbers as multipliers:*

III. *Multiply each item by its multiplier; then take the difference of the sums of these products and divide it by the merchandise balance: the quotient is the number of days, which, carried backward or forward from the latest date, will give the equated date.*

Note.—When the greater sum of the *items* and the greater sum of the *products* fall on the *same* side of the account, the quotient is to be carried *backward* from the latest date: and forward, when these sums are found on different sides.

1. Equate the following account:

Dr. JAMES MARTIN. *Cr.*

1861. Jan. 16,	To merch.,	$716.75	1861. Jan. 19	By cash,	$500.15
" 25,	" "	900.00	Feb. 1,	" merch.,	1915.25
Feb. 7,	" "	2765.50	Mar. 7,	" cash,	1200.00
Mar. 19,	" cash	791.25	April 2,	" merch.,	712.00

2. What is the balance of the following account—when due?

Dr. ISRAEL JENKINS. *Cr.*

1835. May	6,	To merch.,	$7150.00	1835. May	9,	By cash,	$2450.00
"	16,	" "	475.00	"	21,	" "	915.00
June	17,	" "	3475.25	June	12,	" merch.,	4165.50
"	21,	" "	1516.50	"	19,	" merch.,	2915.50
July	5,	" "	279.00				

3. What is the equated date for the payment of the balance of the following account?

Dr. JACOB PARTON. *Cr.*

1861. June	6,	To merch.,	$8000.00	1861. June	2,	By merch.,	$7450.75
"	23,	" "	1756.50	"	19,	" "	2695.25
"	30,	" cash,	2890.75	July	10,	" "	1865.50
July	12,	" note,	3000.15	"	16,	" "	970.00

327. Account of Sales.

An Account of Sales is an account of the goods sold, with their prices, the charges thereon, and the net proceeds. Such an account a consignee transmits to the consignor. The net proceeds of such sale is nothing but the cash balance, due at the equated date. We will illustrate by the following example:

ACCOUNT OF SALES OF FLOUR FOR A. MATTHEWS, CHICAGO.

Date.	Purchaser.	Description.	Price.	
1868.				
Nov. 5,	James Jackson,	75 bbls. superfine,	$6.90	$517.50
Dec. 6,	Robert Fisk,	89 bbls. Excelsior,	7.20	640.80
Dec. 19,	Francis Sutton,	120 bbls. fine,	6.30	756.00
Dec. 23,	James Lyon,	66 bbls. ordinary,	5.90	389.40
		350 bbls.		$2303.70

CHARGES.

Nov. 10th, cash paid transportation, - - -	$76
Nov. 6th, insurance, - - - - - - - - - - -	9
Dec. 23d, storage, - - - - - - - - - - -	10
Commission on $2303.70, at 2½%, - - - -	57.58
Total, - - - - -	$152.58

ALLIGATION.

328. **ALLIGATION** is the process of mixing substances in such a manner that the value of the compound shall be equal to the sum of the values of the several ingredients. It is divided into two parts: Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL.

329. **ALLIGATION MEDIAL** is the method of finding the price or quality of a mixture of several simple ingredients whose prices and quantities are known.

1. A grocer would mix 200 pounds of lump sugar, worth 13 cents a pound, 400 pounds of Havana, worth 10 cents a pound, and 600 pounds New Orleans, worth 7 cents a pound: what should be the price of the mixture?

ANALYSIS.—The quantity, 200 lb., at 13 cents a pound, costs $26; 400 pounds, at 10 cents a pound, costs $40; and 600 lb. at 7 cents a pound, costs $42: hence, the entire mixture, consisting of 1200 lb., costs $108. Now, the price of the mixture will be as many cents as 1200 is contained times in 10800 cents, viz., 9 times: hence, to find the price of the mixture,

OPERATION.

$$\begin{array}{rcrcl} 200 & \times & 13 & = & 26.00 \\ 400 & \times & 10 & = & 40.00 \\ \underline{600} & \times & 7 & = & \underline{42.00} \\ 1200 & & & &)108.00(9 \text{ cts.} \end{array}$$

Rule.—I. *Find the cost of the mixture:*

II. *Divide the cost of the mixture by the sum of the simples, and the quotient will be the price of the mixture.*

Examples

1. If 1 gallon of molasses, at 75 cents, and 3 gallons, at 50 cents, be mixed with 2 gallons, at 37½, what is the mixture worth a gallon?

2. If teas at 37½, 50, 62½, 80, and 100 cents per pound, be mixed together, what will be the value of a pound of the mixture?

3. If 5 gallons of alcohol, worth 60 cents a gallon, and 3 gallons, worth 96 cents a gallon, be diluted by 4 gallons of water, what will be the price of one gallon of the mixture?

4. A farmer sold 50 bushels of wheat, at $2 a bushel; 60 bushels of rye, at 90 cents; 36 bushels of corn, at 62½ cents; and 50 bushels of oats, at 39 cents a bushel: what was the average price per bushel of the whole?

5. During the seven days of the week, the thermometer stood as follows: 70°, 73°, 73½°, 77°, 70°, 80½°, and 81°: what was the average temperature for the week?

6. If gold 18, 21, 17, 19, and 20 carats fine, be melted together what will be the fineness of the compound?

7. A grocer bought 34 lb. of sugar at 5 cents a pound, 102 lb., at 8 cents, 136 lb. at 10 cents a pound, and 34 lb. at 12 cents a pound. He mixed it together, and sold the mixture so as to make 50 per cent. on the cost: what did he sell it for per pound?

8. A merchant sold 8 lb. of tea, 11 lb. of coffee, and 25 lb. of sugar, at an average of 15 cents a pound. The tea was worth 30 cents a pound; the coffee, 25 cents a pound; and the sugar 7 cents a pound: did he gain or lose, and how much?

ALLIGATION ALTERNATE.

330. ALLIGATION ALTERNATE is the method of finding what proportion of several simples, whose prices or qualities are known, must be taken to form a mixture of any required price or quality. It is the reverse of Alligation Medial, and may be proved by it.

The process of Alligation Alternate is founded on an equal of gain and loss. In selling a mixture at an average pr there is a *gain* on each simple below that price, and a *loss* each simple above that price. The gain must be exactly eq to the loss, otherwise the value of the compound would not *an average price.*

CASE I.

331. To find the proportional parts.

1. A miller would mix wheat, worth 12 shillings a bushel; corn, worth 8 shillings; and oats, worth 5 shillings, so as to make a mixture worth 7 shillings a bushel: what are the proportional parts of each?

OPERATION.

		A.	B.	C.	D.	E.
7s.	oats, 5s.	$\frac{1}{2}$	$\frac{1}{2}$	5	1	6 or 3
	corn, 8s.		1		2	2 " 1
	wheat, 12s.	$\frac{1}{5}$		2		2 " 1

ANALYSIS.—On every bushel put into the mixture, whose price is *less* than the mean price, there will be a *gain;* on every bushel whose price is *greater* than the mean price, there will be a *loss;* and since the gains and losses must balance each other, we must connect an ingredient on which there is a gain with one on which there is a loss.

A bushel of oats, when put into the mixture, will bring 7 shillings, giving a gain of 2 shillings; and to gain 1 shilling, we must take half as much, or $\frac{1}{2}$ a bushel, which we write opposite 5s. in column A.

On 1 bushel of wheat there will be a loss of 5 shillings; and to make a loss of 1 shilling, we must take $\frac{1}{5}$ of a bushel, which we write in column A: $\frac{1}{2}$ and $\frac{1}{5}$, are called *proportional numbers.*

Again, comparing the oats and corn, there is a gain of 2 shillings on every bushel of oats, and a loss of 1 shilling on every bushel of corn: to *gain* 1 shilling on the oats, and *lose* 1 shilling on the corn, we must take $\frac{1}{2}$ a bushel of the oats, and 1 bushel of the corn: these numbers are written in column B. Two simples, thus compared, are called a *couplet:* in one, the price of 1 is less than the mean price, and in the other it is greater.

If every time we take $\frac{1}{2}$ a bushel of oats we take $\frac{1}{5}$ of a bushel of wheat, the gain and loss will balance; and if every time we take $\frac{1}{2}$ a bushel of oats we take 1 bushel of corn, the gain *and loss will balance:* hence, *if the* proportional numbers *of a couplet be multiplied by any number, the gain and loss denoted by the products will balance.*

When the proportional numbers, in any column, are fractional (as in columns A and B), multiply them by the least common multiple of their denominators, and write the products in new columns C and D. Then add the numbers in columns C and D standing opposite each simple, and if their sums have a common factor, divide by it: the last result will be the proportional numbers.

NOTE.—The answers to the last, and to all similar questions, will be infinite in number, for two reasons:

1st. If the proportional numbers in column E be multiplied by any number, integral or fractional, the products will denote proportional parts of the simples.

2d. If the proportional numbers of *any couplet* be multiplied by any number, the gain and loss in that couplet will still balance, and the proportional numbers in the final result will be changed.

Rule.—I. *Write the prices or qualities of the simples in a column, beginning with the lowest, and the mean price or quality at the left:*

II. *Opposite the first simple write the part which must be taken to gain* 1 *of the mean price, and opposite the other simple of the couplet write the part which must be taken to lose* 1 *of the mean price, and do the same for each simple:*

III. *When the proportional numbers are fractional, reduce them to integral numbers, and then add those which stand opposite the same simple; if the sums have a common factor, reject it: the result will denote the proportional parts.*

Examples.

1. What proportions of coffee, at 8 cents, 10 cents, and 14 cents per pound, must be mixed together so that the compound shall be worth 12 cents per pound?

2. A merchant has teas worth 40 cents, 65 cents, and 75 cents a pound, from which he wishes to make a mixture worth

60 cents a pound: what is the smallest quantity of each that he can take and express the parts by whole numbers?

3. A farmer sold a number of colts at $50 each, oxen at $40, cows at $25, calves at $10, and realized an average price of $30 per head: what was the smallest number he could sell of each?

4. What is the smallest quantity of water that must be mixed with wine worth 14s. and 15s. a gallon, to form a mixture worth 13s. a gallon, when all the parts are expressed by whole numbers?

CASE II.

332. When the quantity of one of the simples is given.

1. A farmer would mix rye worth 80 cents a bushel, and corn worth 75 cents a bushel, with 66 bushels of oats worth 45 cents a bushel, so that the mixture shall be worth 50 cents a bushel: how much must be taken of each sort?

OPERATION.

		A.	B.	C.	D.	E.	F.
50	45	$\frac{1}{5}$	$\frac{1}{5}$	6	5	11	66
	75		$\frac{1}{25}$		1	1	6
	80	$\frac{1}{30}$		1		1	6

ANALYSIS.—Find the proportional parts as in Case I.: they are 11, 1 and 1. But we are to take 66 bushels of oats in the mixture; hence, each proportional number is to be taken 6 times; that is, as many times as there are units in the quotient of 66 ÷ 11.

Rule.—I. *Find the proportional numbers as in Case I., and write each opposite its simple:*

II. *Find the ratio of the proportional number corresponding to the given simple, to the quantity of that simple to be taken, and multiply each proportional number by it.*

NOTE.—*If we multiply* the numbers in either or both of the *columns C or D by any* number, the proportion of the numbers in

column E will be changed Thus, if we multiply column D by 12 we shall have 60 and 12, and the numbers in column E become 66, 12 and 1, numbers which will fulfil the conditions of the question.

Examples.

1. What quantity of teas at 12s. 10s. and 6s. must be mixed with 20 pounds, at 4s. a pound, to make the mixture worth 8s. a pound?

2 How many pounds of sugar, at 7 cents and 11 cents a pound, must be mixed with 75 pounds, at 12 cents a pound, so that the mixture may be worth 10 cents a pound?

3. How many gallons of oil, at 7s., 7s. 6d., and 9s. a gallon, must be mixed with 24 gallons of oil, at 9s. 6d. a gallon, so as to form a mixture worth 8s. a gallon?

4. Bought 10 knives at \$2 each: how many must be bought at \$$\frac{3}{4}$ each, that the average price of the whole shall be \$1$\frac{1}{4}$?

5. A grocer mixed 50 lb. of sugar worth 10 cents a pound, with sugars worth 9$\frac{1}{4}$ cents, 7$\frac{1}{2}$ cents, 7 cents, and 5 cents a pound, and found the mixture to be worth 8 cents a pound; how much did he take of each kind?

CASE III.

333. When the quantity of the mixture is given.

1. A silversmith has four sorts of gold, viz., of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine; he would make a mixture of 42 ounces of 17 carats fine: how much must he take of each sort?

OPERATION.

		A.	B.	C.	D.	E	F.	G.	H.
17	15	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	7	5	3	15	30
	20			$\frac{1}{3}$			2	2	4
	22		$\frac{1}{5}$			2		2	4
	24	$\frac{1}{7}$			2			2	4

PROPORTIONAL PARTS.

$$15 + 2 + 2 + 2 = 21 ; \quad 42 \div 21 = 2.$$

Rule.—I. *Find the proportional parts as in Case I.:*

II. *Divide the quantity of the mixture by the sum of the proportional parts, and the quotient will denote how many times each part is to be taken. Multiply the parts separately by this quotient, and each product will denote the quantity of the corresponding simple.*

Examples.

1. A grocer has teas at 5s., 6s., 8s., and 9s. a pound, and wishes to make a compound of 88 lb., worth 7s. a pound: how much of each sort must be taken?

2. A liquor dealer wishes to fill a hogshead with water, and with two kinds of brandy at $2.50 and 3.00 per gallon, so that the mixture may be worth $2.25 a gallon: in what proportions must he mix them?

3. A person sold a number of sheep, calves, and lambs, 40 in all, for $48: how many did he sell of each, if he received for each calf 1\frac{3}{4}$, each sheep 1\frac{1}{4}$, and each lamb $$\frac{3}{4}$?

4. A merchant sold 20 stoves for $180; for the largest size he received $19 each, for the middle size, $7, and for the small size, $6: how many did he sell of each kind?

5. A vintner has wines at 4s., 6s., 8s., and 10s. per gallon; he wishes to make a mixture of 120 gallons, worth 5s. per gallon: what quantity must he take of each?

6. A tailor has 24 garments, worth $144. He has coats, pantaloons, and vests, worth $12, $5, and $2 each, respectively: how many has he of each?

7. A merchant has 4 pieces of calico, each worth 24, 22 20, and 15 cents a yard: how much must he cut from each piece to exchange for 42 yards of another piece, worth 17 cents a yard?

8. A man paid $70 to 3 men for 35 days' labor; to the first *he paid $5 a day, to* the second, $1 a day, and to the *hird, $$\frac{1}{4}$ a day: how* many days did each labor?

CUSTOM HOUSE BUSINESS.

334. All merchandise imported into the United States, must be landed at certain ports, called Ports of Entry. On such merchandise the General Government has imposed a greater or less tax, called a *duty*.

335. A Port of Entry is a port where foreign merchandise may be delivered, and where there is a Custom-house for appraisement and the payment of duties.

336. Tonnage Duties are taxes levied on vessels, according to their size, for the privilege of entering ports.

337. All duties, levied by law, on imported goods, are of two kinds: *specific* and *ad valorem*.

338. Specific Duty is a certain sum levied on a particular kind of goods named; as so much per square yard on cotton or woolen goods, so much per ton weight on iron, &c.

339. Ad Valorem Duty is a certain rate per cent. on the invoice.

340. An Invoice is an inventory of goods to be landed, directed to the person who imports them, and stating their cost at the place from which they were exported. Thus, an *ad valorem* duty of 15% on English cloths, is a duty of 15% on the cost of the cloths imported from England.

341. The revenues of the country are under the general direction of the Secretary of the Treasury, and to secure their faithful collection, the government has appointed various officers at each port of entry, or place where goods may be landed.

342. The laws of Congress provide, that the cargoes of al vessels freighted with foreign goods or merchandise, shall b weighed or gauged by the custom-house officers at the po *to which they* are consigned. As duties are only to be pai *on the articles,* and not on the boxes, casks, and bags whi contain them, certain deductions are made from the weights and measures, called *Allowances.*

GROSS WEIGHT is the entire weight of the goods, together with that of the casks, bags, and boxes which contain them.

NET WEIGHT is what remains after all deductions are made

DRAFT was an allowance formerly made from the gross weight, but it is now abolished by law.

TARE is an allowance made for the weight of the boxes, barrels, or bags containing the commodity, and is of three kinds: 1st, Legal tare; 2d, Customary tare; and, 3d, Actual tare, or such as is found by removing the goods and actually weighing the casks and boxes in which they are contained.

In estimating the allowance for tare on all chests, boxes, cases, casks, bags, or other envelope or covering, of all articles imported liable to pay duty, where the original invoice is produced at the time of making the entry, and the tare shall be specified therein, it shall be lawful for the Collector, if he shall see fit, to estimate the said tare according to the said invoice: but in all other cases the real tare shall be allowed, and may be ascertained by such regulations as the Secretary of the Treasury may from time to time prescribe.

On liquors in casks, *customary tare* is sometimes allowed on he supposition that the cask is not full, or what is called its *ctual wants;* and then an allowance of 5 per cent. for leakage.

A tare of 10 per cent. is allowed on porter, ale, and beer, bottles, on account of breakage, and 5 per cent. on all other quors in bottles. At the custom-house, bottles of the com on size are estimated to contain 2¾ gallons the dozen. For bles of Tare and Duty, see Ogden on the Tariff of 1842.

Examples.

1. What is the net weight of 25 hogsheads of sugar, the ross weight being 66 cwt. 3 qr. 14 lb.; the actual tare, 1 lb. per *hogshead.*

		cwt.	*qr.*	*lb.*	
		66	3	14	gross.
25 × 11 = 275 lb.	- -	2	3	0	tare.
	Ans.	64	0	14	net.

2. If the actual tare be 4 lb. per hundred, what will be the tare on 6 T. 2 cwt. 3 qr. 14 lb.?

Tare for 6 T. or 120 cwt. = 480 lb.
2 cwt. = 8
3 qr. = 3
14 lb. = $0\frac{14}{25}$
Tare - - $491\frac{14}{25}$ lb.

3. What will be the cost of 3 hogsheads of tobacco at $9.47 per cwt. net, the gross weight and actual tare being of

		cwt.	*qr.*	*lb.*			*lb.*
No. 1	- -	9	3	24	- -	tare	146
" 2	- -	10	2	12	- -	"	150
" 3	- -	11	1	24	- -	"	158

4. At 21 cents per lb., what will be the cost of 5 hhd. of coffee, the actual tare and gross weight being as follows:

		cwt.	*qr.*	*lb.*			*lb.*
No. 1	- - - -	6	2	14	- - - -	tare	94
" 2	- - - -	9	1	20	- - - -	"	100
" 8	- - - -	6	2	22	- - - -	"	88
" 4	- - - -	7	2	24	- - - -	"	89
" 5	- - - -	8	0	13	- - - -	"	100

5. What is the net weight of 18 hhd. of tobacco, each weighing gross 8 cwt. 3 qr. 14 lb.; actual tare 16 lb. to the cwt.?

6. What is the net weight and value of 80 kegs of figs, gross weight 7 T. 11 cwt. 3 qr., actual tare 12 lb. per cwt., at $2.31 per cwt.?

7. A merchant bought 19 cwt. 1 qr. 24 lb. gross, of tobacco, at $24.28 per cwt.; and 12 cwt. 3 qr. 19 lb. gross, at $28.56 per cwt.; the actual tare of the former was 149 lb., and the latter, 49 lb.: what did the tobacco cost him, net?

8. A grocer bought 17¼ hhd. of sugar, each 10 cwt. 1 qr. 14 lb., tare 11 lb. per cwt.: what is the value at $7.50 per cwt. net?

9. A merchant bought 7 hogsheads of molasses, each weighing 4 cwt. 3 qr. 14 lb. gross; tare 7 lb. per cwt. in addition to 8 lb. per hogshead, and damage in the whole 99¾ lb.: what is the value at $8.45 per cwt. net?

10. The net value of a hogshead of Barbadoes sugar was $22.50; the custom and fees $12.49, freight $5.11, factorage $1.31; the gross weight was 11 cwt. 1 qr. 15 lb., actual tare 11¼ lb. per cwt.: what was the sugar rated at per cwt. net, in the bill of parcels?

11. I imported 87 jars of Lucca oil, each containing 47 gallons: what did the freight come to at $1.19 per cwt. net, reckoning 1 lb. in 11 lb. for actual tare, and 9 lb. to the gallon?

12. A grocer bought 5 hhd. of sugar, each weighing 13 1 qr. 12 lb., at 7½ cents a pound; the tare was 1½ lb. per cwt., in addition to 5½ per cent.: what was the cost of the net weight?

13. A wholesale merchant receives 450 bags of coffee, each weighing 76 lbs.; the actual tare was eight per cent., and the invoice price 10½ cents per pound. He sold at an advance of 33⅓ per cent.: what was his gain, and what his selling price?

14. A merchant imported 176 pieces of broadcloth, each piece measuring 46¼ yd., at $3.25 a yard: what will be the duty at 30 per cent.?

15. What is the duty on 54 T. 13 cwt. 3 qr. 20 lb. of iron, invoiced at $15 a ton, and the duty 33⅓ per cent.?

16. What will be the duty on 225 bags of coffee, each weighing gross 160 lb., invoiced at 6 cents per pound; 2 per cent. being the actual tare, and 20 per cent. the duty?

17. What duty must be paid on 275 dozen bottles of claret, estimated to contain 2¾ gallons per dozen, 5 per cent. being allowed for breakage, and the duty being 35 cents per gallon?

18. A merchant imports 175 cases of indigo, each case weighing 196 lb. gross; 15 per cent. is the actual tare, and the duty 5 cents per pound: what duty must he pay on the whole?

19. What is the tare and duty on 75 casks of Epsom salts, each weighing gross 2 cwt. 2 qr. 24 lb., and invoiced at 1⅞ cents per pound, the actual tare being 11 per cent. and the rate of duty 20 per cent.?

TONNAGE OF VESSELS.

343. There are certain custom-house charges on vessels, which are made according to their tonnage. The tonnage of a vessel is the number of tons weight she will carry, and this is determined by measurement.

[From the "Digest," by Andrew A. Jones, of the N. Y. Custom-house.]

Custom-house charges on all ships or vessels entering from any foreign port or place.

Ships or vessels of the United States, having three-fourths of the crew and all the officers American citizens, *per ton*, -	$0.06
Ships or vessels of nations entitled by treaty to enter at the same rate as American vessels, - - - - - - - -	.06
Ships or vessels of the United States not having three-fourths of the crew as above, - - - - - - - - - -	.50
On foreign ships or vessels other than those entitled by treaty,	.50
Additional tonnage on foreign vessels, denominated light-money, - - - - - - - - - - - - -	.50

Licensed coasters are also liable once in each year to a duty of 50 cents per ton, being engaged in a trade from a port in one State to a port in another State, other than an adjoining State, unless the officers and three-fourths of the crew are American citizens; to ascertain which, the crews are always liable to an examination by an officer.

A foreign vessel is not permitted to carry on the coasting trade; but having arrived from a foreign port with a cargo consigned to more than one port of the United States, she may proceed coastwise with a certified manifest until her voyage is completed.

344. The government estimate the tonnage according to one rule, while the ship-carpenter, who builds the vessel, uses another.

Government Rule.—I. *Measure, in feet, above the upper deck the length of the vessel, from the fore-part of the main stem to the after-part of the stern-post. Then measure the breadth taken at the widest part above the main wale on the outside, and the depth from the under-side of the deck-plank to the ceiling in the hold:*

II. *From the length take three-fifths of the breadth, and multiply the remainder by the breadth and depth, and the product divided by 95 will give the tonnage of a single-decker; and the same for a double-decker, by merely making the depth equal to half the breadth.*

Carpenters' Rule.—*Multiply together the length of the keel, the breadth of the main beam, and the depth of the hold, and the product divided by 95 will be the carpenters' tonnage for a single-decker; and for a double-decker, deduct from the depth of the hold half the distance between decks.*

Examples.

1. What is the government tonnage of a single-decker, whose length is 75 feet, breadth 20 feet, and depth 17 feet?

2. What is the carpenters' tonnage of a single-decker, the length of whose keel is 90 feet, breadth 22 feet 7 inches, and depth 20 feet 6 inches?

3. What is the carpenters' tonnage of a steamship, double decker, length 154 feet, breadth 30 feet 8 inches, and depth, after deducting half between decks, 14 feet 8 inches?

4. What is the carpenters' tonnage of a double-decker, its length 125 feet, breadth 25 feet 6 inches, depth of hold 34 feet, and distance between decks 8 feet?

GENERAL AVERAGE.

345. AVERAGE is a term of commerce signifying a contribution by individuals, where the goods of a particular merchant are thrown overboard in a storm, to save the ship from sinking; or where the masts, cables, anchors, or other furniture of the ship are cut away or destroyed, for the preservation of the vessel. In these and like cases, where any sacrifices are deliberately made, or any expenses voluntarily incurred, to prevent a total loss, such sacrifice or expense is the proper subject of a general contribution, and ought to be ratably borne by the owners of the ship, the freight, and the cargo, so that the loss may fall proportionably on all. The amount sacrificed is called the *jettison.*

346. Average is either *general or particular;* that is, it is either chargeable to all the interests, viz., the ship, the freight, and the cargo, or only to some of them. As when losses occur from ordinary wear and tear, or from the perils incident to the voyage, without being *voluntarily* incurred; or when any particular sacrifice is made for the sake of the *ship only,* or the *cargo only,* these losses must be borne by the parties immediately interested, and are consequently defrayed by a *particular* average. There are also some small charges, called *petty* or *accustomed* averages, one-third of which is usually charged to the ship, and two-thirds to the cargo.

No general average ever takes place, except it can be shown that *the danger was imminent, and that the sacrifice was made indispensable, or was supposed to be so, by the captain and officers, for the safety of the ship.*

347. In different countries different modes are adopted of valuing the articles which are to constitute a general average. In general, however, the value of the freightage is held to be the clear sum which the ship has earned after seamen's wages, *pilotage,* and all such other charges as come under the name

petty charges, are deducted; one-third, and in some cases e-half, being deducted for the wages of the crew.

The goods lost, as well as those saved, are valued at the ice they would have brought, in ready money, at *the place* *delivery*, on the ship's arriving there, freight, duties, and other charges being deducted: indeed, they bear their prortions, the same as the goods saved. The ship is valued at e price she would bring, on her arrival at the port of de ery. But when the loss of masts, cables, and other furnire of the ship is compensated by general average, it is ual, as the new articles will be of greater value than the d, to deduct one-third, leaving two-thirds only to be charged the amount to be contributed.

Examples.

1. The vessel Good Intent, bound from New York to New rleans, was lost on the Jersey beach the day after sailing. he cut away her cables and masts, and cast overboard a part f her cargo, by which another part was injured. The ship as finally got off, and brought back to New York.

AMOUNT OF LOSS.

Goods of A cast overboard, - - - - -		$500
Damage of the goods of B by the jettison, -		200
Freight of the goods cast overboard, - -		100
Cable, anchors, mast, &c., worth -	$300	200
Deduct one-third, - - - -	100	
Expenses of getting the ship off the sands, -		56
Pilotage and port duties going in and out of the harbor, commissions, &c., -		100
Expenses in port, - - - - - - -		25
Adjusting the average, - - - - - -		4
Postage, - - - - - - - -		1
Total loss, - - - - -		$1186

ARTICLES TO CONTRIBUTE.

Goods of A cast overboard, - - - - - -	$500
Value of B's goods at N. O., deducting freight, &c.,	1000
" of C's " " " "	500
" of D's " " " "	2000
" of E's " " " "	5000
Value of the ship, - - - - - - - -	2000
Freight, after deducting one-third, - - - -	800
	$11800

Then,

Total value : total loss : : 100 : per cent. of loss.
$11800 : 1180 : : 100 : 10;

hence, each loses 10 per cent. on the value of his interest in the cargo, ship, or freight. Therefore, A loses $50; B, $100; C, 50; D, $200; E, $500; the owners of the ship, $280—in all, $1180. Upon this calculation, the owners are to lose $280; but they are to receive their disbursements from the contribution: viz., freight on goods thrown overboard, $100; damages to ship, $200; various disbursements in expenses, $180; total, $480; and deducting the amount of contribution, they will actually receive $200. Hence, the account will stand:

The owners are to receive - - - - - -	$200
A loses $500, and is to contribute $50; hence, he receives - - - - - - - - -	450
B loses $200, and is to contribute $100; hence, he receives - - - - - -	100
Total to be received, - -	$750

C, D, and E, have lost nothing, and are to pay	C,	50
	D,	200
	E,	500
Total actually paid, - - -		$750

COINS AND CURRENCIES.

348. Coins are pieces of metal, of gold, silver, or copper, of fixed values, and impressed with a public stamp prescribed by the country where they are made. These are called specie, and are generally declared to be a legal tender in payment of debts. The Constitution of the United States provides, that the value of gold and silver coins shall be fixed by act of Congress.

The coins of a country, and those of foreign countries having a fixed value established by law, together with bank-notes redeemable in specie, make up what is called the *Currency.*

349. A Foreign coin may be said to have four values:

1st. The intrinsic value, which is determined by the amount of pure metal which it contains:

2d. The Custom-house, or legal value, which is fixed by law:

3d. The mercantile value, which is the amount it will sell for in open market:

4th. The exchange value, which is the value assigned to it in buying and selling bills of exchange between one country and another.

Let us take, as an example, the English pound sterling, which is represented by the gold sovereign. Its intrinsic value, as determined at the Mint in Philadelphia, compared with our gold eagle, is $4.861. Its legal or custom-house value is $4.84. Its commercial value, that is, what it will bring in Wall-street, New York, varies from $4.83 to $4.86, seldom reaching either the lowest or highest limit. The exchange value of the English pound, is $$4.44\frac{4}{9}$, and was the legal value before the change in our standard. This change raised the legal value of the pound to $4.84; but merchants, and dealers in exchange, preferred to retain the old value, which became nominal, and to add the difference in the form of a *premium on exchange*, which is explained in Art. 365. For the values of the *various coins, see* Table, page 406

EXCHANGE.

350. Exchange is a term which denotes the payment of money by a person residing in one place to a person residing in another. The payment is generally made by means of bill of exchange.

351. A Bill of Exchange is an open letter of request from one person to another, desiring the payment to a third party named therein, of a certain sum of money to be paid at a specified time and place. Of a bill of exchange three copies are made, and are called a *set of exchange.* They are sent by different ways to the drawee, so that in case one is lost, another may reach him. There are always three parties to a bill of exchange, and generally four:

1. He who writes the open letter of request, is called the *drawer* or *maker* of the bill;

2. The person to whom it is directed, is called the *drawee;*

3. The person to whom the money is ordered to be paid is called the *payee;* and

4. Any person who purchases a bill of exchange is called the *buyer* or *remitter.*

352. Bills of exchange are the proper money of commerce. Suppose Mr. Isaac Wilson, of the city of New York, ships 1000 bags of cotton, worth £6000, to Samuel Johns & Co., of Liverpool; and at about the same time William James, of New York, orders goods from Liverpool, of Ambrose Spooner to the amount of six thousand pounds sterling. Now, Mr Wilson draws a bill of exchange on Messrs. Johns & Co., in the following form, viz.:

Exchange for £6000. New York, July 30th, 1846.

Sixty days after sight of this my first Bill of Exchange (second and third of the same date and tenor unpaid), pay to

David C. Jones, or order, six thousand pounds sterling, with or without further notice. Isaac Wilson.

Messrs. Samuel Johns & Co.,
Merchants, Liverpool.

Let us now suppose that Mr. James purchases this bill of David C. Jones, for the purpose of sending it to Ambrose Spooner, of Liverpool, whom he owes. We shall then have all the parties to a bill of exchange; viz., Isaac Wilson, the *maker* or *drawer;* Messrs. Johns & Co., the *drawees;* David C. Jones, the *payee;* and William James, the *buyer* or *remitter.*

353. A bill of exchange is called an *inland bill,* when the drawer and drawee both reside in the same country; and when they reside in different countries, it is called a *foreign bill.* Thus, all bills in which the drawer and drawee reside in the United States, are inland bills; but if one of them resides in England or France, the bill is a foreign bill.

354. The time at which a bill is made payable varies, and is a matter of agreement between the drawer and buyer. They may either be drawn *at sight,* or at a certain number of days *after sight,* or at a certain number of days *after date.*

355. Days of Grace are a certain number of days granted to the person who pays the bill, after the time named in the bill has expired. In the United States and Great Britain three days are allowed.

356. In ascertaining the time when a bill, payable so many days after sight, or after date, actually falls due, the day of presentment, or the day of the date, is not reckoned. When the time is expressed in months, *calendar months* are always understood.

If the month in which a bill falls due is shorter than the one in which it is dated, it is a rule not to go on into the next month. Thus, a bill drawn on the 28th, 29th, 30th, or 31st of December, payable two months after date, falls due on the last of February, except for the days of grace, and would be actually due on the third of March.

INDORSING BILLS.

357. In examining the bill of exchange drawn by Isaac Wilson, it will be seen that Messrs. Johns & Co. are requested to pay the amount to David C. Jones, or order; that is, either to Jones or to any other person named by him. If Mr. Jones simply writes his name on the back of the bill, he is said to indorse it in *blank*, and the drawees must pay it to any rightful owner who presents it. Such rightful owner is called the *holder*, and Mr. Jones is called the *indorser.*

If Mr. Jones writes on the back of the bill, over his signature, "Pay to the order of William James," this is called a *special indorsement*, and William James is the *indorser*, and he may either indorse in blank, or write over his signature, "Pay to the order of Ambrose Spooner," and the drawees, Messrs. Johns & Co., will then be bound to pay the amount to Mr. Spooner.

A bill drawn payable to bearer, may be transferred by mere delivery.

ACCEPTANCE.

358. When the bill drawn on Messrs. Johns & Co. is presented to them, they must inform the holder whether or not they will pay it at the expiration of the time named. Their agreement to pay it is signified by writing across the face of the bill, and over their signature, the word "accepted," and they are then called the *acceptors.*

LIABILITIES OF THE PARTIES.

359. The drawee of a bill does not become responsible for its payment until after he has accepted. On the presentation of the bill, if the drawee does not accept, the holder should *immediately* take means to have the drawer and all the indorsers notified. Such notice is called a *protest*, and is given by a public officer called a *notary*, or *notary public.* If the indorsers are not notified in a reasonable time, they are not responsible for the amount of the bill.

If the drawee accepts the bill, and fails to make the payment when it becomes due, the parties must be notified as before, and this is called *protesting the bill for non-payment.* If the indorsers are not notified in a reasonable time, they are not responsible for the amount of the bill.

PAR OF EXCHANGE—COURSE OF EXCHANGE.

360. The intrinsic *par of exchange*, is a term used to compare the coins of different countries with each other, with respect to their intrinsic values; that is, with reference to the amount of pure metal in each. Thus, the English sovereign, which represents the pound sterling, is intrinsically worth $4.861 in our gold, taken as a standard, as determined at the Mint in Philadelphia. This, therefore, is the value at which the sovereign should be reckoned, in estimating the par of exchange.

361. The *commercial par of exchange* is a comparison of the coins of different countries according to their market value. Thus, as the market value of the English sovereign varies from $4.83 to $4.85 (Art. 349), the commercial par of exchange will fluctuate. It is, however, always determined when we know the value at which the foreign coin sells in open market.

362. The *course of exchange* is the variable price which is paid at one place for bills of exchange drawn on another. The course of exchange differs from the intrinsic par of exchange, and also from the commercial par, in the same way that the market price of an article differs from its natural price. The commercial par of exchange would at all times determine the course of exchange, if there were no fluctuations in trade.

363. When the market price of a foreign bill is *above* the commercial par, the exchange is said to be at a *premium*, or in favor of the foreign place, because it indicates that the foreign place has sold more than it has bought, and that specie must be shipped to make up the difference. When the market price is *below* this par, exchange is said to be *below par*, or in favor of the place where the bill is drawn. Such place will then be a creditor, and the debt must be paid in specie or other property. It should be observed, that a favorable state of exchange is advantageous to the buyer, but not to the seller, whose interest, as a dealer in exchange, is identified with that of the place on which the bill is drawn.

INLAND BILLS.

364. We have seen that inland bills are those in which the drawer and drawee both reside in the same country (Art. 353).

Examples.

1. A merchant at New Orleans wishes to remit to New York $8465, and exchange is 1½ per cent. premium: how much must he pay for such a bill?

2. A merchant in Boston wishes to pay in Philadelphia $8746.50; exchange between Boston and Philadelphia is 1¼ per cent. below par: what must he pay for a bill?

3. A merchant in Philadelphia wishes to pay $9876.40 in Baltimore, and finds exchange to be 1 per cent. below par: what must he pay for the bill?

4. What must be paid for a draft of $10000, payable 60 days after sight, on St. Louis, exchange being at a premium of ¾ %, interest being charged at 6 %?

5. What amount of exchange on New Orleans can be bought for $14875, the discount being ⅞ %?

6. For what amount must a bill of exchange, at 30 days, be drawn, for which I paid $9650, discount 1 %, and the interest being 6 %?

ENGLAND.

365. It has been stated that exchanges between the United States and England are made in pounds, shillings and pence, and that the exchange value of the pound sterling is reckoned at $\$4.44\frac{4}{9} = 4.4444 +$; that is, this value is the base on which the bills of exchange are drawn. Now this value being below both the commercial and intrinsic value, the drawers of bills increase the course of exchange so as to make up this deficiency.

For example, if we add to the exchange value of the pound, 9 per cent., we shall have its commercial value, very nearly.

Thus, exchange value, - - - - = $4.4444 +
Nine per cent., - - - - - - = .3999 +
which gives, - - - - - $4.8443

and this is the average of the commercial value, very nearly. Therefore, when the course of exchange is at a premium of 9 per cent., it is at the *commercial par;* and as between England and this country, it would stand near this point but for the fluctuations of trade and other accidental circumstances

Examples.

1. A merchant in New York wishes to remit to Liverpool £1167 10s. 6d., exchange being at 8½ per cent. premium; how much must he pay for the bill in United States money?

First, £1167 10s. 6d. - - - = £1167.525
Multiply by 8½ per cent., - - .085
The product is the premium - - = 99.239625
This product added, gives - - £1266.764625

which, reduced to dollars and cents, at the rate of $\$4.44\frac{4}{9}$ to the pound, gives $5630.008+, the amount which must be paid for the bill in dollars and cents.

2. A *merchant has to* remit £36794 8s. 9d. to London;

how much must he pay for a bill in dollars and cents, exchange being 7¾ per cent. premium?

3. A merchant in New York wishes to remit to London $67894.25, exchange being at a premium of 9 per cent.: what will be the amount of his bill in pounds shillings and pence?

Note.—Add the amount of the premium to the exchange value of the pound; viz., $4.44⅑, which, in this case, gives $4.84444; and then divide the amount in dollars by this sum, and the quotient will be the amount of the bill in pounds and the decimals of a pound

4. A merchant in New York owes £1256 18s. 9d. in London; exchange at a nominal premium of 7½ per cent.: how much money, in United States currency, will be necessary to purchase the bill?

5. I have $947.86, and wish to remit to London £364 18s. 8d., exchange being at 8¼ per cent.: how much additional money will be necessary?

6. Received, on consignment from London, an invoice of English cloths amounting to £1569 10s. The duties thereon amounted to $416; storage, cartage, and insurance, amounted to $85. The cloths were sold at an advance of 26 per cent. on the invoice. Supposing the commission 2½ per cent., and the premium of exchange 12 per cent., what would be the face of the bill of exchange that would cover the net proceeds?

FRANCE.

366. Accounts in France, and the exchange between France and other countries, are all kept in francs and centimes, which are hundredths of the franc. We see, from the table, that the value of the franc is 8.6 cents, which gives, very nearly, 5 francs and 38 centimes to the dollar. The rate of exchange is computed on the value 18.6 cents, but is often quoted by stating the value of the dollar in francs. Thus, exchange on Paris is said to be 5 francs 40 centimes; that is, one dollar will buy *a bill on Paris of* 5 francs and 40 hundredths of a franc

Examples.

1. A merchant in New York wishes to remit 167556 francs to Paris, exchange being at a premium of 1½ per cent.: what will be the cost of his bill in dollars and cents?

Commercial value of the franc, - - 18.6 cents.
Add 1½ per cent., - - - - - .279
Gives value for remitting, - - - 18.879 cents

then, $167556 \times 18.879 = \$31632.89724$,

which is the amount to be paid for the bill.

2. What amount, in dollars and cents, will purchase a bill on Paris for 86978 francs, exchange being at the rate of 5 francs and 2 centimes to the dollar?

First, $86978 \div 5.02 = \$17326.29$, the amount, nearly.

Is this bill above or below par? What per cent.?

3. How much money must be paid to purchase a bill of exchange on Paris for 68097 francs, exchange being 3 per cent. below par?

4. A merchant in New York wishes to remit $16785.25 to Paris; exchange gives 5 francs 4 centimes to the dollar: how much can he remit in the currency of Paris?

HAMBURG.

367. Accounts and exchanges with Hamburg, are generally made in the marc banco, valued, as we see in the table, at 35 cents.

Examples.

1. What amount, in dollars and cents, will purchase a bill of exchange on Hamburg for 18649 marcs banco, exchange being at 2 per cent. premium?

2. What amount will purchase a bill for 3678 marcs banco, reckoning the exchange value of the marc banco at 34 cents? Will this be *above or below* the par of exchange?

ARBITRATION OF EXCHANGE

368. Arbitration of Exchange is the method by which the currency of one country is changed into that of another, through the medium of one or more intervening currencies, with which the first and last are compared.

369. When there is but one intervening currency, it is called *Simple Arbitration;* and when there is more than one, it is called *Compound Arbitration.* The method of performing this is called the *Chain Rule.*

370. The principle involved in arbitration of exchange is simply this: To pass from one system of values through several others, and find the true proportion between the first and last.

1. Let it be required to remit $6570 to London, by the way of Paris, exchange on Paris being 5 francs 15 centimes for $1, and the exchange from Paris to London 25 francs and 80 centimes for £1: what will be the value of the remittance to London?

ANALYSIS.—$1 = 5.15 francs; and 1 franc $= \frac{£1}{25.80}$.

If $1 were remitted to Paris, it would produce there 5.15 francs; and if 1 franc were remitted from Paris to London, it would produce there $\frac{£1}{25.80}$.

But $6570 are remitted to Paris; hence, they produce there 6570 × 5.15 francs; and this amount is remitted to London; hence, it produces there,

$$6570 \times 5.15 \times \frac{£1}{25.80} = £1311\ 9s.\ 0\tfrac{3}{4}d.$$

Rule.—I. *Find the value of a single unit of each of the moneys named, in the money of the place next named:*

II. *Multiply the sum to be remitted by these values in succession, and the product will be the equivalent in the money of the place to which the remittance is to be made.*

Examples.

1. A merchant wishes to remit $4888.40 from New York to London, and the exchange is at a premium of 10 per cent. He finds that he can remit to Paris at 5 francs 15 centimes to the dollar, and to Hamburg at 35 cents per marc banco Now, the exchange between Paris and London is 25 francs 80 centimes for £1 sterling, and between Hamburg and London 13¾ marcs bancc for £1 sterling: how had he better remit?

OPERATION.

1st. To London direct.

$$\$4888.40 \times \frac{1}{4.8884} = £1000.$$

2d. Through Paris.

$$4888.40 \times \frac{\overset{1.03}{\cancel{5.15}}}{1} \times \frac{1}{\underset{5.16}{\cancel{25.80}}} = £975.7852 = £975\ \ 15s.\ \ 8\tfrac{1}{4}d.$$

3d. Through Hamburg.

$$\$4888.40 \times \frac{1}{.35} \times \frac{1}{13.75} = £1015.771 = £1015\ \ 15s.\ \ 5d.$$

Hence, the best way to remit is through Hamburg, then direct; and the least advantageous, through Paris.

2. A merchant in New York wishes to transmit $1500 to Vienna, through London and Hamburg: what will be the value when received, if £1 = $4.86, £1 = 14 marcs banco, and 6 marcs banco = 8 florins?

3. A merchant at Natchez wishes to pay $10000 in Boston He transmits through New Orleans and New York. From Natchez to New Orleans exchange is ½ % premium, from New Orleans to New York ⅝ % discount, and from New York to Boston ¼ % *discount: by this* exchange, what amount at Natchez *will pay the debt?*

4. A, of London, draws a bill of £862 10s. on B, of Cadiz, and remits the same to C, of Havre, who, in turn, remits to D, of Amsterdam, and D remits to B, of Cadiz: how much will pay the bill, if 1 Spanish dollar = 2 florins and .75, 12 florins = 26 francs, and 24 f. 15 c. = £1?

INVOLUTION.

371. A POWER OF A NUMBER is any product which arises from multiplying the number continually by itself.

THE ROOT, or simple factor, is called the *first power:*

THE SECOND POWER is the product of the root by itself:

THE THIRD POWER is the product, when the root is taken 3 times as a factor:

THE FOURTH POWER is the product, when it is taken 4 times:

THE FIFTH POWER is the product, when it is taken 5 times.

372. The number denoting how many times the root is taken as a factor, is called the *exponent* of the power. It is written a little at the right and over the root: thus, if the equal factor or root is 3,

$3^1 = 3$, the 1st power, root, or base.

$3^2 = 3 \times 3 = 9$, the 2d power of 3.

$3^3 = 3 \times 3 \times 3 = 27$, the 3d power of 3.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power of three.

373. INVOLUTION is the operation of finding the powers of numbers.

NOTE.—1. There are three things connected with every power: 1st, The root; 2d, The exponent; and 3d, The power or result of the multiplication.

2. In finding any power, one multiplication gives the 2d power: hence, *the number of multiplications is* 1 *less than the exponent.*

Rule.—*Multiply the number into itself as many times less 1 as there are units in the exponent, and the last product will be the power.*

Examples.

Find the power of the following numbers:

1. The square of 4?
2 The square of 15?
3. The square of 142?
4. The square of 463?
5. The square of 1340?
6. The square of 24.6?
7. The square of .526?
8. The square of 3.125?
9. The square of .0524?
10. The square of $\frac{3}{4}$?
11. The square of $\frac{6}{7}$?
12. The square of $\frac{7}{9}$?
13. The square of $\frac{35}{84}$?
14. The square of $\frac{125}{247}$?
15. The square of $7\frac{5}{8}$?
16. The square of $15\frac{9}{11}$?
17. The square of $225\frac{9}{10}$?
18. The cube of 6?
19. The cube of 24?
20. The cube of 125?
21. The cube of 136?
22. The 4th power of 12?
23. The 5th power of 9?
24. The value of $(4.25)^3$?
25 The value of $(1.8)^4$?
26. The value of $(.45)^5$?
27. The value of $(\frac{15}{16})^3$?
28. The cube of $(\frac{5}{6})$?
29. The 4th power of $\frac{3}{8}$?
30. The value of $(2\frac{1}{4})^5$?
31. The value of $(\frac{35}{57})^4$?
32. The value of $(24\frac{3}{5})^3$?
33. The value of $(.25)^6$?
34. The value of $(142.5)^3$?

EVOLUTION.

374. **EVOLUTION** is the operation of finding the root of a number; that is, of finding one of its equal factors.

375. The **SQUARE ROOT** of a number is the factor which, multiplied by itself *once*, will produce the number.

Thus, 8 is the square root of 64, because $8 \times 8 = 64$.

The sign $\sqrt{\ }$ is called the *radical sign*. When placed before a number, it denotes that its square root is to be extracted: Thus, $\sqrt{36} = 6$.

376. *The* ***CUBE ROOT*** *of* a number is the factor which, multiplied by itself *twice*, will produce the number.

Thus, 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$.

We denote the cube root by the sign $\sqrt{\;}$, with 3 written over it: thus, $\sqrt[3]{27}$, denotes the cube root of 27, which is equal to 3. The small figure 3, placed over the radical, is called the *index* of the root.

The terms *Power* and *Root*, are dependent on each other; thus, the *power* is the product of equal factors; and the *root* is one of the equal factors.

EXTRACTION OF THE SQUARE ROOT.

377. The Square Root of a number is one of its two equal factors. To extract the square root is to find this factor The first ten numbers and their squares are:

1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

The numbers in the first line are the *square roots* of those in the second. The numbers 1, 4, 9, 16, 25, 36, &c., having two *exact equal factors*, are called *perfect squares*.

A perfect square is a number which has two *exact equal factors*.

Note.—The square root of a number less than 100 will be less than 10; while the square root of a number greater than 100 will be greater than 10: hence, the square root of a number expressed by one or two figures, is a number expressed by one figure.

378. To find the law of the square of a number.

Any number expressed by two or more figures may be regarded as composed of tens and units.

1. What is the square of 36 = 3 tens + 6 units?

ANALYSIS.—The square of 36 is found by taking 36, thirty-six times. This is done by first taking it 6 units times, and then 3 tens times, and adding the products. 36 taken 6 units times, gives $6^2 + 3 \times 3$; and taken 3 tens times, gives $3 \times 6 + 3^2$; and their sum is, $3^2 + 2\ (3 \times 6) + 6^2$: that is,

OPERATION.

$$\begin{array}{r} 3 + 6 \\ \underline{3 + 6} \\ 3 \times 6 + 6^2 \\ \underline{3^2 + 3 \times 6 } \\ 3^2 + 2\ (3 \times 6) + 6^2 \end{array}$$

Rule.—*The square of a number is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

379. To find the square root of any number.

1. Let it now be required to extract the square root of 2025.

ANALYSIS.—Since the number contains more than two places of figures, its root will contain tens and units. But as the square of one ten is one hundred, it follows that the square of the tens of the required root must be found in the figures on the left of 25. Hence, beginning at the right, we point off the number into periods of two figures each.

We then find the root contained in 20 hundreds, which is 4 tens or 40. We then square 4 tens, which gives 16 hundred, and then place 16 under the first period, and subtract; this takes away the square of the tens, and leaves 425, *which is twice the product of the tens by the units plus the square of the units.*

OPERATION.

$$\begin{array}{rl} & 2\dot{0}\ 2\dot{5}(45 \\ & 16 \\ 85) & \overline{42\ 5} \\ & 42\ 5 \\ & \overline{} \end{array}$$

If, now, we double the tens, and then divide the remainder, exclusive of the right-hand figure (since that figure cannot enter into the product of the tens by the units), by it, the quotient will be the units figure of the root. If we annex this figure to the root and to the augmented divisor, and then multiply the whole divisor thus increased by it, the product will be twice the tens by the units, plus *the square of the* units · and hence, we have found both figures *of the root.*

Rule.—I. *Separate the given number into periods of two figures each, by writing a dot over the place of units, a second over the place of hundreds, and so on for each alternate figure to the left:*

II. *Note the greatest square contained in the period on the left, and place its root on the right, after the manner of a quotient in division. Subtract the square of this root from the first period, and to the remainder bring down the second period for a dividend:*

III. *Double the root thus found for a trial divisor, and place it on the left of the dividend. Find how many times the trial divisor is contained in the dividend, exclusive of its right-hand figure, and place the quotient in the root, and also annex it to the divisor:*

IV. *Multiply the divisor thus increased, by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend:*

V. *Double the whole root thus found, for a new trial divisor, and continue the operation as before, until all the periods are brought down.*

Examples.

1. What is the square root of 425104?

OPERATION.

```
     42̇ 51̇ 04̇(652
     36
    -----
 125)651
     625
    ------
1302)2604
     2604
     ----
```

Analysis.—We first place a dot over the 4, making the right-hand period 04. We then put a dot over the 1, and also over the 2, making three periods.

The greatest perfect square in 42 is 36, the root of which is 6. Placing 6 in the root, subtracting its square from 42, and bringing down the next period 51, we have 651 for a dividend; and by doubling the root, we have 12 for a trial divisor. Now, 12 is contained in 65, 5 times. Place 5 *both in the root* and in the divisor; then multiply 125 by 5; *subtract the product*, and bring down the next period.

We must now double the whole root 65 for a new trial divisor; or we may take the first divisor, after having doubled the last figure 5; then dividing, we obtain 2, the third figure of the root.

NOTES.—1. The left-hand period may contain but one figure; each of the others will contain two.

2. If any trial divisor is greater than its dividend, the corresponding root figure will be a cipher.

3. If the product of the divisor by any figure of the root exceeds the corresponding dividend, the root figure is too large, and must be diminished.

4. There will be as many figures in the root as there are periods in the given number.

5. If the given number is not a perfect square, there will be a remainder after all the periods are brought down. In this case, periods of ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2. What is the square root of 758692?

ANALYSIS.—After using all the periods of the given number, we annex periods of decimal ciphers, each of which gives one decimal place in the quotient.

OPERATION.

```
             .  .  .
            75 86 92(871.029 +
            64
    167)11 86
        11 69
     1741)17 92
          17 41
     174202)510000
            348404
     1742049)16159600
             15678441
               481159 Rem.
```

What are the square roots of the following numbers:

3. Square root of 49?
4. Square root of 144?
5. Square root of 225?
6. Square root of 2304?
7. Square root of 7994?
8. Square root of 6275025?
9. $\sqrt{19000}$ = what No.?
10. $\sqrt{2768456}$ = what No.
11. $\sqrt{36754}$ = what No.?
12. $\sqrt{1000000}$ = what No.?
13. $\sqrt{96728}$ = what No.?
14. $\sqrt{30225}$ = what No.?

380. To extract the square root of a fraction.

1. What is the square root of .6?

Analysis.—We first annex one cipher, to make *even* decimal places; for, one decimal multiplied by itself will give two places in the product. We then extract the root of the first period, and to the remainder annex a decimal period; and so on, till we have found a sufficient number of decimal places.

OPERATION.

$$\begin{array}{rl} & .60(.774+ \\ & \underline{49} \\ 147) & 1100 \\ & \underline{1029} \\ 1544) & 7100 \\ & \underline{6176} \\ & 924 \text{ rem.} \end{array}$$

2. What is the square root of $\frac{16}{25}$?

Analysis.—The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

OPERATION.

$$\sqrt{\tfrac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \tfrac{4}{5}.$$

3. What is the square root of $\frac{3}{4}$?

Analysis.—When the terms are not perfect squares, reduce the common fraction to a decimal, and then extract the square root of the decimal.

OPERATION.

$$\tfrac{3}{4} = .75;$$

$$\sqrt{\tfrac{3}{4}} = \sqrt{.75} = .8545+$$

Rule.—I. *If the fraction is a decimal, point off the periods from the decimal point to the right, annexing ciphers if necessary, so that each period shall contain two places, and then extract the root as in integral numbers:*

II. *If the fraction is a common fraction, and its terms perfect squares, extract the square root of the numerator and denominator separately:*

III. *If, after being reduced to their lowest terms, the numerator and denominator are not perfect squares, reduce the fraction to a decimal, and then extract the square root of the result.*

Examples.

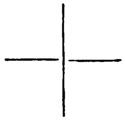
What are the square roots of the following numbers?

4. Square root of $\frac{36}{81}$?
5. Square root of $\frac{225}{2304}$?
6. Square root of .0196?
7. Square root of 6.25?
8. Square root of 278.89?
9. Square root of .205209?
10. Square root of $\frac{7}{8}$?
11. Square root of $\frac{15}{16}$?
12. Square root of $\frac{1}{40}$?
13. Square root of $5\frac{4}{9}$?
14. Square root of .7994?
15. Value of $\sqrt{222\frac{2}{9}}$?
16. Square root of .60794?
17. Value of $\sqrt{.022201}$?
18. Value of $\sqrt{25.1001}$?
19. Value of $\sqrt{196.425}$?
20. Value of $\sqrt{1.5}$?
21. Value of $\sqrt{\frac{2809}{6241}}$?
22. Value of $\frac{\sqrt{9}}{\sqrt{49}}$?
23. Value of $\sqrt{\frac{2}{25}}$?
24. Value of $\sqrt{135}$?
25. Value of $\sqrt{.784}$?
26. Square root of 5647.5225?
27. Square root of 160048.0036?

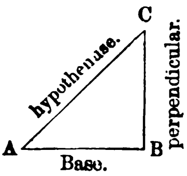
Applications in Square Root.

381. A TRIANGLE is a plain figure which has three sides and three angles.

If a straight line meets another straight line, making the adjacent angles equal, each is called a right angle; and the lines are said to be perpendicular to each other.

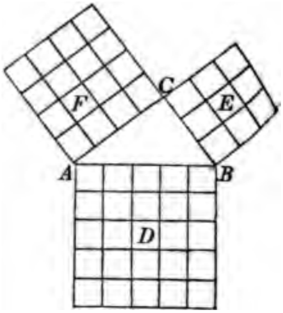


382. A RIGHT-ANGLED triangle is one which has one right angle. In the right-angled triangle ABC, the side AC, opposite the right angle B, is called the *hypothenuse;* the side AB, the *base;* and the side BC, the *perpendicular.*



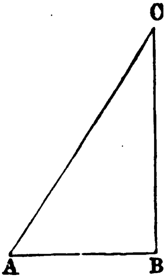
383. A SQUARE is a figure bounded by four equal sides, at right angles to each other.

384. *In a right-angled* triangle the square described on the hypothenuse is equal to the sum of the squares described on the other two sides.



Thus, if ACB be a right-angled triangle, right-angled at C, then will the large square, D, described in the hypothenuse AB, be equal to the sum of the squares F and E, described on the sides AC and CB. This is called the carpenter's theorem. By counting the small squares in the large square D, you will find their number equal to that contained in the small squares F and E. In this triangle the hypothenuse AB = 5, AC = 4, and CB = 3. Any numbers having the same ratio, as 5, 4, and 3, such as 10, 8, and 6; 20, 16, and 12, &c., will represent the sides of a right-angled triangle.

385. When the base and perpendicular are known, to find the hypothenuse.



ANALYSIS.—Wishing to know the distance from A to the top of a tower, I measured the height of the tower, and found it to be 40 feet; also the distance from A to B, and found it 30 feet: what was the distance from A to C?

$$AB = 30; \qquad AB^2 = 30^2 = 900$$
$$BC = 40; \qquad BC^2 = 40^2 = 1600$$
$$AC^2 = AB^2 + BC^2 = 900 + 1600$$
$$AC = \sqrt{2500} = 50 \text{ feet.}$$

Rule.—*Square the base and square the perpendicular, add the results, and then extract the square root of their sum.*

386. To find one side, when we know the hypothenuse and the other side.

1. The length of a ladder which will reach from the middle of a street 80 feet wide to the eaves of a house, is 50 feet: what is the height of the house?

ANALYSIS.—Since the square of the length of the ladder is equal to the sum of the squares of half the width of the street and the height of the house, the square of the length of the ladder diminished by the square of half the width of the street, will be equal to the square of the height of the house: hence,

Rule.—*Square the hypothenuse and the known side, and take the difference; the square root of the difference will be the other side.*

Examples.

1. A general having an army of 117649 men, wished to form them into a square: how many should he place on each front?

2. In a square piece of pavement there are 48841 stones, of equal size, one foot square: what is the length of one side of the pavement?

3. In the center of a square garden, there is an artificial circular pond, covering an area of 810 square feet, which is $\frac{1}{10}$ of the whole garden: how many rods of fence will inclose the garden?

4. Let it be required to lay out 67 A. 2 R. of land in the form of a rectangle, the longer side of which is to be three times as great as the less: what is its length and width?

5. A farmer wishes to set out an orchard of 3200 dwarf pear-trees. He has a field twice as long as it is wide, which he appropriates to this purpose. He sets the trees 12 feet apart, and in rows that are likewise 12 feet apart: how many rows will there be, how many trees in a row, and how much land will they occupy?

6. There is a wall 45 feet high, built upon the bank of a stream 60 feet wide: how long must a ladder be that will reach from the one side of the stream to the top of the wall on the other?

7. A boy having lodged his kite in the top of a tree, finds that by letting out the whole length of his line, which he knows to be 225 feet, it will reach the ground 180 feet from the foot of the tree: what is the height of the tree?

8. There are two buildings standing on opposite sides of the street, one 39 feet, and the other 49 feet from the ground to the eaves. The foot of a ladder 65 feet long rests upon the ground at a point between them, from which it will touch the eaves of either building: what is the width of the street?

9. A tree 120 feet high was broken off in a storm, the top striking 40 feet from the roots, and the broken end resting upon the stump: allowing the ground to be a horizontal plane, what was the height of the part standing?

10. What will be the distance from corner to corner, through the center of a cube, whose dimensions are 5 feet on a side?

11. Two vessels start from the same point, one sails due north at the rate of 10 miles an hour, the other due west at the rate of 14 miles an hour: how far apart will they be at the end of 2 days, supposing the surface of the earth to be a plane?

12. How much more will it cost to fence 10 acres of land, in the form of a rectangle, the length of which is four times its breadth, than if it were in the form of a square, the cost of the fence being $2.50 a rod?

13. What is the diameter of a cylindrical reservoir containing 9 times as much water as one 25 feet in diameter, the height being the same?

NOTE.—If two volumes have the same altitude, their contents will be to each other in the same proportion as their bases; and if the bases are similar figures (that is, of like form), they will be to each other as the *squares of their diameters*, or other like dimensions.

14. If a cylindrical cistern eight feet in diameter will hold 120 barrels, what must be the diameter of a cistern of the *same depth* to hold 1500 barrels?

15. If a pipe 3 inches in diameter will discharge 400 gallons in 3 minutes, what must be the diameter of a pipe that will discharge 1600 gallons in the same time?

16. What length of rope must be attached to a halter 4 feet long, that a horse may feed over $2\frac{1}{2}$ acres of ground?

17. Three men bought a grindstone, which was 4 feet in diameter: how much of the radius must each grind off to use up his share of the stone?

CUBE ROOT.

387. The Cube Root of a number is one of its three equal factors.

Thus, 2 is the cube root of 8; for, $2 \times 2 \times 2 = 8$: and 3 is the cube root of 27; for, $3 \times 3 \times 3 = 27$.

To extract the cube root of a number, is to find one of its three equal factors.

1,	2,	3,	4,	5,	6,	7,	8,	9,
1	8	27	64	125	216	343	512	729

The numbers in the first line are the cube roots of the corresponding numbers of the second. The numbers of the second line are called *perfect cubes.*

A Perfect Cube is a number which has three exact equal factors. By examining the numbers in the two lines, we see,

1st. *That the cube of units cannot give a higher order than hundreds:*

2d. That since the cube of one ten (10) is 1000, and the cube of 9 tens (90), 729,000, *the cube of tens will not give a lower denomination than thousands, nor a higher denomination than hundreds of thousands.*

Hence, if a number contains more than three figures, its cube root will contain more than one; if it contains more than six, its root will contain more than two, and so on; every additional three figures giving one additional figure in the root.

and the figures which remain at the left hand, altho than three, will also give a figure in the root. This plains the reason for pointing off into periods of thr each.

388. Let us see how the cube of any number, formed. Sixteen is composed of 1 ten and 6 units, be written, 10 + 6. To find the cube of 16 = 1C must multiply the number by itself twice.

To do this we place the number thus, 16 =

Product by the units, - - - -			
Product by the tens, - - - -		100 +	
Square of 16, - - - - -		100 +	1
Multiply again by 16, - - - -			
Product by the units, - - - -		600 +	7
Product by the tens, - - -	1000 +	1200 +	3
Cube of 16, - - - -	1000 +	1800 +	10

1. By examining the parts of this number, it is the first part 1000 is the *cube of the tens;* that is,

$$10 \times 10 \times 10 = 1000:$$

2. The second part 1800 is *three times the squa tens multiplied by the units;* that is,

$$3 \times (10)^2 \times 6 = 3 \times 100 \times 6 = 1800:$$

3. The third part 1080 is *three times the squa units multiplied by the tens;* that is,

$$3 \times 6^2 \times 10 = 3 \times 36 \times 10 = 1080:$$

4. The fourth part is the *cube of the units;* that

$$6^3 = 6 \times 6 \times 6 = 216.$$

1. What is the cube root of the number 4096?

Analysis.—Since the number contains more than three figures, we know that the root will contain at least units and tens.

OPERATION.

$$\dot{4}\ 09\dot{6}(1$$
$$1$$
$$1^2 \times 3 = 3)3\ 0\quad(9$$
$$16^3 = 4\ 096$$

Separating the three right-hand figures from the 4, we know that the cube of the tens will be found in the 4; and 1 is the greatest cube in 4.

Hence, we place the root 1 on the right, and this is the tens of the required root. We then cube 1, and subtract the result from 4, and to the remainder we bring down the first figure 0 of the next period.

We have seen that the second part of the cube of 16, viz., 1800, is *three times the square of the tens multiplied by the units*, and hence, it can have no significant figure of a less denomination than hundreds. It must, therefore, make up a part of the 30 hundreds above. But this 30 hundreds also contains all the hundreds which come from the 3d and 4th parts of the cube of 16. If it were not so, the 30 hundreds, divided by three times the square of the tens, would give the unit figure exactly.

Forming a divisor of three times the square of the tens, we find the quotient to be ten; but this we know to be too large. Placing 9 in the root, and cubing 19, we find the result to be 6859. Then trying 8, we find the cube of 18 still too large; but when we take 6, we find the exact number. Hence, the cube root of 4096 is 16.

389. Hence, to find the cube root of a number:

Rule.—I. *Separate the given number into periods of three figures each, beginning at the right, by placing a dot over the place of units, a second over the place of thousands, and so on over each third figure to the left: the left-hand period will often contain less than three places of figures:*

II. *Note the greatest perfect cube in the first period, and set its root on the right, after the manner of a quotient in division. Subtract the cube of this number from the first period, and to the remainder bring down the first figure of the next period for a dividend:*

III. *Take three times the square of the root just found for a trial divisor, and see how often it is contained in the dividend, and place the quotient for a second figure of the root. Then cube the figures of the root thus found, and if their cube be greater than the first two periods of the* *given number, diminish the last figure; but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period for a new dividend:*

IV. *Take three times the square of the whole root for a second trial divisor, and find a third figure of the root as before. Cube the whole root thus found, and subtract the result from the first three periods of the given number when it is less than that number; but if it is greater, diminish the last figure of the root: proceed in a similar way for all the periods.*

Examples.

1. What is the cube root of 20796875?

OPERATION.

$$\dot{2}\dot{0}\ 79\dot{6}\ 87\dot{5}(275$$

$$2^3 = 8$$

$$2^2 \times 3 = 12)127$$

First two periods, - - - - $2\dot{0}\ 79\dot{6}$

$(27)^3 = 27 \times 27 \times 27 =$ $19\ 683$

$$3 \times (27)^2 = 2187)1\dot{1}\ 13\dot{8}$$

First three periods, - - - - $2\dot{0}\ 79\dot{6}\ 87\dot{5}$

$(275)^3 = 275 \times 275 \times 275 =$ $20\ 796\ 875$

Find the cube roots of the following numbers:

1. Cube root of 1728?
2. Cube root of 117649?
3. Cube root of 46656?
4. Cube root of 15069223?
5. Cube root of 5735339?
6. Cube root of 48228544?
7. Cube root of 84604519?
8. Cube root of 289910292481?

390 **To extract the cube root of a decimal fraction.**

Rule.—*Annex ciphers to the decimal, if necessary, so that it shall consist of* 3, 6, 9, *&c., decimal places. Then put the first point over the place of thousandths, the second over the* *place of millionths, and so on over every third place to the right; after which, extract the root as in whole numbers.*

NOTES.—1. There will be as many decimal places in the root as there are periods of decimals in the given number.

2. If, in extracting the root of a number, there is a remainder after all the periods have been brought down, periods of ciphers may be annexed by considering them as decimals.

Examples.

Find the cube roots of the following numbers:

1. Cube root of 8.343?
2. Cube root of 1728.729?
3. Cube root of .0125?
4. Cube root of 19683.46656?
5. Cube root of .387420489?
6. Cube root of .000003375?
7. Cube root of .0066592?
8. Value of $\sqrt[3]{81.729}$?

391. To extract the cube root of a common fraction.

Rule.—I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms:*

II. *Extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in the last case.*

Examples.

Find the cube roots of the following fractions:

1. Cube root of $\frac{64}{125}$?
2. Cube root of $\frac{343}{729}$?
3. Cube root of $31\frac{15}{343}$?
4. Cube root of $91\frac{1}{8}$?
5. Cube root of $\frac{343}{512}$?
6. Cube root of $\frac{729}{15625}$?
7. Cube root of $\frac{19683}{262144}$?
8. Cube root of $\frac{13824}{42875}$?
9. Cube root of $7\frac{6}{7}$?
10. Cube root of $56\frac{3}{5}$?

Applications.

1. What must be the dimensions of a cubical bin, that its volume or capacity may be 19683 feet?

2. If a cubical body contains 6859 cubic feet, what is the length of one side? what the area of its surface?

3 The volume of a globe is 46656 cubic inches; what would be the side of a cube of equal solidity?

4. A person wishes to make a cubical cistern, which shall hold 150 barrels of water: what must be its depth?

5. A farmer constructed a bin that would contain 1500 bushels of grain; its length and breadth were equal, and each half the height: what were its dimensions?

6 What is the difference between half a cubic yard, and a cube whose edge is half a yard?

7. A merchant paid $911.25 for some pieces of muslin. He paid as many cents a yard as there were yards in each piece, and there were as many pieces as there were yards in one piece: how many yards were there, and how much did he pay a yard?

8. If a sphere 3 feet in diameter contains 14.1372 cubic feet, what are the contents of a sphere 6 feet in diameter?

$$3^3 : 6^3 :: 14.1372 : 113.0976. \textit{ Ans.}$$

9. If a ball $2\frac{1}{2}$ inches in diameter weighs 8 pounds, how much will one of the same kind weigh, that is 5 inches in diameter?

10. What must be the size of a cubical bin, that will contain 8 times as much as one that is 4 feet on a side?

11. How many globes, 6 inches in diameter, would be required to make one 12 inches in diameter?

12 If a ball of silver, one unit in diameter, is worth $8, what will be the value of one $5\frac{1}{2}$ units in diameter?

13. If a plate of silver, 6 inches long, 3 inches wide and

$\frac{1}{2}$ inch thick, is worth $100, what will be the dimensions of a similar plate, of the same metal, worth $800?

14. If a man can dig a cellar 12 feet long, 10 feet wide, and $4\frac{1}{2}$ feet deep, in 3 days, what will be the dimensions of a similar cellar, requiring 24 days to dig it, working at the same rate, and the ground being of the same degree of hardness?

15 If I put 2 tons of hay in a stack 10 feet high, how high must a similar stack be to contain 16 tons?

16. Four women bought a ball of yarn 6 inches in diameter, and agreed that each should take her share separately from the outer part of the ball: how much of the diameter did each wind off?

ARITHMETICAL PROGRESSION.

392. An Arithmetical Progression is a series of numbers in which each is derived from the one preceding, by the addition or subtraction of the same number.

The common difference is the number which is added or subtracted.

393. When the series is formed by the continued addition of the common difference, it is called an *increasing* series; and when it is formed by the subtraction of the common difference, it is called a *decreasing* series: thus,

2, 5, 8, 11, 14, 17, 20, 23, is an increasing series;
23, 20, 17, 14, 11, 8, 5, 2, is a decreasing series.

The several numbers are called *terms* of the progression. The first and last terms are called the *extremes*, and the intermediate terms are called the *means*.

394. In every arithmetical progression there are five parts, any three of which being given or known, the remaining two can be determined. They are,

1st, The first term;
2d, The last term;
3d, The common difference;
4th, The number of terms;
5th, The sum of all the terms.

CASE I.

395. Having given the first term, the common difference, and the number of terms, to find the last term.

1. The first term of an increasing progression is 4, the common difference 3, and the number of terms 10: what is the last term?

Analysis.—By considering the manner in which the increasing progression is formed, we see that the 2d term is obtained by adding the common difference to the 1st term; the 3d, by adding the common difference to the 2d; the 4th, by adding the common difference to the 3d, and so on; *the number of additions, in every case, being one less than the number of terms found.* Instead of making the additions, we may multiply the common difference by the number of additions, that is, by 1 less than the number of terms, and *add the first term* to the product.

OPERATION.

9	No. less 1
3	com. diff.
27	
4	1st term.
31	last term.

Rule.—*Multiply the common difference by* 1 *less than the number of terms: if the progression is increasing, add the product to the first term, and the sum will be the last term; if it is decreasing, subtract the product from the first term, and the difference will be the last term.*

Examples.

1. What is the 18th term of an arithmetical progression, of which the first term is 4, and the common difference 5?

2. A man is to receive a certain sum of money in 12 payments: the first payment is $300, and each succeeding pay-

ment is less than the previous one by $20: what will be the last payment?

3. What will $200 amount to in 15 years, at simple interest, the increase being $14 for the first year, $28 for the second, and so on?

4. Mr. Jones has 12 children. He gives, by will, $1000 to the youngest, $50 more to the next older, and so on to each next older $50: how much did the oldest receive?

5. A man has a piece of land 35 rods in length, which tapers to a point, and is found to increase ½ rod in width, for every rod in length: what is the width of the wide end?

6. James and John have 100 marbles. It is agreed between them that John shall have them all, if he will place them in a straight line half a foot apart, and so that he shall be obliged to travel 300 feet to get and bring back the furthest marble; and also, if he will tell, without measuring, how far he must travel to bring back the nearest. How far?

CASE II.

396. Knowing the two extremes of an arithmetical progression, and the number of terms, to find the common difference.

1. The two extremes of a progression are 4 and 68, and the number of terms 17: what is the common difference?

ANALYSIS.—Since the common difference multiplied by 1 less than the number of terms gives a product equal to the difference of the extremes, if we *divide* the difference of the extremes by 1 less than the number of terms, the quotient will be the *common difference*: hence,

OPERATION.

$$\begin{array}{r} 68 \\ \underline{4} \\ 17 - 1 = 16)\overline{64}(4 \end{array}$$

Rule.—*Subtract the less extreme from the greater, and divide the remainder by* 1 *less than the number of terms: the quotient will be the common difference.*

Examples.

1. A man started from Chicago and traveled 15 days; ea day's journey was longer than that of the preceding day the distance which he traveled the first day: what was daily increase if he traveled 75 miles the last day?

2. A merchant sold 14 yards of cloth, in pieces of 1 ya each; for the first yard he received $½, and for the last $26 what was the difference in the price per yard?

3. A board is 17 feet long; it is 2½ inches wide at o end, and 14½ at the other: what is the average increase width per foot in length?

4. The fourth term of a series is 12, and the eleventh 33: find the intermediate terms.

CASE III.

397. To find the sum of the terms of an arithmetical progr sion.

1. What is the sum of the series whose first term is common difference 3, and the number of terms 8?

Given series,	2	5	8	11	14	17	20
Same, order inverted,	23	20	17	14	11	8	5
Sum of both series,	25 +	25 +	25 +	25 +	25 +	25 +	25 +

ANALYSIS.—The two series are the same; hence, their sun equal to twice the given series But their sum is equal to sum of the two extremes, 2 and 23, taken as many times as tl are terms; and the given series is equal to half this sum, or to sum of the extremes multiplied by half the number of terms.

Rule.—*Add the extremes together, and multiply their s by half the number of terms; the product will be the s of all the terms.*

Examples.

1. What debt could be discharged in a year, by weekly payments in arithmetical progression, the first payment being $5, and the last $100?

2. A person agreed to build 56 rods of fence; for the first rod he was to receive 6 cents, for the second, 10 cents, and so on: what did he receive for the last rod, and how much for the whole?

3. If a person travels 30 miles the first day, and a quarter of a mile less each succeeding day, how far will he travel in 30 days?

4. If 120 stones be laid in a straight line, each at a distance of a yard and a quarter from the one next to it, how far must a person travel who picks them up singly and places them in a heap, at the distance of 6 yards from the end of the line and in its continuation?

CASE IV.

398. **Having given the first and last terms, and the common difference, to find the number of terms.**

1. The first term of an arithmetical progression is 5, the common difference 4, and the last term 41: what is the number of terms?

ANALYSIS.—Since the last term is equal to the first term added to the product of the common difference, by ne less than the number of terms Art. 395), it follows that, if the first erm be taken from the last term, the difference will be equal to he product of the common difference by 1 less than the number f terms: if this be divided by the common difference, the quotient *ill be 1 less than the* number of terms.

OPERATION.

$41 - 5 = 36$

$4)36(= 9$

$9 + 1 = 10$ No. terms

Rule.—*Divide the difference of the two extremes by the common difference, and add* 1 *to the quotient: the sum will be the number of terms.*

Examples.

1. A farmer sold a number of bushels of wheat; it was agreed that, for the first bushel, he should receive 50 cents, and an increase of 9 cents for each succeeding bushel, and for the last, he received $500 : how many bushels did he sell?

2. A person proposes to make a journey, and to travel 15 miles the first day, and 33 miles the last, with a daily increase of 1½ miles : in how many days did he make the journey, and what was the whole distance traveled?

3. I owe a debt of $2325, and wish to pay it in equal installments, the first payment to be $575, the second, $500, and decreasing by a common difference, until the last payment which is $200 : what will be the number of installments?

GEOMETRICAL PROGRESSION.

399. A Geometrical Progression is a series of terms, each of which is derived from the preceding one, by multiplying it by a constant number. The constant multiplier, is called the *ratio* of the progression.

400. An increasing series is one whose ratio is greater than 1 :

A decreasing series is one whose ratio is less than 1 Thus,

1 2, 4, 8, 16, 32, &c.—ratio 2—increasing series :
32, 16, 8, 4, 2, 1, &c.—ratio ½—decreasing series.

The several numbers resulting from the multiplication, are *called terms of* the progression. The first and last terms are called the *extremes*, and the intermediate terms are called *means*.

401. In every Geometrical, as well as in every Arithmetical Progression, there are five parts:

1st, The first term;
2d, The last term;
3d, The common ratio;
4th, The number of terms;
5th, The sum of all the terms;

If any three of these parts are known, or given, the remaining ones can be determined.

CASE I.

402. **Having given the first term, the ratio, and the number of terms, to find the last term.**

1. The first term is 4, and the common ratio 3: what is the 5th term?

OPERATION.

$$3 \times 3 \times 3 \times 3 = 81$$
$$\underline{\quad 4}$$
Ans. 324

ANALYSIS.—The second term is formed by multiplying the first term by the ratio; the third term, by multiplying the second term by the ratio, and so on; the number of multiplications *being* 1 *less than the number of terms:* thus,

$$4 = 4, \text{ 1st term,}$$
$$3 \times 4 = 12, \text{ 2d term,}$$
$$3 \times 3 \times 4 = 36, \text{ 3d term.}$$
$$3 \times 3 \times 3 \times 4 = 108, \text{ 4th term.}$$
$$3 \times 3 \times 3 \times 3 \times 4 = 324, \text{ 5th term.}$$

Therefore, *the last term is equal to the first term multiplied by the ratio raised to a power whose exponent is* 1 *less than the number of terms.*

Rule.—*Raise the ratio to a power whose exponent is* 1 *less than the number of terms, and then multiply this power by the first term.*

Examples.

1. The first term of a decreasing progression is 2187; the ratio is $\frac{1}{3}$, and the number of terms 8: what is the last term?

2. The first term of an increasing geometrical series is 8, the ratio 5: what is the 9th term?

3. The first term of a decreasing geometrical series is 729, the ratio $\frac{1}{3}$: what is the 10th term?

4. If a farmer should sell 15 bushels of wheat, at 1 mill for the first bushel, 1 cent for the second, 1 dime for the third, and so on; what would he receive for the last bushel?

5. A man dying left 5 sons, and bequeathed his estate in the following manner: to his executors, $100; to his youngest son twice as much as to the executors, and to each son double the amount of the next younger brother: what was the eldest son's portion?

6. A merchant engaging in business, trebled his capital once in 4 years: if he commenced with $2000, what would his capital amount to at the end of the 12th year?

7. A farmer wishing to buy 16 oxen of a drover, finally agreed to give him for the whole the cost of the last ox only. He was to pay 1 cent for the first, 2 cents for the second, and doubling on each one to the last: how much would they cost him?

8. What is the amount of $500 for 3 years at 6 per cent. compound interest?

NOTE.—The ratio is 1.06.

CASE II.

403. Knowing the two extremes and the ratio, to find the sum of the terms.

1. What is the sum of the terms of the progression 2, 6, *18, 54, 162?*

OPERATION.

$$\begin{array}{r} 6 + 18 + 54 + 162 + 486 = 3 \text{ times.} \\ \underline{2 + 6 + 18 + 54 + 162 \qquad\quad = 1 \text{ time.}} \\ 486 - 2 = 2 \text{ times} \end{array}$$

$$\frac{486 - 2}{2} = \frac{484}{2} = 242 \text{ sum.}$$

ANALYSIS.—If we multiply the terms of the progression by the ratio 3, we have a second progression, 6, 18, 54, 162, 486, which is 3 *times as great* as the first. If from this we subtract the first, the remainder, 486 − 2, will be 2 *times* as great as the first; and if this remainder be divided by 2, the quotient will be the sum of the terms of the first progression.

But 486 is the product of the last term of the given progression multiplied by the ratio; 2 is the first term; and the divisor 2, 1 less than the ratio: hence,

Rule.—*Multiply the last term by the ratio; take the difference between this product and the first term, and divide the remainder by the difference between* 1 *and the ratio.*

NOTE.—When the progression is *increasing,* the first term is subtracted from the product of the last term by the ratio, and the divisor is found by subtracting 1 from the ratio. When the progression is *decreasing,* the product of the last term by the ratio is subtracted from the first term, and the ratio is subtracted from 1.

Examples.

1. The first term of a progression is 4, the ratio 3, and the last term 78722: what is the sum of the terms?

2. The first term of a progression is 1024, the ratio $\frac{1}{4}$, and the last term 4: what is the sum of the series?

3. What debt can be discharged in one year by monthly payments, the first being $2, the second $8, and so on to the end of the year; and what will be the last payment?

4. *A gentleman being* importuned to sell a fine horse, said

that he would sell him on the condition of receiving 1 cent for the first nail in his shoes, 2 cents for the second, and so on, doubling the price of every nail: the number of nails in each shoe being 8, how much would he receive for his horse?

5. A laborer agreed to thresh 64 days for a farmer, on the condition that he should give him 1 grain of wheat for the first day's labor, 2 grains for the second, and double each succeeding day: what number of bushels would he receive, supposing a pint to contain 7680 grains; and what number of ships, each carrying 1000 tons burden, might be loaded, allowing 40 bushels to a ton?

ANALYSIS.

404. An Analysis is an examination of the separate parts of a proposition, and of the connection of those parts with each other.

In analyzing, we generally reason from a *given number* to its *unit*, and then from this unit to the *required number*.

The process is indicated by the relations which exist between the given and the required numbers, and pursued, step by step independently of set rules.

1. If 12 yards of cloth cost \$48.36, what will 7 yards cost?

Analysis.—One yard of cloth will cost $\frac{1}{12}$ as much as 12 yards: since 12 yards cost \$48.36, one yard will cost $\frac{1}{12}$ of \$48.36 = \$4.03; 7 yards will cost 7 times as much as 1 yard, or 7 times $\frac{1}{12}$ of \$48.36 = \$28.21; therefore, if 12 yards of cloth cost \$48.36 7 yards will cost \$28.21.

OPERATION.

$$\left.\begin{array}{l}\frac{1}{12} \text{ of } 48.36 = \$4.03 = \text{price of 1 yd.,}\\ 4.03 \times 7 = \$28.21 = \text{price of 7 yd.,}\end{array}\right. \quad \text{or} \quad \left\{ \frac{48.36 \times 7}{12} = \$28.21 \right.$$

2. If 27 pounds of butter will buy 45 pounds of sugar, how much butter will 36 pounds of sugar buy?

Analysis.—One pound of sugar will buy $\frac{1}{45}$ of 27 lb. of butter; and 36 lb. of sugar will buy 36 times $\frac{1}{45}$ of 27 lb.

OPERATION.

$$\left.\begin{array}{l}\frac{1}{45} \text{ of } 27 = \frac{27}{45} = \text{value of 1 lb. of sugar,} \\ \frac{27}{45} \times 36 = 21\frac{3}{5}\text{ lb.} = \text{value of 36 lb. ``}\end{array}\right. \text{ or } \left\{ \frac{27 \times 36}{45} = 21\frac{3}{5}\text{ lb}\right.$$

3. What will $6\frac{3}{4}$ cords of wood cost, if $2\frac{3}{8}$ cords cost $\$7\frac{1}{8}$?

Analysis.—Price divided by quantity, or $7\frac{1}{8} \div 2\frac{3}{8} = \frac{57}{8} \div \frac{19}{8} =$ $3 = price of 1 cord; $\$3 \times 6\frac{3}{4} = \$20\frac{1}{4}$ = cost of $6\frac{3}{4}$ cords.

OPERATION.

$$7\tfrac{1}{8} \div 2\tfrac{3}{8} \times 6\tfrac{3}{4} = \frac{57}{8} \times \frac{8}{19} \times \frac{27}{4} = \frac{81}{4} = 20\tfrac{1}{4} = \$20.25. \textit{ Ans.}$$

4. A farmer sold a number of cows, and had 12 left, which was $\frac{1}{3}$ of the number sold; if the number sold be divided by $\frac{3}{4}$ of $9\frac{1}{3}$, the quotient will be $\frac{1}{5}$ the number of dollars he received per head: how much did he receive per head for his cows?

Analysis.—12 is $\frac{1}{3}$ of 3 times $12 = 36$; $36 \div \frac{3}{4}$ of $9\frac{1}{3} = 36 \times \frac{1}{7} = 5\frac{1}{7} = \frac{1}{5}$ of the price per head; $5\frac{1}{7} \times 5 = \$25\frac{5}{7}$ = price per head.

OPERATION.

$$12 \times 3 \div \tfrac{3}{4} \text{ of } 9\tfrac{1}{3} \times 5 = \frac{12 \times 3}{1} \times \frac{4}{3} \text{ of } \frac{3}{28} \times \frac{5}{1} = \tfrac{180}{7} = \$25\tfrac{5}{7}$$

Ans.

5. What will 20 bushels of barley cost, in dollars and cents at 7 shillings a bushel, New York currency?

Analysis.—Twenty bushels will cost $7 \times 20 = 140$s.: and $140 \div 8 = \$17.50$, since in New York 8s. = 1 dollar.

OPERATION.

$$\frac{7 \times 20}{8} = \$17.50. \textit{ Ans.}$$

6. What will $12\frac{1}{2}$ pounds of tea cost at 6s. 8d. a pound, Pennsylvania currency?

Analysis. — 6s. 8d. = 80d.; $80 \times 12\frac{1}{2} = 1000$d. = price in pence of $12\frac{1}{2}$ lbs.; and since 7s. 6d. = 90d. Pennsylvania currency, equal 1 dollar, $1000 \div 90 = \$11.11\frac{1}{9}$ = cost of $12\frac{1}{2}$ lb. in Federal money.

OPERATION.

$$\frac{80 \times 12\frac{1}{2}}{90} = \frac{80 \times 25}{90 \times 2} = \frac{100}{9} = \$11.11\frac{1}{9}.$$

7. How many days' work, at 10s. 6d. a day, must be given for 18 bushels of corn at 5s. 10d. a bushel?

Analysis.—18 bushels are worth 5s. 10d. × 18 = 70d. × 18 = 1260d.; and for this as many days' labor must be given as 10s. 6d. = 126d. is contained times in 1260d.: $1260 \div 126 = 10$ days.

OPERATION.

$$\begin{array}{l} \text{5s. 10d.} = 70\text{d.};\\ \text{10s. 6d.} = 126\text{d.}; \end{array} \quad \frac{70 \times 18}{126} = 10 \text{ days. } \textit{Ans.}$$

8. A merchant bought a number of bales of cloth, each containing $133\frac{1}{3}$ yards, at the rate of 12 yards for \$11, and sold it at the rate of 8 yards for \$7, by which he lost \$100 in the trade: how many bales were there?

Analysis.—One yard costs $\frac{1}{12}$ of $\$11 = \$\frac{11}{12}$; one yard was sold for $\frac{1}{8}$ of $\$7 = \$\frac{7}{8}$. The difference between $\frac{11}{12}$ and $\frac{7}{8}$, which is $\$\frac{1}{24}$, is the loss on each yard.

Since the total loss was \$100, he must have sold as many yards as $\$\frac{1}{24}$ is contained times in \$100.

OPERATION

$$\frac{11}{12} - \frac{7}{8} = \$\frac{1}{24} = \text{loss per yard.}$$

$$100 \div \frac{1}{24} = 2400 \text{ yds.}; \qquad 2400 \div 133\frac{1}{3} = 18 \text{ bales. } \textit{Ans.}$$

Proof.

$$\frac{100}{1} \div \frac{1}{24} \div 133\frac{1}{3} = \frac{100}{1} \times \frac{24}{1} \times \frac{3}{400} = 18 \text{ bales. } \textit{Ans.}$$

9. A can mow an acre of grass in $7\frac{1}{2}$ hours; B, in 5 hours: C, in $5\frac{5}{8}$ hours: how many days, working $6\frac{3}{4}$ hours, would they require to mow $13\frac{4}{5}$ acres?

ANALYSIS.—Since A can mow an acre of grass in $7\frac{1}{2}$ hours, B in 5 hours, and C in $5\frac{5}{8}$, A can mow $\frac{2}{15}$, B, $\frac{1}{5}$, and C, $\frac{8}{45}$ of an acre in 1 hour. Together, they can mow $\frac{2}{15}+\frac{1}{5}+\frac{8}{45}=\frac{23}{45}$ of an acre in one hour; and to mow $13\frac{4}{5}$ acres, they will require as many hours as $\frac{23}{45}$ is contained times in $13\frac{4}{5}$, which is 27; and working $6\frac{3}{4}$ hours each day, it will take as many days as $6\frac{3}{4}$ is contained in 27; $27 \div 6\frac{3}{4}=4$ days; hence, it will require 4 days.

OPERATION.

$\frac{2}{15}+\frac{1}{5}+\frac{8}{45}=\frac{23}{45}$ acres = what they all do in 1 hour.

$$13\tfrac{4}{5} \div \tfrac{23}{45} \div 6\tfrac{3}{4} = \frac{69}{5} \times \frac{45}{23} \times \frac{4}{27} = 4 \text{ days. } \textit{Ans.}$$

10. A person employed three men, A, B, and C, to do a piece of work for \$132.66. A can do the work alone in $23\frac{1}{3}$ days, working 12 hours a day; B can do it in 25 days, working 8 hours a day; and C can do it in 16 days, working $11\frac{1}{4}$ hours a day. In what time can the three do it, working together, 10 hours a day, and what share of the money should each receive?

ANALYSIS.—Since A can do the work in $23\frac{1}{3}$ days, working 12 hours each day; B, in 25 days, working 8 hours each day; and C, in 16 days, working $11\frac{1}{4}$ hours each day; A can do the work in 280 days; B, in 200 days; and C, in 180 days, working 1 hour each day. Then A, B, and C, can do $\frac{1}{280}+\frac{1}{200}+\frac{1}{180}=\frac{356}{25200}$ of the work in 1 day, working 1 hour. By working 10 hours they will do 10 times as much; or, the work done by each in 1 day of 10 hours, will be denoted by $\frac{10}{280}$, $\frac{10}{200}$, and $\frac{10}{180}$; and the whole work done in 1 day by $\frac{356}{2520}$; hence, the number of days will be denoted by the number of times which 1 contains $\frac{356}{2520}=\frac{2520}{356}=7\frac{7}{89}$ days.

If the part which each does in 1 day be multiplied by the *number of days, viz.,* $7\frac{7}{89}$, the product will be the part done each; viz., A, $\frac{10}{280} \times 7\frac{7}{89} = \frac{45}{178}$; B, $\frac{10}{200} \times 7\frac{7}{89} = \frac{63}{178}$; and C, $\frac{10}{180} \times 7\frac{7}{89} = \frac{70}{178}$; therefore, A must have $\frac{45}{178}$; B, $\frac{63}{178}$; and C $\frac{70}{178}$ of \$132.66.

OPERATION.

$\frac{10}{280} + \frac{10}{200} + \frac{10}{180} = \frac{356}{2520} =$ the work done in one day.

$1 \div \frac{356}{2520} = 7\frac{7}{89}$ days, in which the three do the work.

$\frac{10}{280} \times 7\frac{7}{89} = \frac{45}{178}$, and $\frac{45}{178}$ of $132.66 = \$33.53\frac{68}{89}$, A's share

$\frac{10}{200} \times 7\frac{7}{89} = \frac{63}{178}$: $\frac{63}{178}$ of $132.66 = \$46.95\frac{24}{89}$, B's share

$\frac{10}{180} \times 7\frac{7}{89} = \frac{70}{178}$: . $\frac{70}{178}$ of $132.66 = \$52.16\frac{86}{89}$, C's share.

11. If 336 men, in 5 days, working 10 hours each day, can dig a trench of 5 degrees of hardness, 70 yards long, 3 yards wide, and 2 yards deep; how many days, of 12 hours each, will 240 men require to dig a trench 36 yards long, 5 yards wide, and 3 yards deep, of 6 degrees of hardness?

ANALYSIS.—The first trench may be represented by the product of its elements, $5 \times 70 \times 3 \times 2$; and the second by the product of its elements, $6 \times 36 \times 5 \times 3$. In 1 day of 1 hour, one man can do $\frac{1}{336}$ of $\frac{1}{5}$ of $\frac{1}{10}$ of $5 \times 70 \times 3 \times 2$; and 240 men, in one day of 12 hours, can do 240×12 times as much; or, $\frac{240}{1} \times \frac{12}{1} \times \frac{1}{336}$ of $\frac{1}{5}$ of $\frac{1}{10}$ of $\frac{5 \times 70 \times 3 \times 2}{1} = 360 =$ work done in one day by 240 men. It will take as many days to dig the second trench as 360 is contained times in $6 \times 36 \times 5 \times 3$. $6 \times 36 \times 5 \times 3 \div 360 = 9$ days.

OPERATION.

$$6 \times 36 \times 5 \times 3 \div \frac{240}{1} \times \frac{12}{1} \times \frac{1}{336} \text{ of } \frac{1}{5} \text{ of } \frac{1}{10} \text{ of } \frac{5 \times 70 \times 3 \times 2}{1} = 9.$$

12. If 20 cords of wood are equal in value to 6 tons of hay, and 5 tons of hay to 36 bushels of wheat, and 12 bushels of wheat to 25 bushels of corn, and 14 bushels of corn to 56 pounds of butter, and 72 pounds of butter to 8 days of labor; how many cords of wood will be equal to 16 days of labor?

ANALYSIS.—1 day's labor is worth $\frac{1}{8}$ of 72 lb. of butter = 9 lb. 1 lb. of butter = $\frac{1}{56}$ of 14 bush. corn = $\frac{14}{56} = \frac{1}{4}$ bush. corn, and 9 lb.

are worth $\frac{1}{4} \times 9 = \frac{9}{4}$ bush. corn. 1 bush. corn is worth $\frac{1}{25}$ of 12 bush. wheat $= \frac{12}{25}$ bush., and $\frac{9}{4}$ bush. corn are worth $\frac{12}{25} \times \frac{9}{4} = \frac{27}{25}$ bush. wheat. 1 bush. wheat is worth $\frac{1}{36}$ of 5 tons hay $= \frac{5}{36}$ tons, and $\frac{27}{25}$ bush. are worth $\frac{5}{36} \times \frac{27}{25} = \frac{3}{20}$ tons of hay. 1 ton is worth $\frac{1}{6}$ of 20 cords $= \frac{20}{6}$ cords, and $\frac{3}{20}$ tons $= \frac{20}{6} \times \frac{3}{20} = \frac{1}{2}$ cord. This is the value of one day's labor, and 16 days' labor will be worth 16 times $\frac{1}{2}$ cord $= 8$ cords.

OPERATION.

$\frac{1}{8}$ of $\frac{72}{1}$ of $\frac{14}{36}$ of $\frac{12}{25}$ of $\frac{5}{36}$ of $\frac{20}{6} \times \frac{16}{1} = 8$ cords. *Ans.*
(See Art. 370.)

13. A, B, and C, put in trade $5626: A's stock was in 5 months, B's, 7 months, and C's, 9 months. They gained $1260, which was so divided that A received $4 as often as B had $5, and as often as C had $3. After receiving $2164.50, B absconded: what was each one's stock in trade, and how much did A and C gain or lose by B's withdrawal?

ANALYSIS.—Since A received $4 as often as B had $5, and as often as C had $3, if the whole gain were divided into 12 equal parts, A would have $\frac{4}{12}$, B, $\frac{5}{12}$, and C, $\frac{3}{12}$, of $1260; or A would have $420, B, $525, and C, $315. Now, if their respective gains be divided by the number of months each one's stock continued in trade, the quotients will represent their monthly gains, viz., A's will be $420 \div 5 =$ $84; B's, $525 $\div 7 =$ $75; and C's, $315 $\div 9 =$ $35, which gives $194 as their whole gain for 1 month.

But since each one's share of the gain for a given time will be to the whole gain for the same time, as each one's stock to the whole stock; it follows, that A will have $\frac{84}{194}$, B, $\frac{75}{194}$, and C, $\frac{35}{194}$, of the whole stock, or A will have $2436, B, $2175, and C, $1015. When B ran away he was entitled to his original stock, $2175, and his share of the gain for 7 months, that is, to $2175 + $525 = $2700; but as he took away only $2164.50, A and C gained $535.50 by his withdrawal, which must be divided between them in the ratio of their investments, or as 4 to 3; therefore, A will have $\frac{4}{7}$, and C $\frac{3}{7}$ of B's unclaimed portion, or A will have $306, and C $229.50.

OPERATION.

$4 + 5 + 3 = 12.$

$$\begin{aligned}
&\text{A's whole gain} = \tfrac{4}{12} \text{ of } \$1260 = \$420\\
&\text{B's} \quad \text{“} \quad \text{“} = \tfrac{5}{12} \quad \text{“} \quad \text{“} = \$525\\
&\text{C's} \quad \text{“} \quad \text{“} = \tfrac{3}{12} \quad \text{“} \quad \text{“} = \$315
\end{aligned}$$

$$\begin{aligned}
&\text{A's monthly gain} = \$420 \div 5 = \$84\\
&\text{B's} \quad \text{“} \quad \text{“} = \$525 \div 7 = \$75\\
&\text{C's} \quad \text{“} \quad \text{“} = \$315 \div 9 = \underline{\$35}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad \$194
\end{aligned}$$

$$\begin{aligned}
&\text{A's stock} = \tfrac{84}{194} \text{ of } \$5626 = \$2436\\
&\text{B's} \quad \text{“} = \tfrac{75}{194} \quad \text{“} \quad \text{“} = \$2175\\
&\text{C's} \quad \text{“} = \tfrac{35}{194} \quad \text{“} \quad \text{“} = \$1015
\end{aligned}$$

$2175 + $525 − $2164.50 = $535.50, what B left.

$\frac{4}{7}$ of $535.50 = $306, A's share of it.

$\frac{3}{7}$ “ “ = $229.50, B's share of it.

14. Mr. Johnson bought goods to the amount of $2400, $\frac{1}{3}$ to be paid in 3 months, $\frac{1}{4}$ in 4 months, $\frac{1}{4}$ in 6 months, and the remainder in 8 months: what is the equated time for the payment of the whole?

ANALYSIS.—$800 to be paid in 3 months, is the same as $1 to be paid in 2400 months; $600, in 4 months, the same as $1 in 2400 months; $600, in 6 months, the same as $1 in 3600 months; and $400 in 8 months, the same as $1 in 3200 months. Then $1, payable in 2400 + 2400 + 3600 + 3200 = 11600 months, is the same as $2400 in $\frac{1}{2400}$ of 11600 months, which is $4\frac{5}{6}$ months = 4 months 25 days, the equated time of payment.

OPERATION.

$$\begin{aligned}
800 \times 3 &= 2400\\
600 \times 4 &= 2400\\
600 \times 6 &= 3600\\
\underline{400} \times 8 &= \underline{3200}\\
2400 \qquad & \quad 11600
\end{aligned}$$

$11600 \div 2400 = 4\frac{5}{6}$ mo. = 4 mo. 25 da. *Ans.*

15. What will be the interest on \$60.48 for 1 year 3 months, at 7 per cent.?

ANALYSIS.—Since the interest on \$1 for 1 year is 7 cents, or seven hundredths of \$1, the interest on \$60.48 for 1 year, will be $\$60.48 \times .07 = \$4.2336$. The interest for 1 month will be $\frac{1}{12}$ as much as for 1 year, or $\frac{1}{12}$ of $\$4.2336 = \$0.3528$; and for 1 yr. 3 mo. = 15 months, it will be 15 times as much as for 1 month or $\$0.3528 \times 15 = \$5.292$.

OPERATION.

$$(\$60.48 \times .07 \div 12) \times 15 = \$5.292. \quad \textit{Ans.}$$

16. What will be the interest on \$88.92, for 8 mo. 20 da., at 7 per cent.?

17. A merchant has three kinds of cloth, worth $\$1\frac{2}{3}$, $\$2\frac{1}{4}$, $\$3\frac{3}{4}$, a yard: what is the least number of whole yards he can sell, to receive an average price of $\$2\frac{1}{2}$ a yard?

ANALYSIS.—If he sells 1 yard worth $\$1\frac{2}{3}$, for $\$2\frac{1}{2}$, he will gain $\frac{5}{6}$ of a dollar; to gain 1 dollar, he must sell as many yards as $\frac{5}{6}$ is contained times in 1, or $\frac{6}{5}$ yards. But since he is neither to gain nor lose by the operation, if he gains on one kind, he must lose an equal sum on some other; hence, he must sell some that is worth more than the average price. If he sells 1 yard worth $\$3\frac{3}{4}$ for $\$2\frac{1}{2}$, he will lose $\frac{5}{4}$ of a dollar; and to lose \$1, he must sell $\frac{4}{5}$ of a yard. Therefore, to make the loss equal to the gain, he must sell $\frac{4}{5}$ of a yard at $\$3\frac{3}{4}$ a yard, as often as he sells $\frac{6}{5}$ of a yard at $\$1\frac{2}{3}$ a yard.

If he sells 1 yard worth $\$2\frac{1}{4}$, for $\$2\frac{1}{2}$, he gains $\frac{1}{4}$ of a dollar, nd to gain \$1 he must sell 4 yards; hence, to keep the average rice, he must lose as much on some other; and as he can only ose on that at $\$3\frac{3}{4}$ a yard, he must sell enough of that to lose 1, which would be $\frac{4}{5}$ of a yard; therefore, as often as he sells yard at $\$1\frac{2}{3}$ a yard, he must sell $\frac{4}{5}$ yard at $\$3\frac{3}{4}$ a yard; and s often as he sells 4 yards at $\$2\frac{1}{4}$ a yard, he must sell $\frac{4}{5}$ yard t $\$3\frac{3}{4}$ a yard.

But since it is desirable to have the proportional parts expressed n the least whole *numbers*, we may multiply the numbers by the least common multiple of their denominators, and divide the products by their greatest common factor; this being done, we obtain in the above example, 3 yards at $\$1\frac{2}{3}$ a yard, 10 yards at $\$2\frac{1}{4}$ a yard, and 4 yards at $\$3\frac{3}{4}$ a yard.

OPERATION.

	$1\frac{2}{3}$	$\frac{6}{5}$		6		6	3
$2\frac{1}{2}$	$2\frac{1}{4}$		4		20	20	10
	$3\frac{3}{4}$	$\frac{4}{5}$	$\frac{4}{5}$	4	4	8	4

18. The hour and minute hands of a clock are together at 12 o'clock: when are they next together?

ANALYSIS.—Since the minute-hand passes over 60 minute spaces while the hour-hand passes over 5, the minute-hand passes over 12 minute spaces while the hour-hand passes over 1, *gaining* 11 minute spaces on the hour-hand in 12 minutes of time, the minute-hand requiring one minute of time to pass over 1 minute of space. Hence, in 1 minute of time, the minute-hand gains on the hour-hand $\frac{11}{12}$ of a minute space.

When the minute-hand has returned to 12, the hour-hand will be at 1, and the minute-hand has then to gain 5 minute spaces. As the minute-hand gains $\frac{11}{12}$ spaces in 1 minute of time, it will take as many minutes as $\frac{11}{12}$ is contained times in 5, viz., $5\frac{5}{11}$ mi. = 5 mi. $27\frac{3}{11}$ sec., which added to 1 o'clock, gives 1 hr. 5 mi. $27\frac{3}{11}$ sec.

SECOND ANALYSIS. — In 12 hours the minute-hand passes the hour-hand 11 times; consequently, if both are at 12, the minute-hand will pass the hour-hand the first time in $\frac{1}{11}$ of 12 hours, or 1 hr. 5 mi. $27\frac{3}{11}$ sec. It will pass it the second time in $\frac{2}{11}$ of 12 hours, and so on.

OPERATION.

$5 \times \frac{12}{11} = \frac{60}{11} = 5\frac{5}{11}$ mi. $= 5$ mi. $27\frac{3}{11}$ sec., which added to 1 hr. = 1 hr. 5 mi. $27\frac{3}{11}$ sec. *Ans.*

19. An apple boy bought a certain number of apples at the rate of 3 for 1 cent, and as many more at 4 for 1 cent; and selling them again at 2 for 1 cent, he found that he had gained 15 cents: how many apples had he?

ANALYSIS.—Since he bought a number of apples at 3 for a cent, and as many more at 4 for a cent, he paid $\frac{1}{3}$ of a cent apiece for the first, and $\frac{1}{4}$ of a cent apiece for the second lot: then, $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ of a cent, what he paid for one of each, and $\frac{7}{12} \div 2 = \frac{7}{24}$ of a cent, the average price for all he bought. Since he sold at 2 for a cent, or $\frac{1}{2}$ a cent apiece, he must have gained on each apple the difference between $\frac{1}{2}$ and $\frac{7}{24} = \frac{5}{24}$ of a cent; hence, to gain 1 cent he must sell as many apples as $\frac{5}{24}$ is contained times in $1 = 4\frac{4}{5}$ apples and to gain 15 cents he must sell 15 times as many, or $4\frac{4}{5} \times 15 = 72$ apples.

OPERATION.

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}, \qquad \frac{7}{12} \div 2 = \frac{7}{24}, \qquad \frac{1}{2} - \frac{7}{24} = \frac{5}{24},$$

$$1 \div \frac{5}{24} = 4\frac{4}{5}, \qquad 4\frac{4}{5} \times 15 = 72 \text{ apples. } \textit{Ans.}$$

20. A gentleman left to his three sons, whose ages were 13, 15, and 17 years, $15000, to be divided in such a manner, that each share being put at interest, at 7 per cent., should give to each son the same amount when he attained the age of 21 years: what was the share of each?

ANALYSIS.—By the question, their respective shares would be at interest 8, 6, and 4 years.

Find the present worth of $1 for 8, 6, and 4 years, respectively: they are $0.6410256 +, $0.7042253 +, and $0.78125. These sums being put at interest at 7 per cent., will each amount to $1 at the expiration of their respective times; and the sum of these numbers, $0.6410256 + $0.7042253 + $0.78125 = $2.1265009 + is the amount, which being so distributed among them, will produce $1 to each. If each number be divided by the sum, $2.1265009, the quotients will denote the parts of $1, which, according to the conditions of the question, each person should receive; therefore, each person will receive for his entire share 15000 like parts of one dollar.

OPERATION.

$1 ÷ 1.56 = $0.6410256 + present worth of $1 for 8 years
$1 ÷ 1.42 = $0.7042253 + " " " 6 "
$1 ÷ 1.28 = $0.78125 " " " 4 "
$2.1265009

$0.6410256 ÷ 2.1265 × 15000 = $4521.694
$0.7042253 ÷ 2.1265 × 15000 = $4967.494
$0.78125 ÷ 2.1265 × 15000 = $5510.815

21. A, B, C, and D, agree to do a piece of work for $312 A, B, and C, can do it in 10 days; B, C, and D, in $7\frac{1}{2}$ days; C, D, and A, in 8 days; and D, A, and B, in $8\frac{4}{7}$ days: in how many days can all do it, working together; in how many days can each do it, working alone; and what part of the pay ought each to receive?

ANALYSIS.—Since A, B, C, can do the work in 10 days, they can do $\frac{1}{10} = \frac{12}{120}$ of it in 1 day; since B, C, D, can do it in $7\frac{1}{2}$ days, they can do $\frac{2}{15} = \frac{16}{120}$ of it in 1 day; since C, D, A, can do it in 8 days, they can do $\frac{1}{8} = \frac{15}{120}$ of it in 1 day; and since D, A, B, can do it in $8\frac{4}{7}$ days, they can do $\frac{7}{60} = \frac{14}{120}$ of it in 1 day; hence, A, B, C, and D, by working 3 days each, will do $\frac{12}{120} + \frac{16}{120} + \frac{15}{120} + \frac{14}{120} = \frac{57}{120}$ of the work, and in 1 day they will do $\frac{1}{3}$ of $\frac{57}{120} = \frac{19}{120}$. It will then take them as many days to do the whole as $\frac{19}{120}$ is contained times in $1 = 6\frac{6}{19}$ days.

By subtracting, in succession, what the three can do in 1 day, when they work together, from what the four can do in 1 day, we shall have what each one will do in 1 day: viz., $\frac{19}{120} - \frac{12}{120} = \frac{7}{120}$, what D will do in 1 day; $\frac{19}{120} - \frac{16}{120} = \frac{3}{120}$ what A can do in 1 day; $\frac{19}{120} - \frac{15}{120} = \frac{4}{120}$, what B can do in 1 day; $\frac{19}{120} - \frac{14}{120} = \frac{5}{120}$, what C can do in 1 day. It will take each as many days to do the whole work as the part which he can do in one day is contained times in 1: viz., $1 \div \frac{3}{120} = 40$ days A's time to do it; $1 \div \frac{4}{120} = 30$ days, B's; $1 \div \frac{5}{120} = 24$ days C's; $1 \div \frac{7}{120} = 17\frac{1}{7}$ days, D's.

Now, each should receive such a part of the whole amount paid viz., $312, as he did of the whole work. This part will be denoted by what he did in 1 day multiplied by the number of days he worked: viz., A, $\frac{3}{120} \times 6\frac{6}{19} = \frac{3}{19}$; B, $\frac{4}{120} \times 6\frac{6}{19} = \frac{4}{19}$; C, $\frac{5}{120} \times 6\frac{6}{19} = \frac{5}{19}$; D, $\frac{7}{120} \times 6\frac{6}{19} = \frac{7}{19}$.

OPERATION.

$\frac{1}{10} = \frac{12}{120}$, what A, B, C, do in 1 day.
$\frac{2}{15} = \frac{16}{120}$, " B, C, D, " "
$\frac{1}{8} = \frac{15}{120}$, " C, D, A, " "
$\frac{7}{60} = \frac{14}{120}$, " D, A, B, " "

$\frac{12}{120} + \frac{16}{120} + \frac{15}{120} + \frac{14}{120} = \frac{57}{120}$, what A, B, C, and D, can do in 3 days.

$\frac{57}{120} \div 3 = \frac{19}{120}$, what A, B, C, and D, can do in 1 day.

$\frac{19}{120} - \frac{16}{120} = \frac{3}{120}$, what A can do in 1 day; $1 \div \frac{3}{120} = 40$ da.
$\frac{19}{120} - \frac{15}{120} = \frac{4}{120}$, " B " " $1 \div \frac{4}{120} = 30$ da.
$\frac{19}{120} - \frac{14}{120} = \frac{5}{120}$, " C " " $1 \div \frac{5}{120} = 24$ da.
$\frac{19}{120} - \frac{12}{120} = \frac{7}{120}$, " D " " $1 \div \frac{7}{120} = 17\frac{1}{7}$ da

Hence, the share of each will be:

$312 × $\frac{3}{19}$ = 49.26\frac{6}{19}$, A's share.
$312 × $\frac{4}{19}$ = 65.68\frac{8}{19}$, B's share.
$312 × $\frac{5}{19}$ = 82.10\frac{10}{19}$, C's share.
$312 × $\frac{7}{19}$ = 114.94\frac{14}{19}$, D's share.
$312.00, amount paid to A, B, C, and D

22. A person owning $\frac{2}{3}$ of a vessel, sold $\frac{5}{8}$ of his share for $1736: what was the value of the whole vessel?

23. If a man performs a journey in $7\frac{1}{8}$ days, traveling $14\frac{2}{3}$ hours a day, in how many days will he perform the same journey by traveling $10\frac{6}{7}$ hours a day?

24. If $\frac{1}{9}$ of a pole stands in the mud, 2 feet in the water, and $\frac{5}{6}$ above the water, what is the length of the pole?

25. After spending $\frac{1}{4}$ of my money, and $\frac{1}{5}$ of the remainder, I had $1062 left: how much had I at first?

26. Suppose a cistern has two pipes, and that one can fill it in 7½ hours, and the other in 4⅙ hours: in what time can both fill it, running together?

27. If 54 yards of ribbon cost $9, what will 26 yards cost?

28 If 2 acres of land cost ¼ of 6/7 of ⅞ of $300, what will ½ of ¾ of 10⅔ acres cost?

29. A regiment of soldiers, consisting of 1000 men, is to be clothed; each suit is to contain 3½ yards of cloth 1⅜ yards wide: how much shalloon that is ⅞ yards wide is necessary for lining?

30. How much tea, at 7s. 6d. a pound, must be given for 234 bushels of oats, at 3s. 9d. a bushel, New York currency?

31. What will 3 pipes of wine cost, at 2s. 9d. per quart. New England currency?

32. A gives B 165 yards of cotton cloth, at 2s. 6d. per yard, Missouri currency, for 625 pounds of lump sugar: how much was the sugar worth a pound?

33. If the expense of keeping 1 horse 1 day is 3s. 4d., Canada currency, what will be the expense of keeping 4 horses 3 weeks, at the same rate?

34. Bought 10 bales of cloth, each bale containing 14 pieces, and each piece 22½ yards, at 10s. 8d. per yard, Illinois currency: what was the cost of the cloth?

35. A has 7½ cwt. of sugar, worth 12 cents a pound, for which B gave him 12½ cwt. of flour: what was the flour worth a pound?

36. Bought 120 yards of cloth, at 6s. 8d. a yard, New York currency, and gave in payment 76 bushels of rye, at 4s. 6d. a bushel, New England currency, and the balance in money: how many dollars will pay the balance?

37. A merchant bought 21 pieces of cloth, each piece containing 41 yards, for which he paid $1260; he sold the cloth *at $1.75 per* yard: did he gain or lose, and how much?

38. The hour and minute hands of a watch are together at 12: at what moment will they be together between 5 and 6?

39. How many yards of carpeting $\frac{3}{4}$ of a yard wide will cover the floor of a room 18 feet long and 15 feet wide?

40. If 9 men can build a house in 5 months, by working 12 hours a day, how many hours a day must the same men work to do it in 6 months?

41. B and C can do a piece of work in 12 days; with the assistance of A they can do it in 9 days: in what time can A do it alone?

42. A can mow a certain field of grass in 3 days, B can do it in 4 days, and C can do it in 5 days: in what time can they do it, working together?

43. Divide the number 480 into 4 such parts that they shall be to each other as the numbers 3, 5, 7, and 9?

44. What length of a board that is $8\frac{4}{7}$ inches broad, will make a square foot?

45. The provisions in a garrison were sufficient for 1800 men, for 12 months; but at the end of 3 months, it was reinforced by 600 men, and 4 months afterward, a second reinforcement of 400 men was sent in: how long would the provisions last after the last reinforcement arrived?

46. A merchant bought a quantity of broadcloth and baize for $488.80; there was $117\frac{1}{2}$ yards of broadcloth, at $\$3\frac{1}{2}$ per yard; for every 5 yards of broadcloth he had $1\frac{1}{2}$ yards of baize: how many yards of baize did he buy, and what did it cost him per yard?

47. If the freight of 40 tierces of sugar, each weighing $8\frac{1}{2}$ cwt., for 150 miles, costs $42, what must be paid for the freight of 10 hhd., each weighing 12 cwt., for 50 miles?

48. If 1 pound of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons, what is the value of a pound of tea, when a lemon is worth 2 cents?

49. What amount must be discounted, at 7 per cent., to make a present payment of a note of $500, due 2 years 8 months hence?

50. If the interest on $225 for 4½ years is $91.12½, what would be the interest on $640, at the same rate, for 2¼ years?

51. A farmer having 1000 bushels of wheat to sell, can have $1.75 a bushel cash, or $1.80 in ninety days: which, at the expiration of ninety days, would be most advantageous to him, money being worth 7 per cent.?

52. A merchant bought goods to the amount of $1575 on 9 months' credit; he sells the same for $1800 in cash: money being worth 6 per cent., what did he gain?

53. Three persons in partnership gain $482.62; A put in ¾ as much capital as B, and B put in ⅚ as much as C: what was each one's share of the gain?

54. A father divided his estate, worth $9268.60, among his 4 children, giving A ¼ of it, B ⅓, and C $5 as often as he gave D $6: how much did each receive?

55. A tax of $475.50 was laid upon 4 villages, A, B, C, and D; it was so distributed, that as often as A and B each paid $5, C paid $7, and D $8; what part of the whole tax did each village pay?

56. There are 1000 men besieged in a town, with provisions for 5 weeks, allowing each man 16 ounces a day. If they are reinforced by 400 men, and no relief can be afforded till the end of 8 weeks, what must be the daily allowance to each man?

57. A reservoir has 3 pipes; the first can fill it in 10 days, the second, in 16 days, and the third can empty it in 20 days: in what time will the cistern be filled if they are all allowed to run at the same time?

58. Two persons, A and B, are on opposite sides of a *wood*, which is 536 yards in circumference; they begin to

travel in the same direction at the same time; A goes at the rate of 11 yards a minute, and B, at the rate of 34 yards in 3 minutes: how many times will B go round the wood before he overtakes A?

59. Two men and a boy were engaged to do a piece of work. One of the men could do it in 10 days, the other in 16 days, and the boy could do it in 20 days: how long would it take them together to do the work?

60. A owes B $500, of which $150 is to be paid in 3 months, $175 in 6 months, and the remainder in 8 months: what would be the equated time for the payment of the whole?

61. If 42 men, in 270 days, working $8\frac{1}{2}$ hours a day, can build a wall $98\frac{3}{4}$ feet long, $7\frac{1}{2}$ feet high, and $2\frac{1}{2}$ feet thick; in how many days can 63 men build a wall $45\frac{1}{3}$ feet long, $6\frac{7}{12}$ feet high, and $3\frac{1}{8}$ feet thick, working $11\frac{1}{3}$ hours a day?

62. After one-third part of a cask of wine had leaked away, 21 gallons were drawn, when it was found to be half full: how much did the cask hold?

63. A man had a bond and mortgage for $2500, dated July 1st, 1854. Not satisfied with 7% interest, he sold the mortgage for its nominal value, and on Sept. 1st, 1854, purchased 10 shares of railroad stock, par $100, at 115. On Nov. 1st, he bought 8 shares more of the same stock, at 98; and on April 1st, 1855, he bought 5 shares more at the same rate. On the first days of August and February, in each year, he received a regular semi-annual dividend of 4 per cent., and at the end of the year (January 1st, 1856) sold his whole stock at 99: did he lose or gain by the investment in stocks, and how much?

64. A landlord being asked how much he received for the rent of his property, answered, that after deducting 9 cents from each dollar, for taxes and repairs, there remained $3014.30: what was the amount of his rents?

65. *If 165 pounds of* soap cost $16.50, for how much will

it be necessary to sell 390 pounds, in order to gain the cost of 36 pounds?

66. What is the height of a wall which is $14\frac{1}{2}$ yards in length, and $\frac{7}{10}$ of a yard in thickness, and which cost $406, it having been paid for at the rate of $10 per cubic yard?

67. A thief escaping from an officer, has 40 miles the start and travels at the rate of 5 miles an hour; the officer in pursuit travels at the rate of 7 miles an hour: how far must he travel before he overtakes the thief?

68. Two families bought a barrel of flour together, for which they paid $8, and agreed that each child should count half as much as a grown person. In one family there were 3 grown persons and 3 children, and in the other, 4 grown persons and 10 children; the first family used from the flour 2 weeks, and the second 3 weeks: how much ought each to pay?

69. At $42 a thousand, how many thousand feet of lumber should be given for a farm containing 33 A. 2 R. 16 P., valued at $125 an acre?

70. A person paid $150 for an insurance on goods, at $3\frac{3}{4}$ per cent., and finds that in case the goods are lost, he will receive the value of the goods, the premium of insurance, and $25 besides: what was the value of the goods?

71. A distiller purchased 5000 bushels of rye, which he could have at 96 cents a bushel, cash, or at $1, 2 months' credit; which would be the more advantageous, at the end of two months, to buy on credit, or to borrow the money at 7 per cent., and pay the cash?

72. A stockholder bought $\frac{2}{3}$ of the capital of a company at par; he sold $\frac{1}{6}$ of his purchase at par, and the remainder for $25000, and by the latter sale made $5000: what was the value of the whole capital?

73. How many bushels of grain will a bin contain, that is 3 ft. 5 in. wide, 2 ft. 6 in. long, and 6 ft. deep?

74. Three travelers have each to make the same journey of 2160 miles; the first travels 30 miles a day, the second 27, and the third 24: how many days should one set out after the other, that they may all arrive together?

75. A house which was resold for $7180, would have given a profit of $420, if the second proprietor had purchased it $130 cheaper than he did: at what price did he purchase it?

76. A piece of land of 188 acres was cleared by two companies of men, working together; the first numbered 25 men, and the second 22; the first company received $84 more than the second: how many acres did each company clear, and what did the clearing cost per acre?

77. I have three notes payable as follow: one for $100, due Feb. 12th; the second for $400, due March 12th; and the third for $300, due April 1st: what is the average time of payment from January 1st?

78. How many marble slabs, 15 in. square, will it take to pave a floor 32 feet long, and 25 feet wide? What will be the cost at $3 a square yard for the marble, and 40 cents a square yard for labor?

79. A man, in his will, bequeathed $500 to A, $425 to B, $300 to C, $250 to D, and $175 to E; but after settling up the estate and paying expenses, there was but $1155 left: what is each one's share?

80. If 3 lb. of tea are worth 7 lb. of coffee, and 14 lb. of coffee are worth 48 lb. of sugar, and 18 lb. of sugar are worth 27 lb. of soap; how many pounds of soap are 6 lb. of tea worth?

81. What is the hour, when the time past noon is $\frac{3}{5}$ the time to midnight?

82. If $\frac{3}{4}$ of a yard of cloth cost $$\frac{2}{3}$, being $\frac{7}{8}$ of a yard wide, what is the value of $\frac{5}{8}$ of a yard $1\frac{3}{4}$ yards wide, of the same quality?

83. A farmer *sold* 60 fowls, a part turkeys, and a part

chickens; for the turkeys he received $1.10 apiece, and for the chickens 50 cents apiece, and for the whole he received $51 60: how many were there of each?

84. A person hired a man and two boys; to the man he gave 6 shillings a day, to one boy 4 shillings, and to the other 3 shillings a day, and at the end of the time he paid them 104 shillings: how long did they work?

85. Divide $6471 among three persons, so that as often as the first gets $5, the second will get $6, and the third $7.

86. Two partners have invested in trade $1600, by which they have gained $300; the gain and stock of the second amount to $1140: what is the stock and the gain of each?

87. What is the height of a tower that casts a shadow 75.75 feet long, at the same time that a perpendicular staff 3 feet high, gives a shade of 4.55 feet in length?

88. A can do a certain piece of work in 3 weeks; B can do 3 times as much in 8 weeks; and C can do 5 times as much in 12 weeks: in what time can they all together do the first piece of work?

89. Two persons pass a certain point, at an interval of 4 hours; the first traveling at the rate of 11¼, and the second 17½ miles an hour: how long, after passing the fixed point, and how far, will the first travel before he is overtaken by the second?

90. Three persons engage in trade, and the sum of their stock is $1600. A's stock was in trade 6 months, B's 12 months, and C's 15 months; at the time of settlement, A receives $120 of the gain, B $400, and C $100: what was each person's stock?

91. A, B, and C, start at the same time, from the same point, and travel in the same direction, around an island 73 miles in circumference. A goes at the rate of 6 miles, B 10 miles, and C 16 miles per day: in what time will they all be together again?

92. What length of wire, ⅛ of an inch in diameter, can be drawn from a cube of copper, of 2 feet on a side, allowing 10 per cent. for waste?

93. A person having $10000 invested in 6 per cent. stocks, sells out at 65, and invests the proceeds in 5 per cents at 82½: what will be the difference in his annual income?

94. In order to take a boat through a lock from a certain river into a canal, as well as to descend from the canal into the river, a volume of water is necessary 46½ yards long, 8 yards wide, and 2⅜ yards deep: how many cubic yards of water will this canal throw into the river in a common year, if 40 boats ascend and 40 descend each day, except Sundays and eight holidays?

95. A company numbering sixty-six shareholders have constructed a bridge which cost $200000: what will be the gain of each partner at the end of 22 years, supposing that 6400 persons pass each day, and that each pays one cent toll, the expense for repairs, &c., being $5 per year for each shareholder?

96. Five merchants were in partnership for four years, the first put in $60, then, 5 months after, $800; the second put in first $600, and 6 months after $1800; the third put in $400, and every six months after, he added $500; the fourth did not contribute till 8 months after the commencement of the partnership; he then put in $900, and repeated this sum every 6 months; the fifth put in no capital, but kept the accounts, for which the others agreed to allow him $800 a year, to be paid in advance, and put in as capital. What is each one's share of the gain, which was $20.000?

97. A general arranged his army in a solid mass, and in a square form, and had 44 men over. If he increases the side of the square by 1 man, and makes a new square on this line. he will require 49 men to complete it: how many men had he?

98. A, B, and C, are to share $987 in the proportion of

$\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively; but by the death of C, it is required to divide the whole sum proportionally between the other two: what will each have?

99. A lady going out shopping, spent at the first place she stopped, one-half her money, and half a dollar more; at the next place, half the remainder, and half a dollar more; and at the next place, half the remainder, and half a dollar more, when she found that she had but three dollars left: how much had she when she started?

100. If a pipe of 6 inches discharges a certain quantity of fluid in 4 hours, in what time will 4 pipes, each of 3 inches bore, discharge twice that quantity?

101. A man bought 12 horses, agreeing to pay $40 for the first, and in an increasing arithmetical progression for the rest, paying $370 for the last: what was the difference in the cost, and what did he pay for them all?

102. A bill for goods, amounting to $15000, is to be paid for in three equal payments without interest; the first in 4 months, the second in 6 months, and the third in 9 months, money being worth 7 per cent.: how much ready money ought to pay the debt?

103. If an iron bar 5 feet long, $2\frac{1}{2}$ inches broad, and $1\frac{3}{4}$ inches thick, weigh 45 pounds, how much will a bar of the same metal weigh, that is 7 feet long, 3 inches broad, and $2\frac{1}{4}$ inches thick?

104. A market woman bought a certain number of eggs at the rate of 4 for 3 cents, and sold them at the rate of 5 for 4 cents, by which she made 4 cents: what did she pay apiece for the eggs? What did she make on each egg sold? How many did she sell to gain 4 cents?

105. A person passed $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ of it in youth, 5 years more than $\frac{1}{7}$ of it in matrimony; he then had a son, whom he survived 4 years, and who reached only $\frac{1}{2}$ the age of his father: at what age did he die?

106. A well is to be stoned, of which the diameter is 6 feet 6 inches, the thickness of the wall is to be 1 foot 6 inches, leaving the diameter of the well within the wall 3 feet 6 inches; if the well is 40 feet deep, how many cubic feet of stone will be required?

107. A surveyor measured a piece of ground in the form of a rectangle, and found one side to be 37 chains, and the other 42 chains 16 links: how many acres did it contain?

108. A farmer bought a piece of land for $1500, and agreed to pay principal and interest in 5 equal annual instalments: if the interest was 7 per cent., how much was the annual payment?

109. A fountain has four receiving pipes, A, B, C, and D; A, B, and C will fill it in 6 hours; B, C, and D in 8 hours; C, D, and A in 10 hours; and D, A, and B in 12 hours: it has also 4 discharging pipes, E, F, G, and H; E, F, and G will empty it in 6 hours; F, G, and H in 5 hours; G, H, nd E in 4 hours; H, E, and F in 3 hours. Suppose the ountain full of water, and all the pipes open, in what time ould it be emptied?

110. If a ball 2 inches in diameter weighs 5 pounds, what ill be the diameter of another ball of the same material that eighs 78.125 pounds?

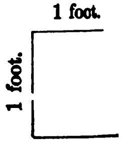
111. A gives B his bond for $5000, dated April 1st, 1861, ayable in ten equal annual instalments, the first payment of 500 to be made April 1st, 1862. Afterward, A agreed to ke up his bond on the first day of April, 1863. He was to ay, on that day, the instalment due on the first of April, 362, with interest at 7 per cent., the instalment due April st, 1863, and to be allowed compound interest, at 7 per nt., to be *computed half-yearly*, on each of the subsequent yments: what sum, on the first day of April, 1863, will ncel the bond?

MENSURATION.

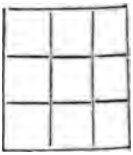
405. **Mensuration** is the art of measuring, and emb. the methods of determining the contents of geometrical It is divided into two parts, the Mensuration of Surfac the Mensuration of Volumes.

MENSURATION OF SURFACES.

406 Surfaces have length and breadth. They are mea by means of a square, which is called the *unit of surface.*



A square is the space included between four equal lines, drawn perpendicular to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot.*

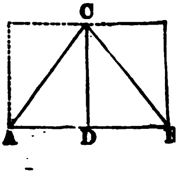


The number of small squares that is contained in any large square, is always equal to the product of two of the sides of the large square. As in the figure, 3 × 3 = 9 square feet. The number of square inches contained in a square foot is equal to 12 × 12 = 144.

If the sides of a square be each four feet, the square wil contain sixteen square feet. For, in the large square there ar sixteen small squares, the sides of which are each one foot Therefore, the square whose side is four feet, contains sixtee square feet.

Triangle.

407. A triangle is a figure bounded by three straight line Thus, ACB is a triangle.



The lines BA, AC, BC, are called *sides;* and the corners, B, A, and C, are called *angles.* The side AB is the *base.*

When a line like CD is drawn, making the angle CDA equal to the angle CDB, *then CD is* said to be at right angles to

AB, and CD is called the *altitude* of the triangle. Each triangle CAD or CDB is called a right-angled triangle. The side BC, or the side AC, opposite the right angle, is called the *hypothenuse.*

The area or contents of a triangle is equal to half the product of its base by its altitude (*Bk.* IV., *Prop.* VI.).

NOTE.—All the references are to Davies' Legendre.

Examples.

1. The base, AB, of a triangle is 50 yards, and the perpendicular, CD, 30 yards: what is the area?

OPERATION.

$$\begin{array}{r} 50 \\ \underline{30} \\ 2\overline{)1500} \\ \hline \textit{Ans.}\ \ 750\ \textit{sq. yards.} \end{array}$$

ANALYSIS.—We first multiply the base by the altitude, and the product is square yards, which we divide by 2 for the area.

2. In a triangular field the base is 60 chains, and the perpendicular 12 chains: how much does it contain?

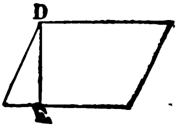
3. There is a triangular field, of which the base is 45 rods, and the perpendicular 38 rods: what are its contents?

4. What are the contents of a triangle whose base is 75 chains, and perpendicular 36 chains?

Rectangle and Parallelogram.

408. A RECTANGLE is a four-sided figure, or quadrilateral, like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.

409. A PARALLELOGRAM is a quadrilateral which has its opposite sides equal and parallel, but its angles not right angles. The line DE, perpendicular to the base, *is called the altitude.*



The area of a square, rectangle, or parallelogram, is equal to the product of the base and altitude.

Examples.

1. What is the area of a square field, of which the sides are each 66.16 chains?

2. What is the area of a square piece of land, of which the sides are 54 chains?

3. What is the area of a square piece of land, of which the sides are 75 rods each?

4. What are the contents of a rectangular field, the length of which is 80 rods, and the breadth 40 rods?

5. What are the contents of a field 80 rods square?

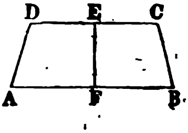
6. What are the contents of a rectangular field, 30 chains long and 5 chains broad?

7. What are the contents of a field, 54 chains long and 18 rods broad?

8. The base of a parallelogram is 542 yards, and the perpendicular height 720 feet: what is the area?

9. The measure of a rectangular field is 24000 square feet, and its length is 200 feet: what is its breadth?

Trapezoid.



410. A TRAPEZOID is a quadrilateral, ABCD, having two of its opposite sides, AB, DC, parallel. The perpendicular, EF, is called the altitude.

The area of a trapezoid is equal to half the product of the sum of the two parallel sides by the altitude (Bk. IV., Prop. VII.).

Examples.

1. Required the area or contents of the trapezoid ABCD, *having given* AB = 643.02 feet, DC = 428.48 feet, and *EF* = 342.32 feet.

ANALYSIS.—We first find the sum of the parallel sides, and then multiply it by the altitude; after which we divide the product by 2 for the area.

OPERATION.

643.02 + 428.48 = 1071.50 = sum of parallel sides. Then, 1071.50 × 342.32 = 366795.88; and $\frac{366795.88}{2} =$ 183397.94 = the area.

2. What is the area of a trapezoid, the parallel sides of which are 24.82 and 16.44 chains, and the perpendicular distance between them 10.30 chains?

3. Required the area of a trapezoid, whose parallel sides are 51 feet and 37 feet 6 inches, and the perpendicular distance between them 20 feet and 10 inches.

4. Required the area of a trapezoid, whose parallel sides are 41 and 24.5, and the perpendicular distance between them 21.5 yards.

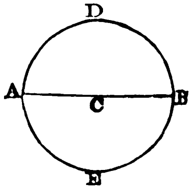
5. What is the area of a trapezoid, whose parallel sides are 15 chains, and 24.5 chains, and the perpendicular height 30.80 chains?

6. What are the contents of a trapezoid, when the parallel sides are 40 and 64 chains, and the perpendicular distance between them 52 chains?

Circle.

411. A CIRCLE is a portion of a plane bounded by a curved line, every point of which is equally distant from a certain point within, called the center

The curved line AEBD is called the *circumference;* the point C, the *center;* the line AB, passing through the center, *diameter;* and CB, a *radius.*



The circumference, AEBD, is 3.1416 times as great as the diameter AB. Hence, if the diameter is 1, the circumference will be 3.1416. Therefore, if the diameter is known, *the circumference is found by multiplying 3.1416 by the diameter* (*Bk.* V., *Prop.* XVI.).

Examples.

1. The diameter of a circle is 8 : what is the circumference?

	OPERATION.
Analysis.—The circumference is found by simply multiplying 3.1416 by the diameter.	3.1416 8 *Ans.* 25.1328

2. The diameter of a circle is 186 : what is the circumference?

3. The diameter of a circle is 40 : what is the circumference?

4. What is the circumference of a circle whose diameter is 57 ?

412. Since the circumference of a circle is 3.1416 times as great as the diameter, it follows, that *if the circumference is known we may find the diameter by dividing it by* 3.1416.

Examples.

1. What is the diameter of a circle whose circumference is 157.08 ?

2. What is the diameter of a circle whose circumference is 23304 3888 ?

3. What is the diameter of a circle whose circumference is 13700 ?

413. To find the area or contents of a circle.

Rule.—*Multiply the square of the radius by* 3.1416 (*Bk* V., *Prop* XV.).

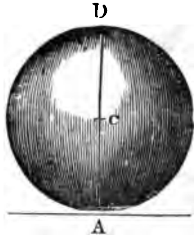
Examples.

1. What is the area of a circle whose diameter is 12 ?
2. What is the area of circle whose diameter is 5 ?
3. What is the area of a circle whose diameter is 14 ?

4. How many square yards in a circle whose diameter is 3½ feet?

5. What is the area of a circle whose diameter is ½ mile?

Sphere.



414. A SPHERE is a portion of space bounded by a curved surface, all the points of which are equally distant from a certain point within, called the center. The line AD, passing through its center C, is called the diameter of the sphere, and AC its radius.

415. To find the surface of a sphere.

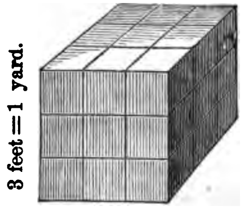
Rule.—*Multiply the square of the diameter by* 3.1416 (*Bk.* VIII., *Prop.* X., *Cor.* 1.).

Examples.

1. What is the surface of a sphere whose diameter is 6?
2. What is the surface of a sphere whose diameter is 14?
3. Required the number of square inches in the surface of a sphere whose diameter is 3 feet or 36 inches.
4. Required the area of the surface of the earth, its mean diameter being 7918.7 miles?

MENSURATION OF VOLUMES.

416. A SOLID OR VOLUME is a portion of space having three dimensions: length, breadth, and thickness. It is measured by a *cube*, called the *cubic unit*, or *unit of volume*.



3 feet = 1 yard.

A CUBE is a volume having six equal faces, which are squares. If the sides of the cube be each one foot long, the figure is called *a cubic foot.* But when the sides of the cube are one yard as in the figure, it is called a cubic yard. The base of the cube, which is the face on which it stands, contains $3 \times 3 = 9$ square feet. Therefore, 9 cubes, of one foot each, can be placed on the base. If the figure were one foot high, it would contain 9 cubic feet; if it were 2 feet high, it would contain two tiers of cubes, or 18 cubic feet; hence, *the contents are equal to the product of the length, breadth, and height.*

417. To find the volume or contents of a sphere.

Rule.—*Multiply the surface by the diameter, and divide the product by* 6 ; *the quotient will be the contents* (*Bk* VIII., *Prop.* XIV., *Sch.* 3).

Examples.

1. What are the contents of a sphere whose diameter is 12?

Analysis.—We find the surface by multiplying the square of the diameter by 3.1416. We then multiply the surface by the diameter, and divide the product by 6.

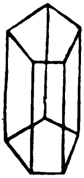
OPERATION.

	$\overline{12}^2 = 144$
multiply by	3.1416
surface	452.3904
diameter	12
	6)5428.6848
solidity	904.7808

2. What are the contents of a sphere whose diameter is 8?
3. Find the contents of a sphere whose diameter is 16 inches.
4. What are the contents of the earth, its mean diameter being 7918.7 miles?
5. Find the contents of a sphere whose diameter is 1.2 feet

Prism.

418. A **Prism** is a volume whose ends or bases are equal plane figures, and whose faces are parallelograms.



The sum of the sides which bound the base, is called the *perimeter* of the base; and the sum of *the parallelograms* which bound the prism, is called *the convex surface.*

419. To find the convex surface of a right prism.

Rule.—*Multiply the perimeter of the base by the perpendicular height, and the product will be the convex surface* (*Bk.* VII., *Prop.* I.).

Examples.

1. What is the convex surface of a prism whose base is bounded by five equal sides, each of which is 35 feet, the altitude being 52 feet?

2 What is the convex surface when there are eight equal sides, each 15 feet in length, and the altitude is 12 feet?

420. To find the volume or contents of a prism.

Rule.—*Multiply the area of the base by the altitude, and the product will be the contents* (*Bk.* VII., *Prop.* XIV.).

Examples.

1. What are the contents of a square prism, each side of the square which forms the base being 16, and the altitude of the prism 30 feet?

Analysis.—We first find the area of the square which forms the base, and then multiply by the altitude.

OPERATION.

$$\begin{array}{lr} & \overline{16}^2 = 256 \\ & 30 \\ \textit{Ans.} & \overline{7680} \end{array}$$

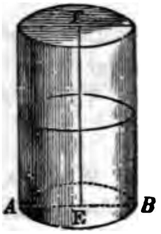
2. What are the contents of a cube, each side of which is 48 inches?

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 5 feet?

4. How many gallons of water will a cistern contain, whose dimensions are the same as in the last example?

5. Required the measure of a triangular prism, whose height *is 20 feet, and area of* the base 691.

Cylinder.



421 A CYLINDER is a volume generated by the revolution of a rectangle, AF, about EF. The line EF is called the axis, or altitude; the circular surface, the *convex surface* of the cylinder; and the circular ends, the *bases*.

422. To find the convex surface of a cylinder.

Rule—*Multiply the circumference of the base by the altitude, and the product will be the convex surface* (*Book* VIII., *Prop.* I.).

Examples.

1. What is the convex surface of a cylinder, the diameter of whose base is 20, and the altitude 40?

2. What is the convex surface of a cylinder whose altitude is 28 feet, and the circumference of its base 8 feet 4 inches?

3. Find the convex surface of a cylinder, in square inches, the diameter of whose base is 15 inches, and altitude 5 feet?

4. What is the convex surface of a cylinder, the diameter of whose base is 40, and altitude 50 feet?

423. To find the volume or contents of a cylinder.

Rule.—*Multiply the area of the base by the altitude: the product will be the contents or volume* (*Book* VIII., *Prop.* II.).

Examples.

1. Required the contents of a cylinder of which the altitude is 11 feet, and the diameter of the base 16 feet.

OPERATION.

$$\begin{array}{r} 16^2 = 256 \\ .7854 \\ \hline \text{area base, } 201.0624 \\ 11 \\ \hline 2211.6864 \end{array}$$

ANALYSIS.—We first find the area of the base, and then multiply by the altitude: the product is the solidity.

2. What are the contents of a cylinder, the diameter of whose base is 40, and the altitude 29?

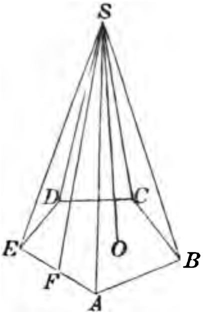
3. What are the contents of a cylinder, the diameter of whose base is 24, and the altitude 30?

4. What are the contents of a cylinder, the diameter of whose base is 32, and altitude 12?

5. What are the contents of a cylinder, the diameter of whose base is 25 feet, and altitude 15?

Pyramid.

424. A **Pyramid** is a volume bounded by several triangular planes united at the same point, S, called the vertex, and by a plane figure or base, ABCDE, in which they terminate. The altitude of the pyramid is the line SO, drawn perpendicular to the base.



425. **To find the volume or contents of a pyramid.**

Rule.—*Multiply the area of the base by the altitude, and divide the product by* 3 (*Bk.* VII., *Prop.* XVII.).

Examples.

1. Required the contents of a pyramid, the area of whose base is 86, and the altitude 24.

OPERATION.

$$\begin{array}{r} 86 \\ \underline{24} \\ 3)\underline{2064} \\ \textit{Ans. } 688 \end{array}$$

Analysis.—We simply multiply the area of the base 86, by the altitude 24, and then divide the product by 3.

2. What are the contents of a pyramid, the area of whose base is 365, and the altitude 36?

3. What are the contents of a pyramid, the area of whose base is 207, and altitude 36?

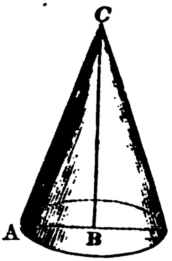
4. What are the contents of a pyramid, the area of whose base is 562, and altitude 30?

5. What are the contents of a pyramid, the area of whose base is 540, and altitude 32?

6. A pyramid has a rectangular base, the sides of which are 50 and 24; the altitude of the pyramid is 36: what are its contents?

7. A pyramid with a square base, of which each side is 15, has an altitude of 24: what are its contents?

Cone.



426. A Cone is a volume generated by the revolution of a right-angled triangle ABC, about the side CB. The point C is the vertex, and the line CB is called the axis, or *altitude.*

427. To find the volume or contents of a cone.

Rule.—*Multiply the area of the base by the altitude, and divide the product by 3; or, multiply the area of the base by one-third of the altitude* (*Bk.* VIII., *Prop.* V.).

Examples.

1. Required the contents of a cone, the diameter of whose base is 6, and the altitude 11.

Analysis.—We first square the diameter, and multiply it by .7854, which gives the area of the base. We next multiply by the altitude, and then divide the product by 3.

OPERATION.

$6^2 = 36$

$36 \times .7854 = 28.2744$

$\underline{11}$

$3\overline{)311.0184}$

Ans. 103.6728

2. What are the contents of a cone, the diameter of whose base is 36, and the altitude 27?

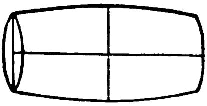
3. What are the contents of a cone, the diameter of whose base is 35, and the altitude 27?

4. What are the contents of a cone, whose altitude is 27 feet, and the diameter of the base 20 feet?

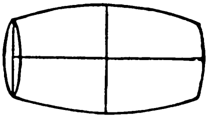
GAUGING.

428. **Cask-Gauging** is the method of finding the number of gallons which a cask contains, by measuring the external dimensions of the cask

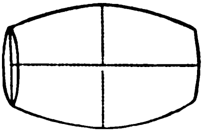
429. Casks are divided into four varieties, according to the curvature of their sides. To which of the varieties any cask belongs, must be judged of by inspection.



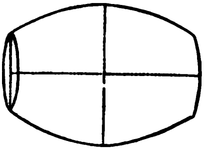
1st Variety—least curvature.



2d Variety—least mean curvature.



3d Variety—greatest mean curvature.



4th Variety—greatest curvature.

430. The first thing to be done is to find the mean diameter. To do this,

Rule.—*Divide the head diameter by the bung diameter and find the quotient in the first column of the following table, marked* Qu. *Then if the bung diameter be multiplied by the number on the same line with it, and in the column answering to the proper variety, the product will be the true mean diameter, or the diameter of a cylinder having the same altitude and the same contents with the cask proposed.*

Qu.	1st Var.	2d Var.	3d Var.	4th Var.	Qu.	1st Var.	2d Var.	3d Var.	4th Var.
50	8660	8465	7905	7637	76	9270	9227	8881	8827
51	8680	8493	7937	7681	77	9296	9258	8944	8874
52	8700	8520	7970	7725	78	9324	9290	8967	8922
53	8720	8548	8002	7769	79	9352	9320	9011	8970
54	8740	8576	8036	7813	80	9380	9352	9055	9018
55	8760	8605	8070	7858	81	9409	9383	9100	9066
56	8781	8633	8104	7902	82	9438	9415	9144	9114
57	8802	8662	8140	7947	83	9467	9446	9189	9163
58	8824	8690	8174	7992	84	9496	9478	9234	9211
59	8846	8720	8210	8037	85	9526	9510	9280	9260
60	8869	8748	8246	8082	86	9556	9542	9326	9308
61	8892	8777	8282	8128	87	9586	9574	9372	9357
62	8915	8806	8320	8173	88	9616	9606	9419	9406
63	8938	8835	8357	8220	89	9647	9638	9466	9455
64	8962	8865	8395	8265	90	9678	9671	9513	9504
65	8986	8894	8433	8311	91	9710	9703	9560	9553
66	9010	8924	8472	8357	92	9740	9736	9608	9602
67	9034	8954	8511	8404	93	9772	9768	9656	9652
68	9060	8983	8551	8450	94	9804	9801	9704	9701
69	9084	9013	8590	8497	95	9836	9834	9753	9751
70	9110	9044	8631	8544	96	9868	9867	9802	9800
71	9136	9074	8672	8590	97	9901	9900	9851	9850
72	9162	9104	8713	8637	98	9933	9933	9900	9900
73	9188	9135	8754	8685	99	9966	9966	9950	9950
74	9215	9166	8796	8732	100	10000	10000	10000	10000
75	9242	9196	8838	8780					

Examples.

1. Supposing the diameters to be 32 and 24, it is required to find the mean diameter for each variety.

Dividing 24 by 32, we obtain .75; which being found in the column of quotients, opposite thereto stand the numbers,

.9242	which being each multiplied by 32, produce respectively,	29.5744	for the corresponding mean diameter required.
.9196		29.4272	
.8838		28.2816	
.8780		28.0960	

2. The head diameter of a cask is 26 inches, and the bung diameter 3 feet 2 inches: what is the mean diameter, the cask being of the third variety?

3. The head diameter is 22 inches, the bung diameter 34

inches: what is the mean diameter of a cask of the fourth variety?

431. Having found the mean diameter, we multiply the square of the mean diameter by the decimal .7854, and the product by the length; this will give the contents in cubic inches. Then, if we divide by 231, we have the contents in wine gallons (see Art. 475); or if we divide by 282, we have the contents in beer gallons (Art. 476).

ANALYSIS.—For wine measure, we multiply the length by the square of the mean diameter, then by the decimal .7854, and divide by 231.

OPERATION.

$$l \times d^2 \times \frac{7854}{231} =$$
$$l \times d^2 \times .0034.$$

If, then, we divide the decimal .7854 by 231, the quotient carried to four places of decimals is .0034; and this decimal, multiplied by the square of the mean diameter and by the length of the cask, will give the contents in wine gallons.

For similar reasons, the content is found in beer gallons by multiplying together the length, the square of the mean diameter, and the decimal .0028.

OPERATION.

$$l \times d^2 \times \frac{7854}{282} =$$
$$l \times d^2 \times .0028.$$

Hence, for gauging or measuring casks,

Rule.—*Multiply the length by the square of the mean diameter; then multiply by* 34 *for wine, and by* 28 *for beer measure, and point off in the product four decimal places. The product will then express gallons, and the decimals of a gallon.*

1. How many wine gallons in a cask, whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches; the cask being of the first variety?

2. How many wine, and how many beer gallons in a cask whose length is 36 inches, bung diameter 35 inches, and head diameter 30 inches, it being of the first variety?

3. How many wine gallons in a cask of which the head diameter is 24 inches, bung diameter 36 inches, and length 3 feet 6 inches, the cask being of the second variety?

OF THE MECHANICAL POWERS.

432. There are six simple machines, which are called *Mechanical powers.* They are, the *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw.*

433. To understand the nature of a machine, four things must be considered.

1st. The power or force which acts. This consists in the efforts of men or horses, of weights, springs, steam, &c.:

2d. The resistance which is to be overcome by the power. This generally is a weight to be moved:

3d. The center of motion, called a *fulcrum* or *prop.* The prop or fulcrum is the point about which all the parts of the machine move:

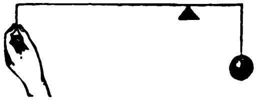
4th. The respective velocities of the power and resistance

434. A machine is said to be in equilibrium when the resistance exactly balances the power; in which case all the parts of the machine are at rest, or in uniform motion, and in the same direction.

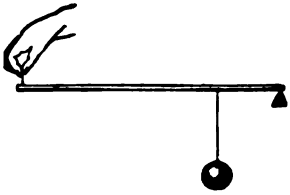
Lever.

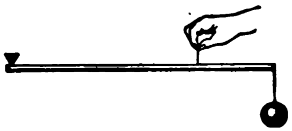
435. The Lever is a bar of wood or metal, which moves around the fulcrum. There are three kinds of levers.

1st. When the fulcrum is between the weight and the power:



2d. When the weight is between the power and the fulcrum:





3d When the power is between the fulcrum and the weight:

The perpendicular distance from the fulcrum to the directions of the weight and power, are called the *arms* of the lever.

436. An equilibrium is produced in all the levers, when the weight, multiplied by its distance from the fulcrum, is equal to the power multiplied by its distance from the fulcrum. That is,

Rule.—*The weight is to the power, as the distance from the power to the fulcrum, is to the distance from the weight to the fulcrum.*

Examples.

1. In a lever of the first kind, the fulcrum is placed at the middle point: what power will be necessary to balance a weight of 40 pounds?

2. In a lever of the second kind, the weight is placed at the middle point: what power will be necessary to sustain a weight of 50 lb.?

3. In a lever of the third kind, the power is placed at the middle point: what power will be necessary to sustain a weight of 25 lb.?

4. A lever of the first kind is 8 feet long, and a weight of 60 lb. is at a distance of 2 feet from the fulcrum: what power will be necessary to balance it?

5. In a lever of the first kind, that is 6 feet long, a weight of 200 lb. is placed at 1 foot from the fulcrum: what power will balance it?

6. In a lever of the first kind, like the common steelyard, the distance from the weight to the fulcrum is one inch; at what *distance from the* fulcrum must the poise of 1 lb. be

placed, to balance a weight of 1 lb? A weight of 1½ lb.? Of 2 lb.? Of 4 lb.?

7. In a lever of the third kind, the distance from the fulcrum to the power is 5 feet, and from the fulcrum to the weight 8 feet: what power is necessary to sustain a weight of 40 lb.?

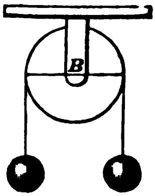
8. In a lever of the third kind, the distance from the fulcrum to the weight is 12 feet, and to the power 8 feet: what power will be necessary to sustain a weight of 100 lb.?

437. Remarks.—In determining the equilibrium of the lever, we have not considered its weight. In levers of the first kind, the weight of the lever generally adds to the power, but in the second and third kinds, the weight goes to diminish the effect of the power.

In the previous examples, we have stated the circumstances under which the power will exactly sustain the weight. In order that the power may overcome the resistance, it must of course be somewhat increased. The lever is a very important mechanical power, being much used, and entering, indeed, into most other machines.

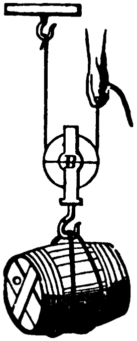
Of the Pulley.

438. The pulley is a wheel, having a groove cut in its circumference, for the purpose of receiving a cord which passes over it. When motion is imparted to the cord, the pulley turns around its axis, which is generally supported by being attached to a beam above.

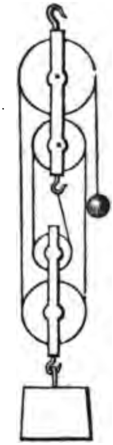


439. Pulleys are divided into two kinds, fixed pulleys and movable pulleys. When the pulley is fixed, it does not increase the power which is applied to raise the weight, but merely *changes the* direction in which it acts

440. A movable pulley gives a mechanical advantage. Thus, in the movable pulley, the hand which sustains the cask actually supports but one-half of the weight of it; the other half is supported by the hook to which the other end of the cord is attached.



441. If we have several movable pulleys, the advantage gained is still greater, and a very heavy weight may be raised by a small power. A longer time, however, will be required, than with the single pulley. It is, indeed, a general principle in machines, that *what is gained in power, is lost in time;* and this is true for all machines. There is also an actual loss of power, viz., the resistance of the machine to motion, arising from the rubbing of the parts against each other, which is called the *friction* of the machine. This varies in the different machines, but must always be allowed for, in calculating the power necessary to do a given work. It would be wrong, however, to suppose that the loss was equivalent to the gain, and that no advantage is derived from the mechanical powers. We are unable to augment our strength, but by the aid of science we so divide the resistance, that by a continued exertion of power, we accomplish that which it would be impossible to effect by a single effort.



If, in attaining this result, we sacrifice time, we cannot but see that it is most advantageously exchanged for power.

442. It is plain, that in the movable pulley, all the parts of *the cord will be equally* stretched; and hence, each cord running from pulley to pulley, will bear an equal part of the weight; consequently,

Rule.—*The power will always be equal to the weight divided by the number of cords which reach from pulley to pulley.*

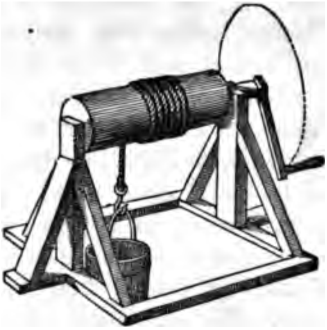
Examples.

1. In a single immovable pulley, what power will support a weight of 60 lb.?

2. In a single movable pulley, what power will support a weight of 80 lb.?

3. In two movable pulleys, with 4 cords (see last fig.), what power will support a weight of 100 lb.?

Winch, or Wheel and Axle.



443. This machine is composed of a wheel or crank, firmly attached to a cylindrical axle. The axle is supported at its ends by two pivots, which are of less diameter than the axle around which the rope is coiled, and which turn freely about the points of support. In order to balance the weight, we must have,

Rule.—*The power to the weight, as the radius of t axle, to the length of the crank, or radius of the wheel.*

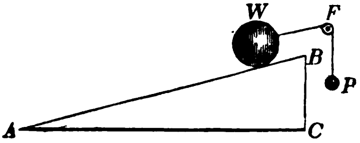
Examples.

1. What must be the length of a crank or radius of wheel, in order that a power of 40 lb. may balance a weig *of 600 lb.* suspended from an axle of 6 inches radius?

2. What must be the diameter of an axle, that a power of 100 lb., applied at the circumference of a wheel of 6 feet diameter, may balance 400 lb.?

Inclined Plane

444. The inclined plane is nothing more than a slope or eclivity, which is used for the purpose of raising weights. It is not difficult to see that a weight can be forced up an inclined plane, more easily than it can be raised in a vertical line. But in this, as in the other machines, the advantage is obtained by a partial loss of power.



Thus, if a weight W, be supported on the inclined plane ABC, by a cord passing over a pulley at F, and the cord from the pulley to the weight be parallel to the length of the plane AB, the power P will balance the weight W., when

P : W :: height BC : length AB.

It is evident that the power ought to be less than the weight, since a part of the weight is supported by the plane: hence,

Rule.—*The power is to the weight, as the height of the plane is to its length.*

Examples.

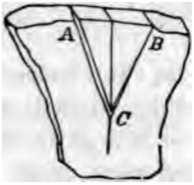
1. The length of a plane is 30 feet, and its height 6 feet: what power will be necessary to balance a weight of 200 lb.?

2. The height of a plane is 10 feet, and the length 20 feet: what weight will a power of 50 lb. support?

3. The height of a plane is 15 feet, and length 45 feet: what power will sustain a weight of 180 lb.?

The Wedge.

445. The wedge is composed of two inclined planes, united together along their bases, and forming a solid ACB. It is used to cleave masses of wood or stone. The resistance which it overcomes is the attraction of cohesion of the body which it is employed to separate The wedge acts, principally, by being struck with a hammer, or mallet, on its head, and very little effect can be produced with it, by mere pressure.



All cutting instruments are constructed on the principle of the inclined plane or wedge. Such as have but one sloping edge, like the chisel, may be referred to the inclined plane; and such as have two, like the ax and the knife, to the wedge.

Rule.—*Half the thickness of the head of the wedge, is to the length of one of its sides, as the power which acts against its head to the effect produced at its side.*

Examples.

1. If the head of a wedge is 4 inches thick, and the length of one of its sides 12 inches, what will measure the effect of a force denoted by 96 pounds?

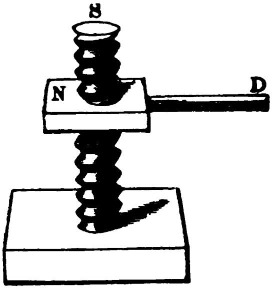
2. If the head of a wedge is 6 inches thick, the length of the side 27 inches, and the force applied measures 250 pounds, what will be the measure of the effect?

3. If the head of a wedge is 9 inches, and the length of the side 2 feet, what will be the effect of a blow denoted by 200 pounds?

4. If the head of a wedge is 10 inches, and the length of the side 30 inches, what will measure the effect of a blow *denoted by 500?*

The Screw.

446. The screw is composed of two parts—the screw, S, and the nut, N.



The screw, S, is a cylinder with a spiral projection winding around it. The nut, N, is perforated to admit the screw, and within it is a grove into which the thread of the screw fits closely.

The handle, D, which projects from the nut, is a lever which works the nut upon the screw. The power of the screw depends on the distance between the threads. The closer the threads of the screw, the greater will be the power; but then the number of revolutions made by the handle, D, will also be proportionably increased: so that we return to the general principle—what is gained in power is lost in time. The power of the screw may also be increased by lengthening the lever, D, attached to the nut.

The screw is used for compression, and to raise heavy weights. It is used in cider and wine presses, in coining, and for a variety of other purposes.

Rule.—*As the distance between the threads of a screw, is to the circumference of the circle described by the power, so is the power employed to the weight raised.*

Examples.

1. If the distance between the threads of a screw is half an inch, and the circumference described by the handle 15 feet, what weight can be raised by a power denoted by 720 pounds?

2. If the threads of a screw are one-third of an inch apart, and the handle is 12 feet long, what power must be applied to sustain 2 tons?

3. What force applied to the handle of a screw 10 feet long, with threads one inch apart, working on a wedge whose head is 5 inches, and length of side 30 inches, will produce an effect measured by 10000 lb.?

4. If a power of 300 pounds applied at the end of a lever 15 feet long will sustain a weight of 282744 lb., what is the distance between the threads of the screw?

QUESTIONS IN NATURAL PHILOSOPHY.

UNIFORM MOTION.

447. If a moving body passes over equal spaces in equal portions of time, it is said to move with uniform motion, or uniformly.

448. The velocity of a moving body is measured by the space passed over in a second of time.

449. The space passed over in any time is equal to the product of the velocity multiplied by the number of seconds in the time.

If we denote the velocity by V, the space passed over by S, and the time by T, we have

$$S = V \times T.$$

LAWS OF FALLING BODIES.

450. A body falling vertically downward in a vacuum, falls through $16\frac{1}{12}$ ft. during the first second after leaving its place of rest, $48\frac{1}{4}$ ft. during the second second, $80\frac{5}{12}$ ft. the third second, and so on: the spaces forming an arithmetical progression of which the common difference is $32\frac{1}{6}$ ft., or double the space fallen through during the first second. This number is called the measure of the force of gravity, and is denoted by g.

451. It is seen from the above, that the velocity of a body is continually increasing. If H denote the height fallen through, T, the time, V, the velocity acquired, and g, the force of gravity, the following formulas have been found to express the relations between these quantities:

$$V = g \times T \quad . \; . \; . \quad (1)$$
$$V^2 = 2g \times H \quad . \; . \quad (2).$$
$$H = \tfrac{1}{2}V \times T \quad . \; . \; . \quad (3).$$
$$H = \tfrac{1}{2}g \times T^2 \quad . \; . \quad (4).$$

From which we see,

1st. *That the velocity acquired at the end of any time, is equal to the force of gravity* ($32\frac{1}{6}$) *multiplied by the time.*

2d. *That the square of the velocity is equal to twice the force of gravity multiplied by the height; or, the velocity is equal to the square root of that quantity.*

3d. *That the space fallen through is equal to one-half the velocity multiplied by the time.*

4th. *That the space fallen through is equal to one-half the force of gravity multiplied by the square of the time.*

452. If a body is thrown vertically upward in a vacuum, its motion will be continually retarded by the action of gravitation. It will finally reach the highest point of its ascent, and then begin to descend. The height to which it will rise may be found by the second formula in the preceding paragraph, when the velocity with which it is projected upward is known; for the times of ascent and descent will be equal.

453. The above laws are only approximately true for bodies falling through the air, in consequence of its resistance. We may measure the depths of wells or mines, and the heights of elevated objects approximately, by using dense bodies, as leaden bullets or stones, which present small surfaces to the air.

Examples.

1. A body has been falling 12 seconds: what space did it describe in the *last second*, and what in the whole time?

2. A body has been falling 15 seconds: find the space described and the velocity acquired.

3. How far must a body fall to acquire a velocity of 120 feet?

4. How many seconds will it take a body to fall through a space of 100 feet?

5 Find the space through which a heavy body falls in 10 seconds, and the velocity acquired.

6. How far must a body fall to acquire a velocity of 1000 feet?

7. A stone is dropped into a well, and strikes the water in 3.2 seconds: what is the depth of the well?

8. A stone is dropped from the top of a bridge, and strikes the water in 2.5 seconds: what is the height of the bridge?

9. A body is thrown vertically upward with a velocity of 160 feet: what height will it reach, and what will be the time of ascent?

10. An arrow shot perpendicularly upward, returned again in 10 seconds. Required the velocity with which it was shot, and the height to which it rose.

11. A ball is let fall from the top of a steeple, and reaches the ground in three seconds and a half: what is the height of the steeple?

12. What time will be necessary for a body falling freely, to acquire a velocity of 2500 feet per second?

13 If a ball be thrown vertically upward with a velocity of 350 feet per second, how far will it ascend, and what will be the time of ascent and descent?

14. How long must a body fall freely to acquire a velocity of 3040 feet per second?

15. If a body falls freely in a vacuum, what will be its velocity after 45 seconds of fall?

16. During how many seconds must a body fall in a vacuum to acquire a velocity of 1970 feet, which is that of a cannon-ball?

17. What time is required for a body to fall in a vacuum, from an elevation of 3280 feet?

18. From what height must a body fall to acquire a velocity of 984 feet?

19. A rocket is projected vertically upward with a velocity of 386 feet: after what time will it begin to fall, and to what height will it rise?

SPECIFIC GRAVITY.

454. The SPECIFIC GRAVITY of a body is the weight of a unit of volume *compared with the weight* of a unit of the standard. Distilled rain-water is the standard for measuring the specific gravity of bodies. Thus, 1 cubic foot of distilled rain-water weighs 1000 ounces avoirdupois. If a piece of stone, *of the same volume*, weighs 2500 ounces, its specific gravity is 2.5; that is, the stone is 2.5 times as heavy as water.

If, then, we denote the standard by 1, the specific gravity of all other bodies will be expressed in terms of this standard; and if we multiply the number denoting the specific gravity of any body by 1000, the product will be the weight in ounces of 1 cubic foot of that body.

If any body be weighed in air and then in water, it will weigh less in water than in air. The difference of the weights will be equal to the sustaining force of the water, which is found to be equal to the weight of an equal volume of water: hence,

Rule.—*If we know or can find the weight of a body in air and in water, the difference of these weights will be equal to that of an equal volume of water; and the weight of the body in air divided by this difference will be the measure of the specific gravity of the body, compared with water as a standard.*

Table

OF SPECIFIC GRAVITIES—WATER, 1.

NAMES OF BODIES.	SPEC. GRAV.	NAMES OF BODIES.	SPEC. GRAV.
METALS.		Porphyry	2.60
Platinum	21.000	Sandstone	2.50
Gold	19.500	Brick	1.86
Quicksilver	13.500		
Lead	11.350	WOODS.	
Silver	10.51	Oak, fresh felled	1.049
Copper	8.800	White Willow	0.9859
Bronze	8.758	Box	0.9822
Brass	8.000	Elm	0.9476
Steel	7.800	Horbeam	0.9452
Iron	7.500	Larch	0.9206
Tin	7.291	Pine	0.9121
Zinc	7.215	Maple	0.9036
		Ash	0.9036
BUILDING STONES.		Birch	0.9012
		Fir	0.8941
Hornblende	3.10	Horse-chestnut	0.8614
Basalt	3.10		
Alabaster	3.00	SOLID BODIES.	
Syenite	3.00		
Dolerite	2.93	Common earth	1.480
Gneiss	2.90	Moist sand	2.050
Quartz	2.75	Clay	2.150
Limestone	2.72	Flint	2.542
Phonolite	2.69	Ice	0.926
Granite	2.66	Lime	1.842
Stone for building	2.62	Tallow	0.942
Trachyte	2.60	Wax	0.969

By inspecting this Table, we see the weight of each body compared with an equal volume of water. Thus, platinum is 21 times as heavy as water; gold, 19 times as heavy; iron, 7½ times as heavy, &c.

Examples illustrating Specific Gravity.

1. A piece of copper weighs 93 grains in air, and 82¼ grains in water: what is its specific gravity?

2. How many cubic feet are there in 2240 pounds of dry oak, of which the specific gravity is .925, a cubic foot of *andard water* weighing 1000 ounces?

3. A piece of pumice-stone weighs in air 50 ounces, and when it is connected with a piece of copper which weighs 390 ounces in air, and 345 ounces in water, the compound weighs 344 ounces in water: what is the specific gravity of the stone?

4. A right prism of ice, the length of whose base is 20.45 yards, breadth 15.75 yards, and height 10.5 yards, floats on the sea; the specific gravity of the ice is .930, and that of the sea-water 1.026: what is the height of the prism above the surface of the water?

5. A vessel in a dock was found to displace 6043 cubic feet of water: what was the weight of the vessel, each cubic foot of the water weighing 63 pounds?

6. A piece of glass was found to weigh in the air 33 ounces, and in the water 21 ounces: what was its specific gravity?

7. A piece of zinc weighed in the air 17 pounds, and lost when weighed in water 2.35 pounds: what was its specific gravity?

8. If a piece of glass weighed in water loses 318 ounces of its weight, and weighed in alcohol loses 250 ounces, what is the specific gravity of the alcohol?

9. A flask filled with distilled water weighed 14 ounces, filled with brandy, it weighed 13.25 ounces; the flask itself weighed 8 ounces: what was the specific gravity of the brandy?

10. What is the weight of a cubic foot of statuary marble, of which the specific gravity is 2.837, the cubic foot of water weighing 1000 ounces?

11. A jar containing air weighed 24 ounces 33 grains; the air was then excluded, and the jar weighed 24 ounces; the jar being then filled with oxygen gas, weighed 24 ounces 36.4 grains: what was the specific gravity of the oxygen, the air *being taken as the standard*?

MARIOTTE'S LAW.

455 This law, which relates to air and all other gases, steam and all other vapors, was discovered by the Abbé Mariotte, a French philosopher, who died in 1684. It will be easily understood from a particular example.

Suppose an upright cylindrical vessel in a vacuum contains a gas which is confined in the vessel by a piston at the upper end. Suppose the gas or vapor fills the whole vessel, and the piston is loaded with a weight of 5 pounds. If now, the piston be loaded with a weight of 10 pounds, the gas will be compressed and occupy only half its former space. If the weight be increased to 15 pounds, the gas will have only one-third of its original volume, and so on. At the same time, the density of the gas or vapor will be doubled, made three times as great, and so on. The law, therefore, may be thus stated:

Rule.—*The temperature remaining the same, the volume of a gas or vapor is inversely proportional to the pressure which it sustains.* Also, the density of a *gas or vapor is directly proportional to the pressure.*

Examples.

1. A vase contains 4.3 quarts of air, the pressure being 10 pounds: what will be the volume of the air when the pressure is 12.3 pounds, the temperature remaining the same?

2. Under a pressure of 15 pounds to the square inch, a certain quantity of gas occupies a volume of 20 quarts: what pressure must be applied to reduce the volume to 8 quarts?

3. A quart of air weighs 2.6 grains under a pressure of 15 pounds: what will be the weight of a quart, if the pressure be reduced to 14.2 pounds?

4. The pressure upon the steam contained in a cylinder is *increased* from 25 pounds upon the square inch to 47 pounds: *what part* of the original volume will be occupied?

APPENDIX.

DIFFERENT KINDS OF UNITS.

I. ABSTRACT UNITS.

456. THE ABSTRACT UNIT 1 is the base of all numbers, and is called a unit of the first order. The unit 1 ten, is a unit of the second order; the unit 1 hundred, is a unit of the third order; and so for units of the higher orders. These are abstract numbers formed from the unit 1, according to the scale of tens. All abstract integral numbers are collections of the unit one.

II. UNITS OF CURRENCY.

457. In all civilized and commercial countries, great care is taken to fix a standard value for money, which standard is called the *Unit of Currency.*

In the United States, the unit of currency is 1 dollar; in Great Britain it is 1 pound sterling, equal to $4.84; in France it is 1 franc, equal to $18\frac{6}{10}$ cents nearly. All sums of money are expressed in the unit of currency, or in units derived from the unit of currency, and having fixed proportions to it.

III. UNITS OF LENGTH.

458. One of the most important units of measure is that for distances, or for the measurement of length. A practical want has ever been felt of some fixed and invariable standard, with which all distances may be compared: such fixed standard has been sought for in nature.

There are two natural standards, either of which affords this desired natural element. Upon one of them, the English have founded their system of measures, from which ours is taken and upon the other, the French have based their syst

These two systems, being the only ones of importance, will be alone considered.

FIRST.—The English system of measures, to which ours conforms, is based upon the law of nature, that the *force of gravity is constant at the same point of the earth's surface* and consequently, that the length of a pendulum which oscillates a certain number of times in a given period, is also con stant. Had this unit been known *before* the adoption and use of a system of measures, it would have formed the natural unit for division and been the natural base of the system of linear measure. But the foot and inch had long been used as units of linear measure; and hence, the length of the pendulum, the new and invariable standard, was expressed in terms of the known units, and found to be equal to 39.1393 inches. The new unit was therefore declared invariable—to contain 39.1393 equal parts, each of which was called an *inch;* 12 of these parts were declared by act of Parliament to be a *standard foot,* and 36 of them an *Imperial yard.* The Imperial yard and the standard foot are marked upon a brass bar, at the temperature of 62½°, and these are the linear measures from which ours are taken. The comparison has been made by means of a brass scale 82 inches long, manufactured by Troughton, in London, and now in the possession of the Treasury Department.

SECOND.—The French system of measures is founded upon the principle of the invariability of the length of an arc of the same meridian between two fixed points. By a very minute survey of the length of an arc of the meridian from Dunkirk to Barcelona, the length of a quadrant of the meridian was computed, and it has been decreed by the French law that the *ten-millionth* part of this length shall be regarded as a standard French *mètre,* and from this, by multiplication and division, the entire system of linear measures has been *established.*

On comparing the two scales very accurately, it has been found that the French mètre is equal to 39.37079 English inches—differing somewhat from the English yard. This relation enables us to convert all measures in either system into the corresponding measures of the other.

IV. UNITS OF SURFACE

459. The linear unit having been established, the most convenient UNIT OF SURFACE is the area of a square, one of whose sides is the unit of length. Thus, the units of surface in common use, are—

A square inch = a square on 1 inch.
A square foot = a square on 1 foot.
A square yard = a square on 1 yard.
A square rod = a square on 1 rod.

V. UNITS OF VOLUME.

460. THE UNIT OF VOLUME, for the measurement of solids, is taken equal to the volume of a cube one of whose edges is equal to the linear unit. The units of volume in common use are—

A cubic inch = a cube whose edge is 1 inch ;
A cubic foot = a cube whose edge is 1 foot = 1728 cubic in.
A cubic yard = a cube whose edge is 1 yard = 27 cubic feet.
A perch of stone = $24\frac{3}{4}$ cubic feet ;

or a block of stone 1 rood long, 1 foot thick, and $1\frac{1}{2}$ ft. wide.

The standard unit of volume for the measurement of liquids is the wine gallon, which contains 231 cubic inches.

The *standard unit of dry measure* is the Winchester bushel, which contains 2150.4 cubic inches, nearly.

VI. UNITS OF WEIGHT.

461. Having fixed an invariable unit of length, we passed easily to an invariable unit of surface, and then, to an invariable unit of volume. We wish now to define an invariable unit of weight.

It has been found that distilled rain-water is the most invariable substance; hence, this has been adopted as the standard.

We have two units of weight, the avoirdupois pound, and the pound troy.

The *standard avoirdupois pound* is the weight of 27.701554 cubic inches of distilled water.

The *standard Troy pound* is the weight of 22.794422 cubic inches. This standard is at present kept in the United States Mint at Philadelphia, and is the *standard unit of weight.*

VII. UNITS OF TIME.

462. Time can only be measured by motion. The diurnal revolution of the earth affords the only invariable motion; hence, the time in which it revolves once on its axis, is the natural unit, and is called a day. From the day, by multiplication, we form the weeks, months, and years; and by division, the hours, minutes, and seconds.

VIII. UNITS OF CIRCULAR OR ANGULAR MEASURE.

463. This measure is used for the measurement of angles, and the natural unit is the right angle. But this is not the most convenient unit. The unit chiefly used is the 360 part of the circumference of a circle, called a *degree*, which is divided into 60 equal parts, called *minutes*, and these again into 60 equal parts, called *seconds*.

Remarks.

464. It is seen that all the units, determined by the pendulum, depend on *time* as the ultimate base; that is, the length of a pendulum which will vibrate seconds determines all the *units of measure and weight.*

Now, time is measured by motion, and the motion of the earth on its axis is the only invariable motion. Hence, we refer to this to fix the unit of time, on which the unit of *length depends*, and from which all the other units are derived

No class of pupils can rightly and clearly apprehend the nature of numbers and the operations performed upon them, without distinct and fixed notions of the units; hence, every teacher should labor to point out their absolute and relative values: this can only be done by means of sensible objects.

Every school-room, therefore, should be provided with a complete set of all the denominate units. The inch, the foot, the yard, the rod, should be accurately marked off on a conspicuous part of the room, together with the principal units of surface, the squre inch, square foot, square yard, &c.

The units of volume should also be exhibited. The cubic inch and the cubic foot will serve as illustrations for one class of the units of volume; and the pint, quart, gallon, and bushel, should be exhibited to illustrate the others.

The unit of weight should also be seen and handled. A child even can apprehend what is meant by an *ounce* or a *pound*, when he takes one of these weights in his hand; and mature years can acquire the idea in no other way.

Let, therefore, every school-room be furnished with a complete set of models to illustrate and explain the *absolute* and *relative* values of the different units.

I. ABSTRACT NUMBERS.

465. An Abstract Number is one whose unit is not named.

Table.

10 Units	**make**	**1 Ten.**
10 Tens		**1 Hundred.**
10 Hundred		**1 Thousand.**
10 Thousand		**1 Ten-thousand.**
&c.,		**&c.**

Table Reversed.

Ten-thous.		Thous.		Hund.		Ten.		Units.
						1	=	10.
				1	=	10	=	100.
		1	=	10	=	100	=	1000.
1	=	10	=	100	=	1000	=	10000.

Scale.—*Uniform—units,* 10.

II. CURRENCY.

I. UNITED STATES MONEY.

466. **United States Money** is the currency established by Congress, A. D. 1786. The names or denominations of its units are, Eagles, Dollars, Dimes, Cents, and Mills.

The coins of the United States are of gold, silver, copper, and nickel, and are of the following denominations :

1. Gold : Double-eagle, eagle, half-eagle, three-dollars, quarter-eagle, dollar.

2. Silver : Dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.

3. Copper : Cent.

4. Nickel : Cent.

Table.

10 Mills	make	1 Cent,	marked *ct.*
10 Cents		1 Dime,	*d.*
10 Dimes		1 Dollar,	*$.*
10 Dollars		1 Eagle,	*E.*
20 Dollars		1 Double Eagle.	2 *E*

Table Reversed.

E.	$	d.	ct.	m.
			1	= 10.
		1	= 10	= 100.
	1	= 10	= 100	= 1000.
1	= 10	100	= 1000	= 10000.

Scale.—Uniform—units, 10.

In all the States the shilling is reckoned at 12 *pence, the pound at* 20 *shillings, and the dollar at* 100 *cents.*

The following table shows the number of shillings in a dollar, the value of £1 in dollars, and the value of $1 in the *fraction of a* pound :

In English currency,	4s. 6d.	- £1 = \$4.84,	and \$1 = £$\frac{1}{4.84}$.
In N. E., Va., Ky., Tenn., Ind., Mo.,	6s.	- £1 = \$3$\frac{1}{3}$,	and \$1 = £$\frac{3}{10}$.
In New York, Ohio, Michigan,	8s.	- £1 = \$2$\frac{1}{2}$.	and \$1 = £$\frac{2}{5}$.
North Carolina,	10s.		
In N. J., Pa., Del., Md.,	7s. 6d.	- £1 = \$2$\frac{2}{3}$,	and \$1 = £$\frac{3}{8}$.
In S. Carolina & Ga.,	4s. 8d.	- £1 = \$4$\frac{2}{7}$,	and \$1 = £$\frac{7}{30}$.
In Canada and Nova Scotia,	5s.	- £1 = \$4,	and \$1 = £$\frac{1}{4}$.

NOTES.—The present *standard* or degree of purity of the coins was fixed by Act of Congress in 1837. It is this:

1. Nine hundred equal parts of pure gold, are mixed with 100 parts of alloy, of copper, and silver, (of which not more than one-half must be silver) thus forming 1000 parts, equal to each other in weight. The silver coins contain 900 parts of pure silver, and 100 parts of pure copper. The copper coins are of pure copper. The nickel cent is 88 parts copper and 12 nickel.

2. The eagle contains 258 grains of standard gold, and the other gold coins in the same proportion. The dollar contains 412½ grains of standard silver, and the others in the same proportion. The cent, 168 grains of pure copper.

3. If a given quantity of gold or silver be divided into 24 equal parts, each part is called a *carat.* If any number of carats be mixed with so many equal carats of a less valuable metal, that there be 24 carats in the mixture, then the compound is said to be as many carats fine as it contains carats of the more precious metal, and to contain as much alloy as it contains carats of the baser.

4. Although the currency of the United States is in dollars, cents and mills, yet in some of the States the old currency of pounds shillings, and pence, is still nominally preserved.

II. ENGLISH MONEY.

467. The units or denominations of English money are *guineas, pounds, shillings,* pence, and farthings.

Table.

4 farthings, marked *far.*, make	1 penny,	marked	*d.*
12 pence, ...	1 shilling,	"	*s.*
20 shillings, ...	1 pound, or sovereign,		£.
21 shillings, ...	1 guinea.		

Table Reversed.

£		s.		d.		far.
				1	=	4.
		1	=	12	=	48.
1	=	20	=	240	=	960.

NOTES.—1. The primary unit in English money is 1 farthing. The units of the scale, in passing from farthings to pence, are 4; in passing from pence to shillings, the units of the scale are 12; in passing from shillings to pounds, they are 20.

2. Farthings are generally expressed in fractions of a penny. Thus, 1 far. = ¼d.; 2 far. = ½d.; 3 far. = ¾d.

3. The standard of the gold coin is 22 parts of pure gold and 2 parts of copper.

The standard of silver coin is 37 parts of pure silver, and 3 parts of copper.

A pound of gold is worth 14.2878 times as much as a pound of silver. In copper coin, 24 pence make 1 pound avoirdupois.

By reading the second table from left to right, we can see the value of any unit expressed in each of the lower denominations. Thus, 1d. = 4 far.; 1s. = 12d. = 48 far. £1 = 20s. = 240d. = 960 far

TABLE OF LEGAL VALUES OF FOREIGN COINS.

	$	*ct.*
Franc of France and Belgium..........................	0	18$\frac{6}{10}$
Florin of the Netherlands..........................		40
Guilder of do.		40
Livre Tournois of France..........................		18½
Milrea of Portugal..........................	1	12
Milrea of Madeira..........................	1	00
Milrea of the Azores..........................		83⅓
Marc Banco of Hamburg..........................		35
Pound Sterling of Great Britain..........................	4	84
Pagoda of India..........................	1	84
Real Vellon of Spain..........................		05
Real Plate of do.		10
Rupee Company..........................		44½

Rupee of British India		44½
Rix Dollar of Denmark	1	00
Rix Dollar of Prussia		68½
Rix Dollar of Bremen		78¾
Rouble, silver, of Russia		75
Tale of China	1	48
Dollar of Sweden and Norway	1	06
Specie Dollar of Denmark	1	05
Dollar of Prussia and Northern States of Germany		69
Florin of Southern States of Germany		40
Florin of Austria and City of Augsburg		48½
Lira of the Lombardo-Venetian Kingdom		16
Lira of Tuscany		16
Lira of Sardinia		18$\frac{6}{10}$
Ducat of Naples		80
Ounce of Sicily	2	40
Pound of Nova Scotia, New Brunswick, Newfoundland, and Canada	4	04

TABLE OF FOREIGN COINS OF USAGE VALUES.

	$	ct.
Berlin Rix Dollar		69½
Current marc		28
Crown of Tuscany	1	05
Elberfeldt Rix Dollar		69¾
Florin of Saxony		48
" Bohemia		48
" Elberfeldt		40
" Prussia		22¾
" Trieste		48
" Nuremburg		40
" Frankfort		40
" Basil		41
" St. Gaul		40$\frac{36}{100}$
" Creveld		40
Florence Livre		15
Genoa do.		18¾
Geneva do.		21
Jamaica Pound	5	00
Leghorn Dollar		90
Leghorn Livre (6½ to the dollar)		15[illegible]
Livre of Catalonia		58½
Neufchatel Livre		26½
Pezza of Leghorn		90
Rhenish Rix Dollar		60¾
Swiss Livre		27
Scuda of Malta		40
Turkish Piastre		05

[The above Tables are taken from a work on the Tariff, by E. D. Ogden, Esq., of the New York Custom-house.]

III. LINEAR MEASURE.

I. LONG MEASURE.

468. This measure is used to measure distances, lengths, breadths, heights and depths.

Table.

12 inches	make	1 foot,	marked *ft.*
3 feet		1 yard,	*yd.*
5½ yards, or 16½ feet		1 rod,	*rd.*
40 rods		1 furlong,	*fur.*
8 furlongs, or 320 rods		1 mile,	*mi.*
3 miles		1 league,	*L.*
69⅛ statute miles, nearly, or 60 geographical miles		1 degree on the equator, or any great circle,	*deg.*, or °
360 degrees		a circumference of the earth.	

Table Reversed.

mi.		fur.		rd.		yd.		ft.	in.
								1	12.
						1	=	3	36.
				1	=	5½	=	16½	198.
		1	=	40	=	220	=	660	7920.
1	=	8	=	320	=	1760	=	5280	63360.

NOTES.—1. A fathom is a length of six feet, and is generally used to measure the depth of water.

2. A hand is 4 inches, and is used to measure the height of horses; a common pace is 3 feet; a military pace, 2½ feet; a geographical mile equals a minute of a great circle; a knot (used by sailors) is a geographical mile.

3. The units of the scale, in passing from inches to feet, are 12; in passing from feet to yards, 3; from yards to rods, 5½; from rod to furlongs, 40; and from furlongs to miles, 8.

ENGLISH SYSTEM.

469. The Imperial yard of Great Britain is the one from which ours is taken. Hence, the units of measure are identical.

FRENCH SYSTEM.

470. The base of the new French system of measure is the measure of the meridian of the earth, a quadrant of which is 10,000,000 *mètres*, measured at the temperature of 32° Fahr. The multiples and divisions of it are decimals, viz.: 1 metre = 10 decimetres = 100 centimetres = 1000 millimetres = 3.280899 United States feet, or 39.37079 inches.

This relation enables us to convert all measures in either system into the corresponding measures of the other.

Austrian,	1 foot	= 12.448 U. S. inches	= 1.03737 foot.	
Prussian, *Rhineland,*	1 foot	= 12.361 " "	= 1.0300 "	
Swedish,	1 foot	= 11.690 " "	= 0.974145 "	
Spanish,	1 foot	= 11.034 " "	= 0.9195 "	
	league (royal)	= 25000 Span. ft.	= 4⅓ miles	nearly
	" (common)	= 19800 "	= 3½ "	nearly

II. CLOTH MEASURE.

471. Cloth measure is used for measuring all kinds of cloth, ribbons, and other things sold by the yard.

Table.

2¼ inches, (in.)	make	1 nail,	marked *na.*
4 nails		1 quarter of a yard,	*qr.*
3 quarters		1 Ell Flemish,	*E. Fl.*
4 quarters		1 yard,	*yd.*
5 quarters		1 Ell English,	*E. E.*

Table Reversed.

E. E.		yd.		E. Fl.		qr.		na.		in.
								1	=	2¼
						1	=	4	=	9.
				1	=	3	=	12	=	27.
		1	=	1⅓	=	4	=	16	=	36.
1	=	1¼	=	1⅔	=	5	=	20	=	45.

NOTE.—The units of the scale, in this measure, are 2¼, 4, 3, $\frac{4}{3}$, and $\frac{5}{4}$.

IV. UNITS OF SURFACE

I. SQUARE MEASURE.

472. Square measure is used in measuring land or any thing in which length and breadth are both considered.

Table.

144 square inches (*sq. in.*) make	1 square foot, ...	*Sq. ft*
9 square feet	1 square yard, ...	*Sq. yd*
30¼ square yards	1 square rod, or 1 perch, .	*P.*
40 square rods, or 40 perches,	1 rood, ...	*R.*
4 roods, or 160 perches, . .	1 acre, ...	*A.*
640 acres	1 square mile .	*M.*

Table Reversed.

A.		R.		P.		sq. yd.		sq. ft.		sq. in.
								1	=	144.
						1	=	9	=	1296.
				1	=	30¼	=	272¼	=	39204.
		1	=	40	=	1210	=	10890	=	1568160.
1	=	4	=	160	=	4840	=	43560	=	6272640.

NOTE.—The units of the scale in this measure are 144, 9, 30¼ 40, and 4.

II. SURVEYORS' MEASURE.

473. The Surveyor's, or Gunter's chain, is generally used in surveying land. It is 4 poles, or 66 feet in length, and is divided into 100 links.

Table.

$\frac{92}{100}$ inches . . . . make	1 link, ... marked	*l*
4 rods = 66 ft = 100 links .	1 chain, ...	*c*
80 chains	1 mile, . .	*mi*
1 square chain	16 square rods, or perches, . .	*P*
10 square chains	1 acre, ...	*A*

NOTES.—1. Land is generally estimated in square miles, acres roods, and square rods, or perches.

2. The units of the scale, in this measure, are 7$\frac{92}{100}$, 4, 80, 1, *and 10*.

V. UNITS OF VOLUME, OR CAPACITY.

I. CUBIC MEASURE.

474. Cubic measure is used for measuring stone, timber, earth, and such other things as have the three dimensions, length, breadth, and thickness.

Table.

1728 cubic inches (*cu. in.*) make	1 cubic foot,	marked	*cu. ft.*
27 cubic feet	1 cubic yard,		*cu. yd.*
40 feet of round, or 50 feet of hewn timber,	1 ton,		*T.*
42 solid feet	1 ton of shipping,		*T*
8 cord feet, or 128 cubic feet,	1 cord,		*C.*
24¾ cubic feet of stone	1 perch,		*P*

NOTES.—1. A cord of wood is a pile 4 feet wide, 4 feet high, and 8 feet long.

2. A cord foot is 1 foot in length of the pile which makes a cord.

3. A CUBE is a solid or volume bounded by six equal squares, called *faces;* the sides of the squares are called *edges.*

4. A cubic foot is a cube, each of whose faces is a square foot; its edges are each 1 foot.

5. A cubic yard is a cube, each of whose edges is 1 yard

6. A ton of *round timber,* when square, is supposed to produce 40 cubic feet: hence, *one-fifth is lost by squaring*

II. LIQUID MEASURE.

475. Liquid, or wine measure, is used for measuring all *liquids except ale, beer,* and milk.

Table.

4 gills (*gi.*)	make 1 pint,	marked	*pt*
2 pints	1 quart,		*qt*
4 quarts	1 gallon,		*gal*
31½ gallons	1 barrel,		*bar.* or *bbl.*
42 gallons	1 tierce,		*tier*
63 gallons	1 hogshead,		*hhd*
2 hogsheads	1 pipe,		*pi*
2 pipes, or 4 hogsheads,	1 tun,		*tun.*

Table Reversed.

tun.	pi.	hhd.	tier.	bar.	gal.	qt.	pt.	gi.
							1	4.
						1	2	8.
					1	4	8	32.
				1	31½	126	252	1008
			1	1⅓	42	168	336	1344.
		1	1½	2	63	252	504	2016.
	1	2	3	4	126	504	1008	4032
1	2	4	6	8	252	1008	2016	8064.

NOTES.—1. The *standard unit*, or gallon of wine measure, in the United States, contains 231 cubic inches, and hence, is equal to the weight, avoirdupois, of 8.339 cubic inches of distilled water, very nearly.

2. The English Imperial wine gallon contains 277.274 cubic inches, and hence, is equal to 1.2 times the wine gallon of the United States, nearly.

III. BEER MEASURE

476 Beer measure was formerly used for measuring ale, beer, and milk. They are now generally measured by wine measure.

Table.

2 pints (*pt.*)	make 1 quart,	marked	*qt*
4 quarts	1 gallon,		*gal*
36 gallons	1 barrel,		*bar*
54 gallons	1 hogshead,		*hhd*

Table Reversed.

hhd.		bar.		gal.		qt.		pt.
						1	=	2.
				1	=	4	=	8.
		1	=	36	=	144	=	288.
1	=	1½	=	54	=	216	=	432.

NOTES.—1. The *standard gallon*, beer measure, contains 282 cubic inches, and hence, is equal to the weight of 10.1799 cubic inches of distilled rain-water.

2. Milk is generally bought and sold by wine measure.

III. DRY MEASURE.

477. Dry measure is used in measuring all dry articles, such as grain, fruit, salt, coal, &c.

Table.

2 pints (*pt.*)	make	1 quart,	marked	*qt*
8 quarts		1 peck,		*pk.*
4 pecks		1 bushel,		*bu.*
36 bushels		1 chaldron,		*ch.*

Table Reversed.

ch.		bu.		pk.		qt.		pt.
						1	=	2.
				1	=	8	=	16.
		1	=	4	=	32	=	64.
1	=	36	=	144	=	1152	=	2304.

NOTES.—1. The *standard bushel* of the United States is the *Winchester* bushel of England. It is a circular measure 18½ inches in diameter, and 8 inches deep, and contains 2150.4 cubic inches, nearly it contains 77.627413 pounds avoirdupois of distilled water.

2. A gallon, dry measure, contains 268.8 cubic inches.

3. Wine measure, beer measure, and dry measure, and all measures of volume, differ from the cubic measure only in the unit which is used as a standard.

VI. UNITS OF WEIGHT.

I. AVOIRDUPOIS WEIGHT.

478. By this weight all coarse articles are weighed, such as hay, grain, chandlers' wares, and all metals except gold and silver.

Table.

16 drams (*dr.*)	make	1 ounce,	marked *oz*
16 ounces		1 pound,	*lb.*
25 pounds		1 quarter,	*qr.*
4 quarters		1 hundred weight	*cwt.*
20 hundred weight		1 ton,	*T.*

Table Reversed.

T.		cwt.		qr.		lb.		oz.		dr.
								1	=	16.
						1	=	16	=	256.
				1	=	25	=	400	=	6400.
		1	=	4	=	100	=	1600	=	25600.
1	=	20	=	80	=	2000	=	32000	=	512000.

NOTES.—1. The standard avoirdupois pound is the weight of 27.7015 cubic inches of distilled water; and hence, 1 cubic foot weighs 1000 ounces, *very nearly.*

2. By the old method of weighing, adopted from the English system, 112 pounds were reckoned for a hundred weight. But now, the laws of most of the States, as well as general usage, fix the hundred weight at 100 pounds.

3. The units of the scale, in passing from drams to ounces, are 16; from ounces to pounds, 16; from pounds to quarters, 25; from quarters to hundreds, 4; and from hundreds to tons, 20.

II. TROY WEIGHT.

479. Gold, silver, jewels, and liquors, are weighed by Troy weight.

Table.

24 grains (*gr.*)	make	1 pennyweight,	marked *pwt.*
20 pennyweights		1 ounce,	*oz*
12 ounces		1 pound,	*lb.*

Table Reversed.

lb.		oz.		pwt.		gr.
				1	=	24.
		1	=	20	=	480.
1	=	12	=	240	=	5760.

NOTES—1. The standard Troy pound is the weight of 22.794377 cubic inches of distilled water. Hence, it is less than the pound avoirdupois.

2. 7000 troy grains = 1 pound avoirdupois.
175 troy pounds = 144 pounds "
175 troy ounces = 192 ounces "
437½ troy grains = 1 ounce "

3. The Troy pound being the one deposited in the Mint at Philadelphia, is generally regarded as the *standard* of weight.

4. The units of the scale are 24, 20, and 12.

III. APOTHECARIES' WEIGHT.

480. This weight is used by apothecaries and physicians in mixing their medicines. But medicines are generally sold, in the quantity, by avoirdupois weight.

Table.

20 grains (*gr.*)	make	1 scruple,	marked	℈.
3 scruples		1 dram,		ʒ.
8 drams		1 ounce,		℥.
12 ounces		1 pound,		℔

Table Reversed.

℔		℥		ʒ		℈		gr.
						1	=	20
				1	=	3	=	60
		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

NOTES.—1. The pound, ounce, and grain, are the same as the pound, ounce, and grain, in Troy weight.

2. The units of the scale, in passing from grains to scruples, are 20; in passing from scruples to drams, 3; from drams to ounces, 8; *and from ounces to pounds*, 12.

IV. FRENCH SYSTEM.

481. The basis of this system of weights is the weight in vacuo of a cubic decimetre of distilled water. This weight is called a kilogramme, and is the unit of the French system. It is equal to 2.204737 pounds avoirdupois. The other denominations are as follows:

10 milligrammes = 1 centigramme; 10 centigrammes = 1 decigramme; 10 decigrammes = 1 gramme; 10 grammes = 1 decagramme; 10 decagrammes = 1 hectogramme; 10 hectogrammes = 1 kilogramme; 10 kilogrammes = 1 quintal; 10 quintals = 1 ton of sea-water.

COMPARISON OF WEIGHTS.

English,	1 pound	=	1.000936	pounds	avoirdupois.
French,	1 kilogramme	=	2.204737	"	"
Spanish,	1 pound	=	1.0152	"	"
Swedish,	1 pound	=	0.9376	"	"
Austrian,	1 pound	=	1.2351	"	"
Prussian,	1 pound	=	1.0333	"	"

VII. UNITS OF TIME.

482. Time is a part of duration. The time in which the earth revolves on its axis is called a *day*. The time in which it goes round the sun is called a *solar year*. Time is divided into parts according to the following

Table

60 seconds, *sec*	make	1 minute,	marked	*m.*
60 minutes		1 hour,		*hr.*
24 hours		1 day,		*da.*
7 days		1 week,		*wk*
52 weeks, nearly		1 year,		*yr*
12 calendar months = 365 *da.*		1 Julian or common year,		*yr.*
366 days	make	1 leap-year		
100 years		1 century,		*cent.*

Table Reversed.

yr.		wk.		da.		hr.		m.		sec.
								1	=	60
						1	=	60	=	3600
				1	=	24	=	1440	=	86400
		1	=	7	=	168	=	10080	=	604800
1	=	52⅐	=	365	=	8760	=	525600	=	31536000

The year is divided into 12 calendar months :

No.		No. days.	No.		No. days.
1st.	January,	31	7th.	July,	31
2d.	February,	28	8th.	August,	31
3d.	March,	31	9th.	September,	30
4th.	April,	30	10th.	October,	31
5th.	May,	31	11th.	November,	30
6th.	June,	30	12th.	December,	31

The number of days in each month may be remembered by the following :

Thirty days hath September,
April, June, and November ;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

NOTES.—1. Days are numbered in each month from the first day of the month.

2. Months are numbered from January to December.

3. The centuries are numbered from the beginning of the Christian Era. The year 30, for example, at its commencement, was called the 30th year of the first century, though neither the century nor the year had elapsed. Thus, June 2d, 1856, was the 6th month of the 56th year of the 19th century.

4. The civil day begins and ends at 12 o'clock at night. In the civil day, the hours are reckoned from that time.

Dates.

1. The length of the solar year is 365 da. 5 hr. 48 m. 48 sec., very nearly. It is desirable to have the periods and dates of the civil year correspond to those of the solar year ; else, the summer months of the

one would in time become the winter months of the other, thereby producing great confusion in dates and history.

2. The common civil year is reckoned at 365 da., and the solar year at 365 da. 6 hr. The 6 hours accumulate for 4 years before they are counted, when they amount to 1 day, and are added to February; and the year is called a *bissextile*, or *leap-year.*

3. The odd 6 hours have been so added, that the leap-years occur in those numbers which are divisible by 4. Thus, 1856, 1860, 1864 &c., are leap-years; and when any number is not divisible by 4, the remainder denotes how many years have passed since a leap-year.

4. This method of disposing of the fractional part of the year would be without error, if the solar year were exactly 365 da. 6 hr. in length; but it is not; it is only 365 da. 5 hr. 48 m. 48 sec. long: hence, the leap-year is reckoned at *too much,* and to correct this error, every *centennial* year is reckoned as a *common year.* But this makes an error again, on the other side, and every fourth centennial year the day is retained. Thus, 1800 was not, and 1900 will not be, reckoned a leap-year: the error will then be on the other side, and 2000 will be a leap-year. This disposition of the fractional part of the year causes the civil and solar years to correspond very nearly, and indicates the following rule for finding the leap-years:

Rule.—*Every year which is divisible by* 4 *is a leap year, unless it is a centennial year, and then it is not a leap-year unless the number of the century is also divisible by* 4.

5. The registration of the days, by reckoning the civil year at 365 da., was established by the Roman emperor, Julius Cæsar, and hence this period is sometimes called the Julian year.

The error, arising from the fractional part, continued to increase until 1582, when it amounted to 10 days; that is, as the year had been reckoned too long, the number of days had been *too few,* and the *count,* in the civil year, was *behind* the count in the solar year.

In this year (1582), Pope Gregory decreed the 4th day of October to be called the 14th, and this brought the civil and the solar years together. The new calendar is sometimes called the *Gregorian Calendar.*

6 The method of dating by the old count, is called *Old Style,* and by the new, *New Style.* The difference is now 12 days. In *Russia, they* still use the old style; hence, their dates are 12 days *behind ours.* Their 4th of January is our 16th.

VIII. UNITS OF CIRCULAR MEASURE.

483. Angular, or circular measure, is used in estimating *latitude* and *longitude*, in measuring the motions of the heavenly bodies, and also in measuring angles.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*. Each degree is divided into 60 minutes, and each minute into 60 seconds.

Table.

60 seconds (″),	make	1 minute,	marked ′
60 minutes		1 degree,	°
30 degrees		1 sign,	s
12 signs, or 360°,		1 circle,	c

Table Reversed.

c		s		°		′		″
						1	=	60.
				1	=	60	=	3600.
		1	=	30	=	1800	=	108000.
1	=	12	=	360	=	21600	=	1296000.

Miscellaneous Table.

12 units, or things	make	1 dozen.
12 dozen		1 gross.
12 gross or 144 dozen,		1 great gross.
20 things		1 score.
.00 pounds		1 quintal of fish.
196 pounds		1 barrel of flour.
200 pounds		1 barrel of pork.
18 inches		1 cubit.
22 inches, nearly,		1 sacred cubit.
14 pounds of iron or lead		1 stone.
21½ stones		1 pig.
8 pigs		1 fother.

BOOKS AND PAPER.

The terms, *folio, quarto, octavo, duodecimo,* &c., ind the number of leaves into which a sheet of paper is fold

A sheet folded in 2 leaves,	is called,	a folio.
A sheet folded in 4 leaves,	"	a quarto, or 4to.
A sheet folded in 8 leaves,	"	an octavo, or 8vo.
A sheet folded in 12 leaves,	"	a 12mo.
A sheet folded in 16 leaves,	"	a 16mo.
A sheet folded in 18 leaves,	"	an 18mo.
A sheet folded in 24 leaves,	"	a 24mo.
A sheet folded in 32 leaves,	"	a 32mo.

24 sheets of paper	make 1 quire.
20 quires, ...	. . . 1 ream.
2 reams, ...	. . . 1 bundle.
5 bundles, ...	. . . 1 bale.

METRIC SYSTEM OF WEIGHTS AND MEASURES.

The primary base, in this system, for all denominations of weights and measures, is the one-ten-millionth part of the distance from the equator to the pole, measured on the earth's surface. It is called a METER, and is equal to 39.37 inches, very nearly.

The change from the base, in all the denominations, is according to the decimal scale of tens: that is, the units increase ten times, at each step, in the ascending scale, and decrease ten times, at each step, in the descending scale.

MEASURES OF LENGTH.

Base, 1 meter = 39.37 inches, nearly.

Table.

Ascending Scale.					Descending Scale.		
1 Myriameter.	1 Kilometer	1 Hectometer.	1 Decameter.	1 METER. UNIT.	1 Decimeter.	1 Centimeter.	1 Millimeter.

The names, in the ascending scale, are formed by prefixing to the base, Meter, the words, Deca (ten), Hecto (one hundred), Kilo (one thousand), Myria (ten thousand), from the Greek numerals; and in the descending scale, by prefixing Deci (tenth), *Centi (hundredth)*, Milli (thousandth), from the Latin numerals.

Hence, the name of a unit indicates whether it is greater or less than the standard; and, also, how many times. The table is thus read:

10 millimeters make 1 centimeter
10 centimeters make 1 decimeter.
10 decimeters make 1 METER.
10 METERS make 1 decameter.
10 decameters make 1 hectometer.
10 hectometers make 1 kilometer.
10 kilometers make 1 myriameter.

Table of Equivalents.

Myriameter	Kilometer	Hectometer	Decameter	Meter.	Decimeter.	Centimeter.	Millimeter
.	.	.	.	.	.	1=	10
.	.	.	.	.	1=	10=	100
.	.	.	.	1=	10=	100=	1,000
.	.	.	1=	10=	100=	1,000=	10,000
.	.	1=	10=	100=	1,000=	10,000=	100,000
.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000
1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000

Table of Equivalents in English Measure.

1 Millimeter = 0.0394 inches, nearly.
1 Centimeter = 0.3937 "
1 Decimeter = 3.9370 "
1 METER = 39.37 in. = 3.280833 ft.
1 Decameter = 32 ft. 9.7 in.
1 Hectometer = 19 rd. 14 ft. 7 in.
1 Kilometer = 4 fur. 38 rd. 13 ft. 10 in.
1 Myriameter = 6 mi. 1 fur. 28 rd. 6 ft. 4 in.

Besides a clear apprehension of the length of the base, *1 meter, it is* well to consider the length of the largest unit, the myriameter, equal to nearly 6 and one-fourth miles; and also the length of the smallest unit, the millimeter, about four-hundredths of an inch. Compare, also, each of the smaller measures, the decimeter and centimeter, with the inch.

When, in the metric system, the value of any single unit is fixed in the mind, the values of all the others may be readily apprehended, since they always arise from multiplying or dividing by 10.

NOTE.—In all the tables, the UNIT is in small capitals, and should be constantly referred to.

Methods of Reading.

The number 25365.897 meters, is read, in English,

Twenty-five thousand three hundred and sixty-five meters, and 897 thousandths of a meter. But in the language of the metric system, it may be read,

Two myriameters, 5 kilometers, 3 hectometers, 6 decameters, 5 meters, 8 decimeters, 9 centimeters, and 7 millimeters. It may also be read, beginning with the lowest denomination, 7 millimeters, 9 centimeters, &c., &c.

In reading, remember that the unit of any place is *ten* times as great as the unit of the place next at the right, and *one-tenth* of the unit of the place next at the left. Hence, the change from one unit to another, and the methods of reduction and reading, are identical with those in the system of decimal currency.

1. Write, numerate, and read, five hundred and ninety-six hectometers.

2. Write, numerate, and read, eighty-nine thousand and forty-one centimeters.

Questions.—What is the primary base of the metric system? To what portion of the earth's surface is it equal? What is its length? What is the ascending scale from the meter? What is the descending scale? What is the length of a myriameter? According to what law do the different units increase and decrease?

MEASURES OF SURFACES, OR SQUARE MEASURE.

Base, 1 Are = the square whose side is 10 meters.
= 119.6 square yards, nearly.
= 4 perches or square rods, nearly.

The unit of surface is a square whose side is 10 meters. It is called an **Are**, and is equal to 100 square meters.

Table.

1 Hectare.	1 Are. Unit.	1 Centare.

The table is thus read:

100 centares	make	1 ARE.
100 ares	make	1 hectare.

Table of Equivalents.

Hectare.	Are.	Centare.
.	1 =	100
1 =	100 =	10,000

Equivalents in acres, roods, and perches.

1 Centare = 1.195985 sq. yards, nearly.
1 Are = 3.95367 perches.
1 Hectare = 2A. 1R. 35.367P.

MEASURES OF VOLUMES.

Base, 1 liter = the cube or the decimeter.
= 61.023378 cubic inches.
= a little more than a wine quart.

Questions.—What is the primary base of the measure for surfaces? To what is it equal, in square yards? What are the denominations, beginning with the least? To what is the centare equal? To what is the hectare equal?

The unit for the measure of volume is the cube whose edge is one-tenth of the meter—that is, a cube whose edge is 3.937 inches. This cube is called a LITER, and is one-thousandth part of the cube constructed on the meter, as an edge.

Table.

Ascending Scale.				Descending Scale.		
Kiloliter, or Stere.	Hectoliter.	Decaliter.	LITER. UNIT.	Deciliter.	Centiliter.	Milliliter.
1	1	1	1	1	1	1

The table is thus read:

10	milliliters	make	1 centiliter.
10	centiliters	make	1 deciliter.
10	deciliters	make	1 liter.
10	liters	make	1 decaliter.
10	decaliters	make	1 hectoliter.
10	hectoliters	make	1 kiloliter, or stere.

Table of Equivalents

Kiloliter, or Stere.	Hectoliter.	Decaliter.	LITER.	Deciliter.	Centiliter.	Milliliter.
.	.	.		.	1 =	10
	.	.	.	1 =	10 =	100
.	.	.	1 =	10 =	100 =	1,000
.	.	1 =	10 =	100 =	1,000 =	10,000
.	1 =	10 =	100 =	1,000 =	10,000 =	100,000
1 =	10 =	100 =	1,000 =	10,000 =	100,000 =	1,000,000

NOTE.—The kiloliter, or stere, is the cube constructed on the meter as an edge. Hence, the liter is one-thousandth part of the kiloliter.

Equivalents in Cubic Measure.

1 milliliter = .061023 cubic inches.
1 centiliter = .610234 cubic inches.
1 deciliter = 6.102338 cubic inches.
1 LITER = 61.023378 cubic inches.
1 decaliter = 610.233779 cubic inches.
1 hectoliter = 6102.337795 cu. in. = 3.5314454 cu. ft.
1 kiloliter, or stere = 61023.377953 cu. in. = 35.314454 cu. ft.

NOTE.—Law of change in the units, and methods of reading, are the same as in linear measure.

DRY MEASURE.

EQUIVALENTS IN THE WINCHESTER BUSHEL.

Since 1 bushel = 2150.4 cu. in.; 1 pk. = 537.6 cu. in.; 1 qt. = 67.2 cu. in; 1 pt. = 33.6 cu. in.; therefore,

1 milliliter = .001816 pints.
1 centiliter = .018161 pints.
1 deciliter = .181611 pints.
1 LITER = 1.816112 pints.
1 decaliter = 1 pk. 1.08056 qt.
1 hectoliter = 2 bu. 3 pk. 2 qt. 1.6112 pt.
1 kiloliter, or stere = 28 bu. 1 pk. 4 qt. 0.112 pt.

NOTE.—The liter, or standard, is a little less than 1 quart, and the stere, nearly 30 Winchester bushels.

LIQUID MEASURE.

EQUIVALENTS IN THE WINE GALLON.

Since 1 wine gallon contains 231 cubic inches, 1 quart will contain 57.75 cubic inches; 1 pint, 28.875 cubic inches; and 1 gill, 7.21875 cubic inches, we have,

---

Questions.—What is the unit for the measure of volumes? To what is it equal in cubic inches? What part is it of the cube on the meter? Name all the denominations of volume. What is the unit of Dry Measure? To what is it equal? To what is the stere or kiloliter equal?

1 milliliter = 0.008453 gills.
1 centiliter = .084534 gills.
1 deciliter = .845345 gills.
1 LITER = 1 qt. .11336 pt.
1 decaliter = 2 gal. 2 qt. 1 pt. .1336 pt.
1 hectoliter = 26 gal. 1 qt. 1 pt. 1.344 gills.
1 kiloliter, or stere = 1 tun, 12 gal. 0 qt. 1 pt. 1.44 gills.

WEIGHTS.

Base, 1 gram = weight of a cubic centimeter of rain-water.
= 15.432 grains, Troy, nearly.
= .0352746 ounces, Avoirdupois, nearly.

The unit of weight is also equal to the one-millionth part of the weight of a cubic meter of pure rain-water, weighed in vacuum. It is called a GRAM, and is equal to 15.432 grains, Troy, which is equal to .0352746 ounces, Avoirdupois, very nearly

Table.

Ascending Scale.							**Descending Scale.**		
1 Millier, tonneau	**1 Quintal.**	**1 Myriagram.**	**1 Kilogram.**	**1 Hectogram.**	**1 Decagram.**	**1 GRAM. UNIT.**	**1 Decigram.**	**1 Centigram.**	**1 Milligram.**

The table is thus read :

10 milligrams make 1 centigram.
10 centigrams make 1 decigram.
10 decigrams make 1 gram.
10 grams make 1 decagram.
10 decagrams make 1 hectogram.
10 hectograms make 1 kilogram.
10 kilograms make 1 myriagram.
10 myriagrams make 1 quintal.
10 quintals make 1 millier, or tonneau.

Table of Equivalents.

Millier, or tonneau.	Quintal.	Myriagram.	Kilogram.	Hectogram.	Decagram.	GRAM.	Decigram.	Centigram.	Milligram.
.	.	.	.	.	.	.	.	1=	10
.	.	.	.	.	.	.	1=	10=	100
.	.	.	.	.	.	1=	10=	100=	1,000
.	.	.	.	.	1=	10=	100=	1,000=	10,000
.	.	.	.	1=	10=	100=	1,000=	10,000=	100,000
.	.	.	1	10=	100=	1,000=	10,000=	100,000=	1,000,000
.	.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000
.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000=	100,000,000
1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000=	100,000,000=	1,000,000,000

Equivalents in Avoirdupois and Troy Weights.

1 Milligram	=	0.0154	grains,	Troy.
1 Centigram	=	0.1543	grains,	"
1 Decigram	=	1.5432	grains,	"
1 GRAM	=	15.4327	grains,	"
1 Decagram	=	0.3527	ounces,	Avoirdupois.
1 Hectogram	=	3.5274	ounces,	"
1 Kilogram	=	2.2046	pounds,	"
1 Myriagram	=	22.046	pounds,	"
1 Quintal	=	220.46	pounds,	"
1 Millier, or ton	=	2204.6	pounds,	"

NOTE.—Law of change in the units, and methods of reading, the same as in Linear Measure.

NATURE OF THE METRIC SYSTEM.

The Metric system is based on the METER. From the meter, three other units are derived; and the four constitute the primary units of the system. They are:

Questions.—What is the unit of weight? To what is it equal in Troy weight? To what is it equal in Avoirdupois? Name all the units of the weight, from the lowest to the highest. To what is the millier, or ton, equal?

Meter = 39.37 inches, nearly: unit of length.
Are = a square on 10 meters: unit of surface.
Liter = a cube whose edge is a decimeter: unit of volume.
Gram = the weight of a cube of rain-water, each edge of which is a centimeter: unit of weight

From these four units all others are derived, according to the decimal scale.

Every system of Weights and Measures *must* have an *invariable unit* for its base; and every other unit of the entire system should be derived from it, according to a fixed law.

The French Government, in order to obtain an invariable unit, measured a degree of the arc of a meridian on the earth's surface; and from this computed the length of the meridional arc from the equator to the pole. This length they divided into ten million equal parts, and then took one of these parts for the unit of length, and called it a Meter. The length of this meter is equal to 1 yard, 3 inches, and 37 hundredths of an inch, very nearly. Thus they obtained the length of the unit which is the base of the Metric System of Weights and Measures.

The next step was to fix the *law*, by which the other units should be obtained from the base. The scale of tens was adopted.

PRONUNCIATION.

Me'ter.	Are.	Li'ter.	Gram.
Mil'li-me-ter.		Mil'li-lĭ-ter.	Mil'li-gram.
Cen'ti-me-ter.	Cen'târe.	Cen'ti-lĭ-ter.	Cen'ti-gram.
Dec'i-me-ter.		Dec'i-lĭ-ter.	Dec'i-gram.
Dee'a-me-ter.		Dee'a-lĭ-ter.	Dee'a-gram.
Hee'-to-me-ter.	Hee'târe.	Hee'to-lĭ-ter.	Hee'to-gram.
Kil'o-me-ter.		Kil'o-lĭ-ter.	Kil'o-gram.
Myr'i-a-me-ter.		Myr'i-a-lĭ-ter.	Myr'i-a-gram.

TO CHANGE FROM ONE SYSTEM TO THE OTHER.

To change, in Linear Measure, from the Metric to the Common system.

Rule.

Multiply the meters and decimals of a meter by 3.280833 (the value of a meter), *and the product will be the result in feet.*

To change from the Common to the Metric system.

Rule.

Reduce the linear measure to feet and decimals of a foot, and then divide by 3.280833*: the quotient will be the result in meters and decimals of a meter.*

Examples.

1. In 5961.874 meters, how many feet and inches?

2. In 874163 meters, and 37 hectometers, how many feet and inches?

3. Express 320 rods, 5 yards and 6 inches in the Metric Measures.

4. Express 1 mile, 3 furlongs, 39 rods and 5 yards in the Metric Measures.

To change, in Square Measure, from the Metric to the Common system.

Rule.

Reduce the number to ARES *and decimals of the* ARE*: then multiply by* 3.95367, *and the product will be the result in perches.*

To change, from the Common system, to the Metric system.

Rule.

Find the value of the number in perches and decimals of a perch: then divide by 3.95367, *and the quotient will be the result in* ARES *and decimals of* the ARE.

Examples.

1. In 6127 ares, 4 hectares and 3 centares, how many acres, roods and perches?

2. In 327 ares, 15 hectares and 89 centares, how many square feet?

3. In 4 acres, 3 perches and 200 square feet, how many hectares, ares and centares?

4. In 1375 square yards and 250 square feet, how many hectares, ares and centares?

To change, in measures of volume, from the Metric to the Common system.

Rule.

Reduce the number to LITERS *and decimals of the* LITER: *then multiply by* 61.023378, *and the product will be the result in cubic inches.*

To change, in measures of volume, from the Common to the Metric system.

Rule.

Reduce the number to cubic inches: then divide by 61.023378, *and the quotient will be the result in* LITERS *and decimals of the* LITER.

Examples.

1. In 6 kiloliters, 9 hectoliters, 6 decaliters, 8 liters and 4 centiliters, how many cubic feet and inches?

2. In 8 hectoliters, 9 decaliters, 27 liters and 15 milliers, how many cubic yards, feet and inches?

3. Change 27 cubic yards, 16 cubic feet and 16 cubic inches, to the Metric measures.

4. Change 40 cubic yards, 25 cubic feet and 1167 inches, to the Metric measures.

To change, in weights, from the Metric to the Common system.

Rule.

Reduce the number to grams and decimals of a gram: then multiply by 15.423, *and the product will be the result in grains Troy; or, multiply by* .0352746, *and the product will be ounces in Avoirdupois.*

To change, in weights, from the Common to the Metric system.

Rule.

Reduce the number to Troy grains, or to Avoirdupois ounces: then divide by 15.423, *or by* .0352746, *and the quotient will be* GRAMS *and decimals of the* GRAM.

Examples.

1. Change 4 quintals, 6 kilograms, 4 decagrams, 7 grams and 6 centigrams, to Avoirdupois and Troy weights.

2. Change 2 milliers, 6 myriagrams, 9 grams, 4 decagrams and 9 milligrams, to Troy and Avoirdupois.

3. Change 1 T. 3 cwt. 3 qr. 20 lb. 6 oz., to the Metric weights.

4. Change 16 lb. 11 oz. 4 pwt. 19 gr., Troy, to the Metric weights.

Ques.—In linear measure, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

In square measure, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

In measures of volume, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

In weights, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

ANSWERS.

PAGE.	EX.	ANS.	EX	ANS.	EX.	ANS.	EX.	ANS.	EX.	AN	
16.	1	XI.	2	XIV.	3	XVI.	4	XVII.	5	XIX	
16.	6	XXII.	7	XXVIII.	8	XXIX.	9	XXXIII			
16.	10	XXXVII.	11	XXXVIII.	12	XLIII.	13				
16.		XLVII.	14	XLIX.	15	LVI.	16	LVIII.	17	LIX.	
16.	18	LXV.	19	LXIX.	20	LXVII.	21	LXXV			
16	22	LXXVI.	23	LXXXI.	24	LXXXVII.	25				
16		LXXXIX.	26	XCIV.	27	XCV.	28	XCVII			
16.	29	XCIX.	30	CXV.	31	DCCL.	32	MLX.			
16	33	MMXL.	34	DLX.	35	DCCCCLX.	36	DCXC.			
16	37	ML.	38	MMMMIV.	39	$\overline{\text{V}}$MIX.	40	I$\overline{\text{X}}$IX.			
16.	41	DCCCVI.	42	DCVIII.	43	$\overline{\text{V}}$MMMVI.	44				
16.		MMI.	19.	1	7	2	80	3	9000	4	93
20.	5	961	6	7408	7	897021	8	86029430	9	4328-	
20.		021063	10	967040932	11	30430208123	12	360-			
20.		030702010	13	5800006000812	14	75605070905008					
20.	15	904000800200720	16	6000900704098020	17						
20.		80510006040900040900	18	6050900001	21.	19					
21.		98765432101 2345678	22.	1	621	2	5702	3	800		
22.	4	10406	5	65029	6	40000241	7	59000310			
22.	8	12111	9	300001006	10	69003000200					
23.	32	47000069000465207	33	8000000000000429006009							

23. ‖ 34 | 950000000000000089089306 ‖ 35 | 6000000451065

23. ‖ 047104 ‖ 36 | 989065841411 ‖ 30. ‖ 1 | 2 ; 7 ‖ 2 | 7 ; 3

30. ‖ 3 | 1 ; 7 ‖ 32. ‖ 6 | 42600 ; 426000 ‖ 7 | 36860 ‖ 8 | $8.75

32. ‖ 9 | 433005 ‖ 10 | 8996 ‖ 11 | £1 12s. 8d. 1 far. ‖ 12 | 15445 lb

32. ‖ 13 | 7 T. 14 cwt. 1 qr. 20 lb ‖ 33. ‖ 14 | 26215 grs

33 ‖ 15 | 122 lb. 2 oz. 18 pwt. 9 gr. ‖ 16 | 29362 gr. ‖ 17 |

33. ‖ 30 lb. 4 ℥ 3 ʒ 2 ℈ 7 gr. ‖ 18 | 249 in. ‖ 19 | 1600 rd. 8800 yd.

33. ‖ 26400 ft. 316800 in. ‖ 20 | 75 yd. 2 ft. 6 in. ‖ 21 |

33. ‖ 6 sq. yd. 2 sq. ft. ‖ 22 | 2 A. 0 R. 35 P. ‖ 23 | 45 A. 6 sq. Ch.

33. ‖ 24 | 568 P. ‖ 25 | 967680 cu. in. ‖ 26 | 3968 cu. ft.

33. ‖ 27 | 440 cords ‖ 28 | 2512 na. ‖ 29 | 144 yd. ‖ 30 |

33. ‖ 78 E. E. 1 qr. ‖ 31 | 1008 qt. ‖ 32 | 15 hhd. ‖ 33 | 3024 pt.

33. ‖ 34 | 129 bar. ‖ 35 | 1984 pt. ‖ 36 | 32 bu. 3 pk. 7 qt.

33. ‖ 37 | 63113856 sec. ‖ 38 | 8 mo. 2 wk. ‖ 37. ‖ 1 | 182630

37. ‖ 2 | 87539 ‖ 3 | 110526 ‖ 4 | 79165 ‖ 5 | 73285 ‖ 6 | 4148-

37. ‖ 907 ‖ 7 | 395873 ‖ 8 | 24177 ‖ 9 | 66395 ‖ 10 | 22099

37. ‖ 11 | 73566 ‖ 12 | 833157 ‖ 38. ‖ 13 | 32921 ‖ 14 | 185876

38. ‖ 15 | 93684 ‖ 16 | 34289 ‖ 17 | 243972 ‖ 18 | $991.546

38. ‖ 19 | $85.465 ‖ 20 | $770.560 ‖ 21 | 525.892 ‖ 22 | $9638.495

38. ‖ 23 | £223 2s. 5d. 1 far. ‖ 24 | 1296 lb. 10 oz. 2 pwt.

38. ‖ 25 | 453 ℔ 9 ℥ 3 ʒ ‖ 26 | 2 cwt. 3 qr. 8 lb. 8 oz. 5 dr

38. ‖ 27 | 43 T. 2 cwt. 0 qr. 7 lb. ‖ 28 | 312 yd. 0 qr. 2 na.

38. ‖ 29 | 251 E. E. 1 qr. 3 na. ‖ 30 | 143 L. 2 mi. 6 fur

39. ‖ 31 | 4 fur. 1 rd. 4 yd. 0 ft. 7 in. ‖ 32 | 322 A. 1 R. 4 P.

39. *‖ 33* | 2224 Tun 0 hhd. 5 gal. ‖ 34 | 339 gal. 3 qt.

39. || 35 | 230 chal. 25 bu. 3 pk. 4 qt. || 36 | 820 yr. 4 mo. 5 da.
39. || 37 | 904 da. 18 hr. 1 mi. || 38 | 2 T. 14 cwt. 1 qr. 20 lb. 15 oz.
39. || 39 | 23592550 || 40 | $137915940 || 41 | 88056
39. || 42 | 121 mi. 4 fur. 8 rd. 5 ft. || **40.** || 43 | $22.009
40. || 44 | $27.740 || 45 | 2 Tun 2 hhd. 29 gal. 2 qt. 0 pt
40. || 46 | $20308675 || 47 | $30569853 || 48 | $29026
40. || 49 | $8209.75 || 50 | $150106 || 51 | 29714 || **41.** || 52 | $50-
41. || 110025 || 53 | 59808512 || 54 | 2 T. 4 cwt. 2 qr. 1 lb.
41. || 55 | 205 acres. || 56 | $75002.295 || 57 | $7425
41. || 58 | 4 lb. 5 oz. 6 pwt. || 59 | 1053420 || **42.** || 60 | 1842 yrs.
42 || 61 | 32341 || 62 | $27131.23 || 63 | $28.105 || 64 | 39 yd. 1 qr.
42. || 65 | $180.825 || 66 | $35068.807 || 67 | £59 2s. 2d. 2 far
42. || 68 | 66585383 || **43.** || 69 | $1019.10 || 70 | $33800
43. || 71 | 380 bu. 1 pk. || 72 | $458.342 || 73 | £57 14s. 2d 3 far.
43. || 74 | $6235 || 75 | 66° 50′ || 76 | 10 cents. || 77 | 5868
47. || 1 | 363296 || 2 | 56579 || 3 | 733071 || 4 | 1711927
47. || 5 | 41923288 || 6 | 7838180 || 7 | 106026 || 8 | 4391
47. || 9 | 62786 || 10 | 198621115 || 11 | 3591757651
48. || 12 | 4199675 || 13 | 8878778 || 14 | 99999977 || 15 |
48. || $8443.641 || 16 | $806.384 || 17 | $4853673.758
48. || 18 | £14 18s. 3d. 1 far. || 19 | 3 T. 8 cwt. 2 qr. 7 lb.
48. || 20 | 117 yd. 2 qr. 1 na. || 21 | 59 L. 1 mi. 3 fur. 28 rd.
48. || 22 | 8 Tun 1 hhd. 53 gal. 3 qt. || 23 | 89 A. 2 R. 37 P.
48. || 24 | 975 bu. 1 pk. 6 qt. || 25 | 124 cords 58 ft. 522 in.
48. || 26 | 25 E. E. 1 qr. 3 na. || 27 | 79 ℔ 10 ℥ 8 ʒ

48. || 28 | 12 ℥ 4 ʒ 2 ℈ || 29 | 124 E. E. 3 qr. 3 na

48. || 30 | 96 E. F. 1 qr. 1 na. || 31 | 12 T. 17 cwt. 3 qr.

48. || 32 | 2 cwt. 2 qr. 22 lb. || 33 | 69 qr. 2 lb. 14 oz.

48. || 34 | 134 lb. 14 oz. 13 dr. || 49. || 35 | 10 A. 2 R. 18 P

49. || 36 | 37 A. 2 R. 34 P. || 37 | 147 da. 21 hr. 56 min

49. || 38 | 52 hr. 50 min. 54 sec. || 39 | $8759.625 || 40 | 183666662

49. || 41 | 6 yr. 9 mo. 3 wk. 1 da. || 42 | 88 ℔ 0 ℥ 6 ʒ

49. || 43 | $8.20 || 44 | $39.868 || 45 | $10.626 || 46 |

49. || £121 17s. 0d. 1 far. || 47 | 6 yr. 0 mo. 0 wk. 6 da. 9 hr. 2 min.

49. || 48 | 6353870 || 49 | 5747 || 50 | $6020 || 51 | 25712808.91

50. || 52 | 36190 || 53 | 683021 || 54 | 107445034 || 55 | 6274

50. || 56 | 4 T. 3 cwt. 2 qr. 23 lb. || 57 | £19 19s. 2d. 3 far.

50. || 58 | 2299 mi. 2 fur. 4 rd. || 59 | $199.625 || 60 | $175.875

50. || 61 | $3.25 || 51. || 62 | 19987563 || 63 | 2899248

51. || 64 | $73675 || 65 | 22815 || 66 | $198.625 || 67 |

51. || 80 yr. 8 mo. 0 da. 3 hr. 30 min. || 68 | 655.125

51. || 69 | 249 yr. 1 mo. 11 da. || 70 | 17877 || 71 | $7310756

52. || 72 | 4 cwt. 1 qr. 18 lb. || 73 | 7398 || 74 | 2360 || 75 | $526

52. || 76 | 6274 || 77 | $356.35 gain. || 78 | 3 A. 2 R. 39 P.

52. || 79 | 41 cords 5 cord ft. || 80 | $3280.105 || 81 | $44161.987

52. || 82 | 2 yr. 8 mo. 19 da. || 53. || 83 | $14352.50 || 84 |

53. | 30 gal. 2 qt. 1 pt. || 85 | 50062 || 86 | 15550 || 87 | 12° 23′ 53′

53. || 88 | $161.175 loss. || 89 | 2271707 || 90 | 32 yd. 0 qr. 2 na

53. || 91 | £950 2s. 8d. || 60. || 1 | 6776368 || 2 | 68653214

60. || 3 | 3422454 || 4 | 1952883 || 5 | 4354224 || 6 | 102854064

60. || 7 | 24668698404 || 8 | 3329480 || 9 | 4036084764

60. || 10 | 129844534245 || 61. || 11 | 810444 || 12 | 2361312

61. || 13 | 72127422 || 14 | 5403312 || 15 | 124409760

61. || 16 | 1990170000 || 17 | 3165172200 || 18 | 582400000000

61. || 19 | $104448.48 || 20 | $2501.136 || 21 | $23121.312

61. || 22 | $71997.312 || 23 | $7019.168 || 24 | $30780.960

61. || 25 | $21597.440 || 26 | $38824.056 || 27 | $278879.364

61. || 28 | $379255.968 || 29 | $9282001.666 || 30 | £81 6s. 8d.

61. || 31 | 24 T. 7 cwt. 3 qr. || 32 | 118 yd. 1 ft. 3 in.

61. || 33 | 114° 26′ 15″ || 34 | 56 hhd. 7 gal. 2 qt. 0 pt.

61. || 35 | 598 E. F. || 62. || 1 | 865 T. 11 cwt. 3 qr. 20 lb.

62. || 2 | 320 yr. 2 mo. 0 wk. 1 da. 15 hr. 12 min. || 3 | 4896

62. || 4 | 670460 ; 6704600 || 5 | 5704900 ; 57049000

62. || 6 | 4980496000 ; 49804960000 || 7 | 9072040000 ;

62. || 907204000000 || 8 | 74040900 ; 740409000 || 9 | 67493600 ;

62. || 67493600000 || 10 | 129359360000 || 11 | 13729103000000

62. || 12 | 664763206000000 || 13 | 8799238229600000

62. || 14 | 2526426017908695000000 || 15 | 1093689368445084-

62. || 378777040 || 16 | 16714410677359581583737 || 17 | $61975

62. || 18 | 3240 || 19 | 2097 || 20 | 133 yd. 3 qr. 2 na

62. || 21 | £3 19s. 4d. 2 far. || 22 | $1031.68 || 63. || 23 | $15

63. || 24 | $506.88 || 25 | $6336 || 26 | $5545 || 27 | $16763832

63. || 28 | 496 mi. 1 fur. 24 rd. || 29 | $657 || 30 | $24.375

63. || 31 | 868 miles || 32 | 7℔ 2℥ 7ʒ 0℈ 12 gr. || 33 |

63. || 411 bu. 1 pk. 0 qt. || 34 | 427816 || 64. || 35 | $84.28

64. || 36 | $16875.60 || 37 | 2 T. 18 cwt. 1 qr. 21 lb. || 38 | $971.04

64. || 39 | 461 left ; $1315 price. || 40 | $1417 || 41 | $65962788.75

64. || 42 | 750 || 43 | 13500 || 44 | $243.00 || 45 | 11914

65. || 46 | $4770.755 || 47 | $61 || 48 | $672 || 49 | 286 yr. 9 mo

65. || 50 | 84 rd. 14 ft. || 51 | 50 || 52 | 24 cords. || 53 | $92 gain

65. || 54 | 216 || 55 | $149.25 || 56 | 37816 || 57 | $34.88

66. || 58 | 669 hhd. 40 gal. 2 qt. || 59 | 13650000 || 60 | $202.50

66. || 61 | $21.475 || 62 | $927.35 || 67. || 63 | $18844.01

67. || 64 | $132.935 || 65 | £175 18s. 6d. || 72. || 1 | 6579

72. || 2 | 36842 || 3 | 269368 || 4 | 275155 || 5 | 7948312

72 || 6 | 1147187 || 7 | [illegible]2331642 || 8 | £15 19s. 9d.

72. || 9 | 4 A. 0 R. 33 P. [illegible] | 9 yd. 2 qr. 1 na. || 11 | $79.3445

72. || 12 | $209.728 || 13 | $6[illegible]862.18 || 73. || 14 | $1531140 9\frac{12}{48}$

73. || 15 | 237132 || 16 | 177242 || 17 | 68 || 18 | 44670

73. || 19 | $27\frac{14}{28}$ || 20 | $17.4512 || 21 | 3.842\frac{86}{129}$ || 22 | $1.125

73. || 23 | $0.375 || 24 | $0.81 || 25 | $5.01 || 26 | $52.88 || 27 | 9

73. || 28 | 95 || 29 | $8 || 30 | 763521 || 31 | $407294\frac{1080}{1754}$

73. || 32 | $13195133\frac{1842}{3574}$ || 33 | $125139201\frac{13019}{45705}$

73. || 34 | $269577255882\frac{5561}{17493}$ || 35 | $14243757748\frac{35411}{47143}$

73. || 36 | $15395919\frac{12214}{37149}$ || 37 | $30001000\frac{6347}{57143}$ || 38 |

73. || $131809655\frac{104990}{374567}$ || 39 | $300335575\frac{273118}{571007}$ || 40 | 9948157

73. || $977\frac{81605}{678957}$ || 41 | $59085714\frac{84}{121}$ || 42 | $1258127\frac{15785}{57149}$

73. || 43 | $119191753\frac{90107}{123456}$ || 44 | 17 A. 3 R. 7 P.

73. || 45 | 1 da. 12 hr. 31 min. 30 sec. || 46 | 35 mi. 0 fur. 29 rd.

73. || 47 | 49 gal. $3\frac{84}{126}$ qt. || 48 | 2 bu. 0 pk. 7 qt. || 74. || 49 | $25.25

74. ‖ 50 | 2s. 4d. ‖ 51 | 22 mi. 1 fur. 8 rd. ‖ 52 | 316 A. 1 R. 35 P.

74. ‖ 53 | $27.397+ ‖ 54 | 98765 ‖ 55 | $11250 ‖ 56 | 148018$\frac{248}{363}$

74. ‖ 57 | $4.75 ‖ 58 | $12.50 ‖ 59 | 757188$\frac{48}{104}$

74. ‖ 60 | $1.625 ‖ 61 | 365 days. ‖ 62 | 800008 ‖ 63 | 47

75. ‖ 64 | 1 T. 13 cwt. 3 qr. ‖ 65 | 45 cu. ft. 995$\frac{12}{14}$ cubic inches

75. ‖ 66 | 30$\frac{7152}{9285}$ tons. ‖ 67 | 4424$\frac{9}{213}$ ‖ 68 | 59′ 10″$\frac{250}{363}$

75. ‖ 69 | 5 doz. ‖ 70 | $4.50 ‖ 71 | £273 7s. 6d. ‖ 72 | 41684$\frac{24}{114}$

75. ‖ 73 | 9 ‖ 74 | $56 ‖ 75 | 666$\frac{408}{432}$ ‖ 76 | 200000

76. ‖ 1 | 7175 ‖ 2 | 4600 ‖ 3 | 168525 ‖ 4 | 76850 ‖ 1 | 2725

76. ‖ 2 | 387321 ‖ 3 | 4413840 ‖ 4 | 15423 ‖ 5 | 2674584

76. ‖ 6 | 280082 ‖ 77. ‖ 1 | 4800 ‖ 2 | 5950 ‖ 3 | 185000

77. ‖ 4 | 8380225 ‖ 1 | 559750066$\frac{2}{3}$ ‖ 2 | 493574666$\frac{2}{3}$

77. ‖ 3 | 355850400 ‖ 4 | 148072400 ‖ 1 | 7408000

77. ‖ 2 | 2199176000 ‖ 3 | 242601500 ‖ 4 | 17573500

79. ‖ 1 | $142 ‖ 2 | $17 ‖ 3 | $14 ‖ 4 | $35 ‖ 5 | $864 ‖ 6 | $172

79. ‖ 7 | $120 ‖ 8 | $90 ‖ 80. ‖ 1 | $121.615 ‖ 2 | $67.50

80. ‖ 3 | $737.88 ‖ 4 | $496.875 ‖ 5 | $118.9145 ‖ 81. ‖ 1 | $3.024

81. ‖ 2 | $12.8915 ‖ 3 | $5.922 ; $6.4575 ; $9.198 ‖ 4 | $18.22765

81. ‖ 5 | 736.68468\frac{3}{4}$ ‖ 6 | $876.434 ‖ 7 | $2423.09925

81. ‖ 8 | $339286.5375 ‖ 82. ‖ 1 | 254 ‖ 2 | 26251$\frac{20}{100}$

82. ‖ 3 | 291147 ‖ 4 | 211488$\frac{69}{100}$ ‖ 5 | 978 ‖ 6 | 954 ‖ 7 | 140848

82. ‖ 8 | 2025 ‖ 9 | 39252 ‖ 10 | 475542 ‖ 11 | 242172

82. ‖ 12 | 484344 ‖ 13 | 951084 ‖ 14 | 2250 ‖ 15 | 48126

82. ‖ 16 | 16215 ‖ 17 | 48645 ‖ 18 | 144378 ‖ 84. ‖ 1 | 387

84. ‖ 2 | 1548 ‖ 3 | 532 ‖ 4 | 804 ‖ 5 | 15911 ‖ 6 | 1935

84. ‖ 7 | 1809 ‖ 8 | 3216 ‖ **85.** ‖ 1 | 1322$\frac{275}{315}$ ‖ 2 | 1740$\frac{226}{462}$

85. ‖ 3 | 218$\frac{3399}{3456}$ ‖ 4 | 83253$\frac{94}{105}$ ‖ 5 | 2459$\frac{328}{385}$ ‖ 6 | 4073$\frac{852}{1153}$

85. ‖ 7 | 9507$\frac{1687}{7560}$ ‖ **86.** ‖ 1 | 4$\frac{292654}{4200000}$ ‖ 2 | 146 ‖ 3 | 91$\frac{2711606}{3011400}$

86. ‖ 4 | 158732$\frac{25800}{72000}$ ‖ 5 | 253$\frac{43400}{146000}$ ‖ 6 | 2247$\frac{53398}{63000}$

86. ‖ 1 | 196$\frac{22}{33}$ ‖ 2 | 3117$\frac{30}{46}$ ‖ 3 | 61096$\frac{52}{57}$ ‖ 4 | 727$\frac{369}{409}$

86. ‖ 5 | 9095$\frac{25}{48}$ ‖ 6 | 6992$\frac{46}{88}$ ‖ 7 | 6150$\frac{75}{140}$ ‖ 8 | 4079$\frac{34}{123}$

89. ‖ 1 | £31 17s. 6d. ‖ 2 | £2 9s. 5$\frac{3}{4}$d. ‖ 3 | £1 16s. 10$\frac{1}{2}$d.

89. ‖ 4 | \$594.50 ‖ 5 | £469 5s. ‖ 6 | £931 ‖ 7 | £587 5s.

89. ‖ 8 | £82 10s. ‖ 9 | \$2.70 ‖ 10 | \$555 ‖ 11 | \$547.50

89. ‖ 12 | \$3.00 ‖ 13 | \$812.25 ‖ 14 | \$24.375 ‖ 15 | \$63.4375

89. ‖ 16 | \$315.40 ‖ 17 | \$469.03 ‖ 18 | £61 0s. ‖ **91.** ‖ 3 | 10° 34′ 0″

91. ‖ 4 | 35° 11′ 0″ ‖ 5 | 13° 23′ 0″ ‖ **92.** ‖ 1 | 1 hr. 2 min. 8 sec. P.M.

92. ‖ 2 | 2 hr. 55 min. 24 sec. P. M. ‖ 3 | 8 hr. 12 min. A. M.

92. ‖ 4 | 1 hr. 2 min. 20 sec. Fast. ‖ 93. ‖ 1 | 33° 55′ W

93. ‖ 2 | 95° 48′ W.; 10 hr. 17 min. 48 sec. P.M. ‖ 3 | 23° 45′ 22″ W.

93. ‖ 4 | 120° W. ‖ 5 | 156° 59′ E. ‖ **94.** ‖ 1 | \$128 ‖ 2 | 2 bu. 1 pk

94. ‖ 3 | 32 ‖ 4 | 463684 ‖ 5 | 41666$\frac{40}{60}$ ‖ 6 | 57979$\frac{278}{400}$

94. ‖ 7 | 7 mo. 1 wk. 4$\frac{1}{2}$ d. ‖ 8 | 12 yr. ‖ 9 | 6 mo. 0 wk. 5 d. 14 hr. 40 min

94. ‖ 10 | 765 ‖ 11 | \$72 ‖ 12 | \$5 ‖ 13 | \$812.25 ‖ 14 | \$147.9375

95. ‖ 15 | £14 14s. ‖ 16 | £166 2s. 8d. ‖ 17 | 6d. ‖ 18 | \$6.95175

95. ‖ 19 | \$8.64 ‖ 20 | \$93 ‖ 21 | 36 ‖ 22 | 45 lb. 6 oz. 14 pwt

95. ‖ 23 | 50 ‖ 24 | \$2480 gain; \$19 per acre. ‖ 25 | 6780 cu. ft

95. ‖ 26 | \$773.395 ‖ 27 | \$4.2408 ‖ 28 | \$16.7025 ‖ **96.** ‖ 29 | 7680

96. ‖ 30 | 1 lb. 7 oz. 12 pwt. 11 gr. ‖ 31 | \$10 ‖ 32 | 2 bu. 1 pk. 7 qt

96. ‖ *33* | *\$0.75* ‖ 34 | 104 \\ 35 \ 16 \\ 36 \ 52 gal. 1 qt. ‖ 37 | 96

96. ‖ 38 | $598281 ‖ 39 | 31680 ‖ **97.** ‖ 40 | 130 ‖ 41 | 119$\frac{7747}{34337}$

97. ‖ 42 | 11 hr. 4 min. 32 sec. A. M. ‖ 43 | 127° 30′

97. ‖ 44 | 67° 35′ A's long.; 9 hr. 19 min. P. M. B's time

97. ‖ 45 | 10 cords 7 C. ft. 15 cu. ft. ‖ 46 | 1 cwt. 3 qr. 9 lb. 10 oz

97. ‖ 47 | $164.475 ‖ 48 | 282 yr. 6 mo. 8 da. ‖ 49

97. ‖ 6 gal. 2 qt. 0 pt. 2 gi. ‖ 50 | 6° 13 mi. 1 fur. 34 rd. 2 yd

97. ‖ 51 | 1000000 ‖ **98.** ‖ 52 | 13824 ‖ 53 | 36100

98. ‖ 54 | 14 mi. 5 fur. 21 rd. 8 ft. ‖ 55 | 10 ‖ 56 | 36

98. ‖ 57 | 3 yd. 1 qr. 3 na. ‖ 58 | 33 ‖ 59 | 13209$\frac{203}{1585}$

98. ‖ 60 | $11.88 ‖ 61 | 1 yr. 205 da. 17 hr. 15 min.

98. ‖ 62 | $10591021.60 ‖ **99.** ‖ 63 | 25 yr. 6 mo. 16 da. 9 hr.

99. ‖ 64 | $2478, Widow's share; $1239, Child's share. ‖ 65 |

99. ‖ 13068 ‖ 66 | 107° 47′; 1 hr. 11 min. 8 sec. P. M

99. ‖ 67 | 4 hr. 56 min. P. M.; 26° east of New York.

99. ‖ 68 | 48 hr. ‖ 69 | 4333$\frac{500}{750}$ ‖ **100.** ‖ 70 | $2 ‖ 71 | 46$\frac{1}{2}$ lbs.

100. ‖ 72 | 14 days. ‖ 73 | 28 bar. 6 gal. ‖ 74 | 24 bar. 19 gal

100. ‖ 75 | 85.33\frac{1}{3}$ ‖ 76 | 11$\frac{24}{60}$ rolls. ‖ 77 | 7 mi. 6 fur. 20 rd.

100. ‖ 78 | 8750 lb. ‖ 79 | $18.025 ‖ 80 | 2500 bbl. ‖ **101.** ‖ 81 |

101. ‖ 482 bu. 1 pk. 2 qt. = 1st; 160 bu. 3 pk. 0 qt. 1$\frac{1}{3}$ pt. = 2d;

101. ‖ 321 bu. 2 pk. 1 qt. 0$\frac{2}{3}$ pt. = 3d. ‖ 82 | 40° 50′ East,

101. ‖ 35$\frac{70}{200}$ ‖ 83 | $2400 = Captain's; $1000 = Lieutenant's

101. ‖ $600 = Midshipman's; and $200 = Sailor's. ‖ 84 | 87° 30

101. ‖ 85 | 9 hr. 33 min. 14 sec. A. M. ‖ 86 | 10 hr. 54 min. 16 sec. A. M

101. ‖ 87 | 19° ‖ 88 | 4800 yd. ‖ 89 | $7410 ‖ **102.** ‖ 90 | 514

102. ‖ 91 | *2011 bu.* ‖ *92* | 1 yr. 338 da. 22 hr. ‖ 93 | 72 = greater;

102. ‖ 26 = less. ‖ 94 | $57 = less : $133 = greater ‖ 95 | 170 da.

102. ‖ 96 | $1.1875 ‖ 97 | $8383\frac{1}{3}$ = A's ; $8520\frac{1}{3}$ = B's ;

102. ‖ $7708\frac{1}{3}$ = C's. ‖ 98 | $11651.25 = 1st ; $11576.25 = 2d;

102. ‖ $11496.25 = 3d ; $11401.25 = 4th. ‖ 104. ‖ 1 | 3 × 3

104. ‖ 2 × 5 ; 2 × 2 × 3 ; 2 × 7 ; 2 × 2 × 2 × 2;

104. ‖ 2 × 3 × 3 ; 2 × 2 × 2 × 3 ; 3 × 3 × 3 ; 2 × 2 × 7

104. ‖ 2 | 2 × 3 × 5 ; 2 × 11 ; 2 × 2 × 2 × 2 × 2;

104. ‖ 2 × 2 × 3 × 3, 2 × 19 ; 2 × 2 × 2 × 5 ; 3 × 3 × 5;

104. ‖ 7 × 7 ‖ 105. ‖ 3 | 2 × 5 × 5 ; 2 × 2 × 2 × 7 ; 2 × 29;

105. ‖ 2 × 2 × 3 × 5 ; 2 × 2 × 2 × 2 × 2 × 2 ; 2 × 3 × 11;

105. ‖ 2 × 2 × 17 ; 2 × 5 × 7 ; 2 × 2 × 2 × 3 × 3

105. ‖ 4 | 2 × 2 × 19 ; 2 × 3 × 13 ; 2 × 2 × 2 × 2 × 5;

105. ‖ 2 × 41 ; 2 × 2 × 3 × 7 ; 2 × 43 ; 2 × 2 × 2 × 11;

105. ‖ 2 × 3 × 3 × 5 ‖ 5 | 2 × 2 × 5 × 5 ; 2 × 3 × 17;

105. ‖ 2 × 2 × 2 × 13 ; 5 × 5 × 11 ; 5 × 2 × 2 × 2 × 2 × 2 × 2 × 3.

105. ‖ 2 × 2 × 2 × 59 ; 2 × 2 × 2 × 2 × 2 × 5 ; 2 × 2 × 11 × 19

105. ‖ 6 | 5 × 3 × 7 ; 2 × 53 ; 2 × 2 × 3 × 3 × 3 ; 2 × 5 × 11;

105. ‖ 5 × 23 ; 2 × 2 × 29 ; 2 × 2 × 2 × 3 × 5;

105. ‖ 5 × 5 × 5 ; 5 × 5 × 5 × 3 × 3 ; 2 × 2 × 2 × 5 × 3 × 3

105. ‖ 1 | 2, 5, 3 ‖ 2 | 2, 3, 7 ‖ 3 | 5, 7, 3 ‖ 4 | 2, 3, 7 ‖ 5 | 2

107. ‖ 1 | 32 ‖ 2 | 3$\frac{3}{4}$ ‖ 3 | 14 ‖ 4 | 48 ‖ 5 | 8$\frac{8}{9}$ ‖ 6 | 4$\frac{2}{3}$ ‖ 7 | 8

107. ‖ 8 | $\frac{7}{45}$ ‖ 9 | 6$\frac{2}{5}$ ‖ 10 | 27 ‖ 11 | 9 ‖ 12 | 36. ‖ 108. ‖ 13 | 46

108. ‖ 14 | 4 ‖ 15 | 16$\frac{2}{3}$ ‖ 16 | .8 ‖ 17 | 471$\frac{1}{2}$ ‖ 18 | 15 ‖ 19 | 6210

108. ‖ 20 | 6$\frac{3}{4}$ ‖ 21 | 17$\frac{1}{2}$ ‖ 22 | 11$\frac{1}{4}$ ‖ 23 | 4$\frac{1}{2}$ ‖ 110. ‖ 1 | 1260

110. ‖ 2 | 7200 ‖ 3 | 1260 ‖ 4 | 1008 ‖ 5 | 10500 ‖ 6 | 10800

110. ‖ 7 | 540 ‖ 8 | 420 ‖ 9 | 336 ‖ 10 | 1176 ‖ 11 |

110. ‖ 144 rods. 16 days = A's time; 12 days = B's time;

110. ‖ 9 days = C's time. ‖ **111.** ‖ 12 | $1680. 112 at $15;

111. ‖ 105 at $16 ; 80 at $21 ; 70 at $24 ‖ 13 | 210 bu. 105 bags

111. ‖ 70 bbls. ; 30 boxes ; 14 hhds. ‖ 14 | 60 days. A = 3 times

111. ‖ B = 4 times ; C = 5 times ; D = 6 times. ‖ **112.** 2 | 18

112. ‖ 3 | 12 ‖ 4 | 5 ‖ 5 | 6 ‖ 6 | 10 ‖ 7 | 28 ‖ 8 | 14

114. ‖ 1 | 16 ‖ 2 | 7 ‖ 3 | 22 ‖ 4 | 124 ‖ 5 | 62 ‖ 6 | 81 ‖ 7 | 45

114. ‖ 8 | 25 ‖ 9 | 12 ‖ 10 | 3 ‖ 11 | $22 per head. 13 = A's

114. ‖ No. ; 21 = B's No. ; 29 = C's No. ‖ **121.** ‖ 1 | $\frac{18}{9}$; $\frac{21}{8}$

121. ‖ 2 | $\frac{28}{8}$; $\frac{63}{8}$ ‖ 3 | $\frac{55}{31}$; $\frac{60}{31}$ ‖ 4 | $\frac{84}{25}$; $\frac{98}{25}$ ‖ 5 | $\frac{141}{15}$; $\frac{188}{15}$

121. ‖ 6 | $\frac{98}{19}$; $\frac{126}{19}$ ‖ 7 | $\frac{235}{28}$; $\frac{470}{28}$ ‖ 8 | $\frac{81}{29}$; $\frac{297}{29}$ ‖ 1 | $\frac{17}{2}$; $\frac{17}{4}$;

121. ‖ $\frac{17}{8}$ ‖ 2 | $\frac{9}{12}$; $\frac{9}{8}$; $\frac{9}{6}$; $\frac{9}{4}$; $\frac{9}{3}$ ‖ 3 | $\frac{7}{5}$; $\frac{7}{6}$; $\frac{7}{3}$; $\frac{7}{2}$ ‖ 4 | $\frac{17}{24}$;

121. ‖ $\frac{17}{16}$; $\frac{17}{12}$; $\frac{17}{8}$; $\frac{17}{6}$ ‖ 5 | $\frac{6}{10}$; $\frac{6}{8}$; $\frac{6}{4}$; $\frac{6}{2}$ ‖ 6 | $\frac{7}{5}$; $\frac{7}{7}$ ‖ 7 | $\frac{6}{2}$;

121. ‖ $\frac{6}{7}$; $\frac{6}{6}$; $\frac{6}{14}$; $\frac{6}{21}$ ‖ 8 | $\frac{19}{12}$; $\frac{19}{9}$; $\frac{19}{6}$; $\frac{19}{4}$; $\frac{19}{3}$ ‖ **122.** ‖ 1 | $\frac{8}{19}$;

122. ‖ $\frac{4}{19}$; $\frac{2}{19}$; $\frac{1}{19}$ ‖ 2 | $\frac{7}{11}$; $\frac{2}{11}$; $\frac{1}{11}$ ‖ 3 | $\frac{10}{19}$; $\frac{4}{19}$; $\frac{5}{19}$; $\frac{2}{19}$

122. ‖ 4 | $\frac{12}{26}$; $\frac{10}{26}$; $\frac{6}{26}$; $\frac{4}{26}$; $\frac{3}{26}$ ‖ 5 | $\frac{9}{19}$; $\frac{6}{19}$; $\frac{3}{19}$; $\frac{2}{19}$ ‖ 6 | $\frac{8}{25}$;

122. ‖ $\frac{4}{25}$; $\frac{3}{25}$; $\frac{2}{25}$ ‖ 7 | $\frac{9}{29}$; $\frac{3}{29}$; $\frac{1}{29}$ ‖ 8 | $\frac{9}{59}$; $\frac{6}{59}$; $\frac{2}{59}$; $\frac{1}{59}$

122. ‖ 1 | $\frac{3}{24}$; $\frac{3}{28}$; $\frac{3}{32}$ ‖ 2 | $\frac{4}{45}$; $\frac{4}{36}$; $\frac{4}{81}$ ‖ 3 | $\frac{14}{51}$; $\frac{14}{68}$; $\frac{14}{204}$

122. ‖ 4 | $\frac{30}{282}$; $\frac{30}{376}$; $\frac{30}{517}$ ‖ 5 | $\frac{15}{119}$; $\frac{15}{85}$; $\frac{15}{51}$ ‖ 6 | $\frac{14}{189}$; $\frac{14}{216}$;

122. ‖ $\frac{14}{162}$ ‖ 7 | $\frac{25}{57}$; $\frac{25}{133}$; $\frac{25}{209}$ ‖ 8 | $\frac{11}{120}$; $\frac{11}{60}$; $\frac{11}{150}$

123. ‖ 1 | $\frac{28}{32}$; $\frac{42}{48}$; $\frac{35}{40}$ ‖ 2 | $\frac{40}{55}$; $\frac{64}{88}$; $\frac{72}{99}$; $\frac{88}{121}$ ‖ 3 | $\frac{112}{133}$; $\frac{128}{152}$;

123. ‖ $\frac{144}{171}$ ‖ 4 | $\frac{70}{145}$; $\frac{112}{232}$; $\frac{84}{174}$; $\frac{168}{348}$ ‖ 5 | $\frac{46}{50}$; $\frac{69}{75}$; $\frac{92}{100}$; $\frac{115}{125}$

124. ‖ 1 | $\frac{2}{4}$; $\frac{1}{2}$ ‖ 2 | $\frac{1}{2}$ ‖ 3 | $\frac{12}{18}$; $\frac{8}{12}$; $\frac{6}{9}$; $\frac{4}{6}$; $\frac{2}{3}$ ‖ 4 | $\frac{24}{32}$; [illegible] ;

124. ‖ $\frac{6}{8}$; $\frac{3}{4}$ ‖ 5 | $\frac{36}{48}$; $\frac{24}{32}$; $\frac{18}{24}$; $\frac{12}{16}$; $\frac{6}{8}$ ‖ 6 | $\frac{18}{72}$; $\frac{12}{48}$; $\frac{9}{36}$; $\frac{6}{24}$;

124. ‖ $\frac{1}{4}$ ‖ 125. ‖ 1 | $\frac{126}{7}$ ‖ 2 | $\frac{300}{12}$ ‖ 3 | $\frac{152}{8}$ ‖ 4 | $\frac{406}{14}$ ‖ 5 | $\frac{2405}{3}$

125. ‖ 6 | $\frac{1305}{9}$ ‖ 7 | $\frac{5400}{12}$ ‖ 8 | $\frac{11772}{36}$ ‖ 9 | $\frac{12416}{128}$ ‖ 10 | $\frac{14863}{89}$

125. ‖ 11 | $\frac{24375}{75}$ ‖ 1 | $\frac{319}{8}$ ‖ 2 | $\frac{1129}{10}$ ‖ 126. ‖ 3 | $\frac{1025}{24}$

126. ‖ 4 | $\frac{34513}{51}$ ‖ 5 | $\frac{38177}{104}$ ‖ 6 | $\frac{148261}{175}$ ‖ 7 | $\frac{5926782}{879}$

126. 8 | $\frac{135187}{200}$ ‖ 9 | $\frac{28278}{151}$ ‖ 10 | $\frac{1346}{9}$ ‖ 11 | $\frac{37219}{99}$

126. ‖ 12 | $\frac{1749383049}{99999}$ ‖ 13 | $\frac{459287}{95}$ ‖ 14 | $\frac{16106}{9}$ ‖ 15 | $\frac{881}{7}$

126. ‖ 16 | $\frac{1503}{4}$ ‖ 17 | $\frac{29251}{63}$ ‖ 18 | $\frac{61451}{640}$ ‖ 19 | $\frac{110249}{112}$

126. ‖ 20 | $\frac{12882}{366}$ ‖ 21 | $\frac{11786}{135}$ ‖ 22 | $77\frac{7}{7}$ ‖ 23 | $333\frac{3}{3}$

127. ‖ 1 | $1\frac{5}{7}$ ‖ 2 | 12 ‖ 3 | $5\frac{144}{324}$ ‖ 4 | $24\frac{700}{800}$ ‖ 5 | 9 ‖ 6 | $56\frac{6}{42}$

127. ‖ 7 | $112\frac{12}{56}$ ‖ 8 | $22\frac{48}{224}$ ‖ 9 | $640\frac{9}{160}$ ‖ 10 | $5\frac{273}{841}$ ‖ 11 | $14\frac{375}{1256}$

127. ‖ 12 | 225 ‖ 13 | $191\frac{15}{25}$ ‖ 14 | 14 ‖ 15 | $376\frac{317}{999}$

127. ‖ 16 | $10731\frac{56}{349}$ ‖ 128. ‖ 1 | $\frac{1}{7}$ ‖ 2 | $\frac{1}{5}$ ‖ 3 | $\frac{1}{3}$ ‖ 4 | $\frac{1}{8}$

128. ‖ 5 | $\frac{5}{8}$ ‖ 6 | $\frac{117}{265}$ ‖ 7 | $\frac{2}{13}$ ‖ 8 | $\frac{7}{9}$ ‖ 9 | $\frac{7}{9}$ ‖ 10 | $\frac{13}{7} = 1\frac{6}{7}$

128. ‖ 11 | $\frac{4}{7}$ ‖ 12 | $\frac{187}{515}$ ‖ 13 | $\frac{41}{51}$ ‖ 14 | $\frac{69}{349}$ ‖ 15 | $\frac{309}{361}$

128. ‖ 16 | $\frac{183}{2381}$ ‖ 17 | $\frac{12}{13}$ ‖ 18 | $\frac{5}{24}$ ‖ 19 | $\frac{11}{37}$ ‖ 20 | $\frac{21}{134}$

128. ‖ 21 | $\frac{1}{25}$ ‖ 22 | $\frac{2}{123}$ ‖ 129. ‖ 1 | $\frac{5}{12}$ ‖ 2 | $\frac{7}{30}$ ‖ 3 | $\frac{9}{14}$

129. ‖ 4 | $\frac{5}{18}$ ‖ 5 | $\frac{3}{16}$ ‖ 6 | $1\frac{11}{64}$ ‖ 7 | 1 ‖ 8 | $35\frac{3}{4}$ ‖ 9 | 147

129. ‖ 10 | $8\frac{1}{6}$ ‖ 11 | $\frac{15}{416}$ ‖ 12 | $\frac{41}{3080}$ ‖ 13 | $13\frac{21}{43}$ ‖ 130. ‖ 1 | $\frac{63}{84}$

130. ‖ $\frac{448}{84}$, $\frac{72}{84}$ ‖ 2 | $\frac{126}{210}$, $\frac{140}{210}$, $\frac{30}{210}$, $\frac{525}{210}$ ‖ 3 | $\frac{1140}{120}$, $\frac{520}{120}$, $\frac{330}{120}$, $\frac{96}{12}$

130. ‖ 4 | $\frac{16}{24}$, $\frac{21}{24}$, $\frac{20}{24}$, $\frac{12}{24}$, $\frac{54}{24}$ ‖ 5 | $\frac{4725}{630}$, $\frac{540}{630}$, $\frac{280}{630}$, $\frac{378}{630}$ ‖ 6 | $\frac{990}{126}$

130. ‖ $\frac{588}{126}$ ‖ 7 | $\frac{30}{105}$, $\frac{714}{105}$ ‖ 8 | $\frac{88}{18}$, $\frac{42}{18}$, $\frac{99}{18}$, $\frac{108}{18}$ ‖ 9 | $\frac{156}{30}$, $\frac{36}{30}$, $\frac{105}{30}$

130. // $\frac{110}{30}$ *// 10 |* $\frac{20}{5}$, $\frac{24}{5}$ *‖ 11 * $\frac{637}{42}$, $\frac{324}{42}$ *\\\\ 12 * $\frac{288}{21}$, $\frac{9}{21}$, $\frac{129}{21}$ \\

130. ‖ $\frac{7}{21}$ ‖ 1 | $\frac{9}{12}$, $\frac{7}{12}$, $\frac{6}{12}$, $\frac{10}{12}$ ‖ 2 | $\frac{18}{21}$, $\frac{8}{21}$, $\frac{14}{21}$ ‖ 3 | $\frac{84}{20}$, $\frac{18}{20}$, $\frac{145}{20}$

130. ‖ 4 | $\frac{190}{18}$, $\frac{15}{18}$, $\frac{132}{18}$ ‖ 5 | $\frac{186}{30}$, $\frac{25}{30}$, $\frac{220}{30}$ ‖ 6 | $\frac{32}{40}$, $\frac{35}{40}$, $\frac{580}{40}$, $\frac{150}{40}$

130. ‖ 7 | $\frac{42}{72}$, $\frac{64}{72}$, $\frac{204}{72}$, $\frac{99}{72}$ ‖ 8 | $\frac{36}{42}$, $\frac{7}{42}$, $\frac{32}{42}$, $\frac{28}{42}$ ‖ 9 | $\frac{36}{44}$, $\frac{33}{44}$, $\frac{38}{44}$

130. ‖ $\frac{22}{44}$ ‖ 10 | $\frac{150}{60}$, $\frac{310}{60}$, $\frac{54}{60}$, $\frac{265}{60}$ ‖ 132. ‖ 1 | $\frac{63}{168}$, $\frac{96}{168}$, $\frac{70}{168}$

132. ‖ 2 | $\frac{15}{42}$, $\frac{18}{42}$, $\frac{32}{42}$ ‖ 3 | $\frac{88}{32}$, $\frac{10}{32}$, $\frac{9}{32}$ ‖ 4 | $\frac{129}{24}$, $\frac{106}{24}$, $\frac{7}{24}$

132. ‖ 5 | $\frac{254}{30}$, $\frac{12}{30}$, $\frac{7}{30}$ ‖ 6 | $\frac{642}{66}$, $\frac{9}{66}$, $\frac{10}{66}$ ‖ 7 | $\frac{105}{42}$, $\frac{136}{42}$, $\frac{3}{42}$

132. ‖ 8 | $\frac{164}{48}$, $\frac{56}{48}$, $\frac{18}{48}$, $\frac{27}{48}$ ‖ 9 | $\frac{96}{108}$, $\frac{20}{108}$, $\frac{21}{108}$ ‖ 10 | $\frac{348}{78}$, $\frac{555}{78}$,

132. ‖ $\frac{10}{78}$ ‖ 11 | $\frac{128}{20}$, $\frac{174}{20}$, $\frac{49}{20}$ ‖ 12 | $\frac{36}{68}$, $\frac{142}{68}$, $\frac{73}{68}$ ‖ 13 | $\frac{416}{72}$,

132. ‖ $\frac{453}{72}$, $\frac{14}{72}$, $\frac{1}{72}$ ‖ 134. ‖ 1 | 720 far. ‖ 2 | $1666\frac{2}{3}$ lb.

134. ‖ 3 | 7840 min. ‖ 4 | 3240 grs. ‖ 5 | $\frac{1}{297}$ rd. ‖ 6 | $\frac{1}{45}$ yd.

134. ‖ 7 | $\frac{11}{72000}$° ‖ 8 | $\frac{15}{3328}$ cord. ‖ 9 | 8s. 9d.; 9s. 4d.

134. ‖ 10 | 6 fur. 8 rd. 4 yd. 2 ft. 8 in.; 6 fur. 34 rd. 1 yd. 1 ft. $8\frac{4}{7}$ in

134. ‖ 11 | 33 rd. 1 yd. 2 ft. 6 in. ‖ 12 | $\frac{1}{336}$ guin. ‖ 13 | $\frac{1}{7776}$

134. ‖ 14 | $\frac{6480}{11}$ ‖ 1 | $\frac{53}{55}$ mi. ‖ 2 | £$\frac{421}{480}$ ‖ 3 | $\frac{1991}{2000}$

135. ‖ 4 | $\frac{309}{400}$ ‖ 5 | $\frac{205}{252}$ ‖ 6 | $\frac{63}{64}$ ‖ 7 | $\frac{139}{144}$ ‖ 8 | $\frac{899}{1416}$ ‖ 9 | $\frac{7}{96}$

135. ‖ 10 | $\frac{43}{960}$ ‖ 136. ‖ 2 | $\frac{5}{6}$, $\frac{7}{10}$, $\frac{16}{63}$, $\frac{19}{90}$ ‖ 3 | $\frac{22}{120}$, $\frac{31}{240}$, $\frac{15}{54}$, $\frac{13}{40}$

137. ‖ 1 | $2\frac{133}{204}$ ‖ 2 | $3\frac{97}{144}$ ‖ 3 | $2\frac{21}{64}$ ‖ 4 | $1\frac{187}{1008}$ ‖ 5 | $4\frac{14}{15}$ ‖ 6 | $4\frac{20}{33}$

137. ‖ 7 | $2\frac{451}{2040}$ ‖ 8 | $9\frac{25}{28}$ ‖ 9 | $15\frac{337}{600}$ ‖ 10 | $11\frac{9}{20}$ ‖ 11 | $26\frac{1}{84}$

137. ‖ 12 | $10\frac{53}{112}$ ‖ 13 | $4\frac{23}{24}$ ‖ 14 | $6\frac{41}{110}$ ‖ 15 | $6\frac{43}{88}$ ‖ 16 | $13\frac{113}{168}$

137. ‖ 17 | $18\frac{5}{24}$ ‖ 18 | $89\frac{5}{21}$ ‖ 19 | $212\frac{7}{32}$ ‖ 20 | $16\frac{17}{18}$ ‖ 21 | $65\frac{61}{120}$

137. ‖ 22 | $100\frac{13}{90}$ ‖ 23 | $89\frac{16}{45}$ ‖ **138.** ‖ 24 | $347\frac{11}{12}$ bus.; $417 [illegible]

139. ‖ 1 | $14\frac{1}{15}$ in. ‖ 2 | 2 da. 14 hr. 30 min. ‖ 3 | 1 cwt. 1 [illegible]

139. ‖ 19 lb. $4\frac{1}{3}$ oz. ‖ 4 | 2 oz. 10 pwt. 12 gr. ‖ 5 | 9 cwt. 1 qr.

139. ‖ 5 lb. $8\frac{8}{9}$ oz. ‖ 6 | 20 bu. 1 pk. $5\frac{6}{7}$ qt. ‖ 7 | 3 hhd. 37 gal

139. ‖ 3 qt. 0 pt. $1\frac{3}{5}$ gil. ‖ 8 | 55 da. 2 hr. 47 min. 30 sec.

139. ‖ 9 | 2 R. 20 P. 11 sq. ft. $58\frac{1}{35}$ sq. in. ‖ 10 | 7 in. ‖ 11

139. ‖ 13s 10$\frac{2}{3}$d. ‖ 12 | 7 fur. 2 ft. 9 in. ‖ 13 | 222 da. 1 hr. 24 min

139. ‖ 14 | 7 oz. 7 pwt. 23 gr. ‖ 15 | 5 s. 16° 16′ $40\frac{20}{133}$′

139. ‖ 16 | 1 yd. 0 qr. $2\frac{5}{6}$ na. ‖ 17 | 1 C. ft. 11 cu. ft. $466\frac{2}{7}$ cu. in.

139. ‖ 18 | 2 C. 4 C. ft. 2 cu. ft. ‖ 19 | 3 yd. 2 qr. $0\frac{2}{3}$ na.

140. ‖ 20 | 3 A. 2 R. $33\frac{1}{4}$ P. ‖ 21 | 11 cwt. 3 qr. 21 lb. 11 oz. $1\frac{1}{3}$dr.

140. ‖ 22 | 3 fur. 0 rd. 2 ft. 6 in. ‖ 141. ‖ 1 | $\frac{2}{7}$ ‖ 2 | $\frac{3}{19}$ ‖ 3 | $\frac{4}{25}$

141. ‖ 4 | $\frac{20}{61}$ ‖ 5 | $\frac{2}{35}$ ‖ 6 | $\frac{5}{48}$ ‖ 7 | $\frac{2}{195}$ ‖ 8 | $35\frac{71}{90}$ ‖ 9 | $\frac{7}{36}$

141. ‖ 10 | $\frac{43}{72}$ ‖ 11 | $24\frac{4}{15}$ ‖ 12 | $1\frac{7}{135}$ ‖ 13 | $\frac{1}{8}$ ‖ 14 | $3\frac{1}{24}$

141. ‖ 15 | $7\frac{3}{5}$ ‖ 16 | $14\frac{3}{5}$ ‖ 17 | $\frac{1}{40}$ ‖ 18 | $3\frac{2}{9}$ ‖ 19 | $9\frac{14}{15}$ ‖ 20 | $6\frac{29}{40}$

141. ‖ 21 | $8\frac{7}{8}$ ‖ 22 | $\frac{14}{45}$ ‖ 23 | $72 ‖ 142. ‖ 24 | \$$\frac{3}{8}$ ‖ 25 | $18\frac{11}{14}$

142. ‖ 26 | $18\frac{13}{18}$ ‖ 27 | $33\frac{3}{20}$ ‖ 28 | $22\frac{3}{16}$ ‖ 29 | \$$\frac{3}{10}$ ‖ 30 | \$$18\frac{2}{3}$

142. ‖ 2 | $\frac{6}{135}$ ‖ 3 | $\frac{1}{210}$ ‖ 4 | $\frac{1}{380}$ ‖ 5 | $\frac{3}{810}$ ‖ 143. ‖ 2 | $2\frac{34}{133}$

143. ‖ 3 | $76\frac{1}{8}$ ‖ 4 | $73\frac{31}{32}$ ‖ 5 | $6\frac{33}{38}$ ‖ 6 | $182\frac{83}{100}$

144. ‖ 1 | 9 oz. 7 pwt. 12 gr. ‖ 2 | 7 cwt. 1 qr. 24 lb. 8 oz.

144. ‖ 3 | 29 gal. $3\frac{5}{8}$ qt. ‖ 4 | 1 mi. 1 fur. 16 rd. ‖ 5 | 1s. 3d.

144. ‖ 6 | 38′ $34\frac{2}{7}$″ ‖ 7 | 563 A. 0 R. $35\frac{5}{9}$ P. ‖ 8 | 10 cwt. 1 qr.

144. ‖ 22 lb. $9\frac{19}{21}$ oz. ‖ 9 | 1 lb. 8 oz. 16 pwt. 16 gr. ‖ 10 |

144. ‖ 2 cords 2 C. ft. 4 cu. ft ‖ 11 | $5\frac{1}{3}$ in. ‖ 12 | 4 ℥ 3 ʒ 2 ℈ 4 gr

144. ‖ 13 | 1 A. 1 R. 17 P. $21\frac{1}{4}$ sq. yd. ‖ 14 | 1 pwt. $18\frac{1}{2}$ gr

146. ‖ 1 | $3\frac{3}{7}$ ‖ 2 | $1\frac{7}{25}$ ‖ 3 | $7\frac{1}{5}$ ‖ 4 | $11\frac{1}{19}$ ‖ 5 | 16 ‖ 6 | 70

146. ‖ 7 | 44 ‖ 8 | 1584 ‖ 9 | $608\frac{7}{12}$ ‖ 10 | $5987\frac{5}{8}$ ‖ 11 | 4536

146. ‖ 12 | 5405 ‖ 13 | 6975 ‖ 14 | 11725 ‖ 15 | $3\frac{5}{9}$ ‖ 16 | $12\frac{6}{7}$

146. ‖ 17 | 63 ‖ 18 | $178\frac{1}{8}$ ‖ 19 | $14\frac{124}{231}$ ‖ 20 | $19\frac{4}{9}$ ‖ 21 | $\frac{21}{40}$

146. ‖ 22 | $\frac{5}{96}$ ‖ 23 | $\frac{1}{24}$ ‖ 24 | $\frac{3}{20}$ ‖ 25 | 14 ‖ 26 | 18 ‖ 27 | $130\frac{1}{2}$

146. ‖ 28 | $\frac{5}{33}$ ‖ 29 | 14 ‖ 30 | $6316\frac{7}{8}$ ‖ 31 | $\frac{10}{21}$ ‖ 32 | $6\frac{48}{55}$

146. ‖ 33 | $\frac{4}{7}$ ‖ 34 | $\frac{1}{15}$ ‖ 35 | $2\frac{2}{3}$ ‖ 36 | 20 ‖ 37 | $\frac{53}{56}$ ‖ 38 | $\frac{23}{84}$

146. ‖ 39 | $2\frac{8}{21}$ ‖ 40 | $5\frac{1}{4}$ ‖ 41 | $14\frac{4}{7}$ ‖ 42 | \$$\frac{2}{3}$ ‖ **147.** ‖ 43 | $11\frac{1}{4}$

147. ‖ 44 | $22\frac{1}{2}$ ‖ 45 | $3\frac{1}{16}$ ‖ 46 | $14\frac{2}{3}$ ‖ 47 | $7\frac{21}{32}$ ‖ 48 | $8\frac{7}{16}$

147. ‖ 49 | 55 cents. ‖ 50 | $34\frac{1}{3}$ ‖ 51 | $43\frac{3}{4}$ ‖ 52 | 325 ‖ 53 | $\frac{5}{6}$

147. ‖ 54 | $\frac{3}{10}$ ‖ 55 | \$$\frac{3}{8}$ ‖ 56 | $\frac{11}{24}$ ‖ 57 | $20\frac{4}{5}$ ‖ 58 | $12\frac{1}{2}$ ‖ 59 | $5\frac{1}{3}$

148. ‖ 60 | $\frac{1}{3}$ ‖ 61 | 120 A. = A's share ; 80 A. = B's share ,

148 ‖ 20 A. = C's share. ‖ **149.** ‖ 1 | $\frac{1}{5}$ ‖ 2 | $\frac{3}{28}$ ‖ 3 | $\frac{13}{135}$

149 ‖ 4 | $\frac{3}{319}$ ‖ 5 | $\frac{23}{832}$ ‖ 6 | $7\frac{1}{7}$ ‖ 7 | 36 ‖ 8 | $\frac{7}{8}$ ‖ 9 | $2\frac{2}{5}$

149. ‖ 10 | $1\frac{23}{40}$ ‖ 11 | $\frac{112}{135}$ ‖ 12 | $1\frac{7}{128}$ ‖ 13 | $\frac{2}{5}$ ‖ 14 | $61\frac{1}{11}$

149. ‖ 15 | $1662\frac{1}{2}$ ‖ 16 | 1363 ‖ 17 | $\frac{7}{8}$ ‖ 18 | $\frac{26}{33}$ ‖ 19 | $1\frac{19}{58}$

149. ‖ 20 | $2\frac{13}{27}$ ‖ 21 | $9\frac{8}{13}$ ‖ 22 | $48\frac{1}{35}$ ‖ 23 | $\frac{40}{133}$ ‖ 24 | $1\frac{37}{200}$

149. ‖ 25 | $\frac{7}{121}$ ‖ 26 | $\frac{3}{17}$ ‖ 27 | $\frac{4}{27}$ ‖ 28 | $\frac{1}{10}$ ‖ 29 | $\frac{9}{521}$ ‖ 30 | $\frac{2}{125}$

149. ‖ 31 | 40 ‖ 32 | 1120 ‖ 33 | $1\frac{1}{5}$ ‖ 34 | $1\frac{1}{5}$ ‖ 35 | $\frac{243}{500}$

149. ‖ 36 | $\frac{112}{135}$ ‖ 37 | $\frac{112}{135}$ ‖ 38 | $1\frac{1}{2}$ ‖ 39 | $825\frac{1}{2}$ ‖ 40 | $4193\frac{7}{17}$

149. ‖ 41 | $16046\frac{3}{32}$ ‖ 42 | $\frac{7}{9}$ ‖ 43 | $\frac{8}{27}$ ‖ 44 | $\frac{1}{3}$ ‖ 45 | $22\frac{6}{7}$

149. ‖ 46 | $68\frac{652}{781}$ ‖ 47 | $3\frac{1}{3}$ ‖ 48 | $1\frac{1}{5}$ ‖ 49 | $9\frac{5}{9}$ ‖ 50 | $72\frac{53}{180}$

150. ‖ 51 | $5\frac{1}{4}$ lbs. ‖ 52 | $1\frac{3}{32}$ yds. ‖ 53 | $1\frac{1}{2}$ ‖ 54 | 4 ‖ 55 | $\frac{7}{10}$

150. ‖ 56 | $3\frac{3}{35}$ ‖ 57 | $1\frac{1}{15}$ ‖ 58 | 6 ‖ 59 | $\frac{5}{6}$ ‖ 60 | 21 ‖ 61 | $27\frac{3}{8}$

150. ‖ 62 | $14\frac{4}{41}$ ‖ 63 | $\frac{56}{363}$ ‖ 64 | $\frac{2}{25}$ ‖ 65 | $\frac{2}{49}$ ‖ **151.** ‖ 66 | \$$5\frac{1}{4}$

151. ‖ 67 | $19\frac{23}{51}$ ‖ 68 | $14\frac{4}{5}$ ‖ 69 | \$$\frac{4}{9}$ ‖ 70 | \$$\frac{61}{64}$ ‖ 71 | $13\frac{1}{3}$

151. ‖ 72 | $108\frac{9}{16}$ ‖ 73 | $\frac{3}{4}$ ‖ 74 | 4 ‖ 75 | $24\frac{1}{2}$ ‖ 76 | $1\frac{1}{6}$ ‖ 77 | 10[illegible]

151. ‖ 78 | $4\frac{1}{2}$ ‖ 79 | 6 ‖ 80 | $13\frac{1}{8}$ ‖ 81 | $\frac{4}{5}$ ‖ 82 | 6096

151. ‖ 83 | $17\frac{7}{15}$ ‖ **152.** ‖ 1 | $1\frac{1}{24}$ ‖ 2 | $\frac{128}{135}$ ‖ 3 | $2\frac{62}{81}$ ‖ 4 | 100

152. ‖ 5 | $\frac{16}{81}$ ‖ 6 | $\frac{5}{7}$ ‖ 7 | $1\frac{1}{4}$ ‖ 8 | 35 ‖ 9 | $\frac{350}{549}$ ‖ 10 | $2\frac{4}{7}$ ‖ 11 |

152. ‖ $53\frac{1}{7}$ ‖ 12 | $\frac{9}{64}$ ‖ **153.** ‖ 1 | 15 ‖ 2 | $17\frac{3}{5}$ ‖ 3 | $7\frac{13}{35}$ ‖ 4 | $42\frac{479}{702}$

153. ‖ 5 | $\frac{41}{840}$ ‖ 6 | $26\frac{7}{18}$ ‖ 7 | 15 ‖ 8 | $16\frac{1}{3}$ ‖ 9 | 8 bu. $1\frac{1}{3}$ pk.

153. ‖ 10 | 1 mi. 2 fur. 16 rd. ‖ 11 | 4 mi. 7 fur. 19 rd. 3 yd. $0\frac{27}{35}$ ft

153. ‖ 12 | $20\frac{1}{3}$ ‖ 13 | 14 ‖ 14 | $20\frac{41}{44}$ ‖ 15 | 2700 = A's share,

153. ‖ 2800 = B's share ; 800 = C's share. ‖ **154.** ‖ 16 | 40

154. ‖ 17 | £7 17s. 5d $0\frac{4}{7}$ far. ‖ 18 | 24 = John's ; 32 = James'

154. ‖ 19 | $285\frac{5}{7}$ ‖ 20 | A, 80 ; B, 24 ; C, 30 ; D, 40 ; 66 rem.

154. ‖ 21 | $467\frac{2}{3}$ ‖ 22 | \$$2\frac{1}{16}$ selling price ; \$$\frac{42}{208}$ = 1st one's gain;

154. ‖ \$$\frac{49}{208}$ = 2d one's gain. ‖ 23 | $257\frac{19}{28}$ ‖ 24 | $7\frac{1}{2}$ ‖ 25 |

154. ‖ $1724\frac{1}{3}$ = A's ; $1231\frac{2}{3}$ = B's ‖ 26 | 165 ‖ **156.** ‖ 1 | 7 ft. 2′

156. ‖ 2 | 5 ft. 2′ 6″ ‖ 3 | 21 ft. 4′ 11″ 4‴ ‖ 4 | 5 ft. 7′

156. ‖ 5 | 3′ 3″ 2‴ ‖ 6 | 2 ft. 7′ 3″ ‖ 7 | 15 ft. 4′ 10″ 4‴

156. ‖ 8 | 3 ft. 6′ 5″ 5‴ ‖ 9 | 87 ft. 10′ 7″ 4‴ ‖ 10 | 183 ft. 5′ 6″ 2‴

156. ‖ 11 | 223 ft. 8′ 4″ 9‴ ‖ 12 | 87 ft. 2′ 7″ 9‴ 6⁗

156. ‖ 13 | 317 ft. 11′ 0″ 4‴ ‖ 14 | 543 ft. 6′ 3″ 2‴ sum ;

156. ‖ 107 ft. 8′ 9″ 2‴ diff. ‖ **160.** ‖ 1 | 41 cu. ft. 3′ 10″

160. ‖ 2 | 43 sq. ft. 6′ 6″ ‖ 3 | 82 sq. ft. 9′ 4″ ‖ 4 | 347 sq. ft. 10′ 3′

160. ‖ 5 | 554 sq. ft. 7′ 8″ 8‴ 3⁗ ‖ 6 | 2917 sq. ft. 0′ 0″ 7‴ 4‴

160. ‖ 7 | 194 sq. ft. 4′ 3″ 6‴ ‖ 8 | 39 sq. ft. 11′ 2″ 3″

160. ‖ 9 | 296 sq. ft. 10′ 6″ ‖ 10 | 96 sq. yd. 2 sq. ft. 8′ 3

160. ‖ 11 | 3150 sq. ft. ‖ 12 | $327\frac{7}{9}$ sq. yd. ‖ 13 | 21 sq. ft

161. ‖ *14* | *\$26.40* ‖ 15 | 10 A. 1 R. 25 P. ‖ 16 | 3119 sq. ft. 6′ 9″

161. ‖ 17 | 99 ‖ 18 | $\$208.01\frac{1}{4}$ ‖ 19 | 89 cu. ft. 3′ ‖ 20 | $\$18.49\frac{1}{3}$

161. ‖ 21 | 504 cu. ft. ‖ 22 | $11\frac{13}{16}$ cords ‖ 23 | $24124\frac{52}{77}$

161. ‖ 24 | 41958 ‖ 25 | 19419 cu. ft. 9′ ‖ 26 | 849 cu. ft. 8′ 8″

161. ‖ 27 | $15.403+ ‖ 28 | $275\frac{46}{243}$ cu. yd. ‖ **162.** ‖ 29 | $\$19.80\frac{8}{9}$

163. ‖ 1 | 4 ft. 7′ ‖ 2 | 5 ft. 3′ 3″ ‖ 3 | 48 ft. 6′ ‖ 4 | 8 ft. 7′

163. ‖ 5 | 12 ft. 6′ ‖ 6 | 37 ft. 3′ ‖ 7 | 1 ft. 7′ ‖ 8 | 8 ft.

163. ‖ 9 | 6 ft. 6′ $3\frac{1}{15232}$″ ‖ **167.** ‖ 1 | .06 ‖ 2 | 1.7 ‖ 3 | .005

167. ‖ 4 | .27 ‖ 5 | .047 ‖ 6 | 6.41 ‖ 7 | 7.008 ‖ 8 | 9.05 ‖ 9 | 11.50

167. ‖ 10 | 44.7 ‖ 1 | 27.4 ‖ 2 | 36.015 ‖ 3 | 99.0027 ‖ 4 | .320

167. ‖ 5 | 200.000320 ‖ 6 | .3600 ‖ 7 | 5.000003 ‖ 8 | 40.0000009

167. ‖ 9 | .4900 ‖ 10 | 59.0067 ‖ 11 | .0469 ‖ 12 | 79.000415

167. ‖ 13 | 67.0227 ‖ 14 | 105.0000095 ‖ 15 | 40.204000

168. ‖ 1 | $37.265 ‖ 2 | $17.005 ‖ 3 | $215.08 ‖ 4 | $275.005

168. ‖ 5 | $9.008 ‖ 6 | $15.069 ‖ 7 | $27.182 ‖ 8 | $3.059

171. ‖ 1 | 1306.1805 ‖ 2 | 528.697893 ‖ 3 | 159.37 ‖ 4 | 1.5415

171. ‖ 5 | 446.0924 ‖ 6 | 27.2087 ‖ 7 | 88.76257 ‖ 8 | 71.01

171. ‖ 9 | 1835.599 ‖ 10 | 397.547 ‖ 11 | 31.02464 ‖ 12 | 90.210129

171. ‖ 13 | 204.0278277 ‖ 14 | 400.33269960 ‖ 15 | .1008879

171. ‖ 16 | $85.463 ‖ **172.** ‖ 17 | $1065.19 ‖ 18 | 3.8896

172. ‖ 19 | $427.835 ‖ 20 | $19.215 ‖ 21 | $670.975 ‖ 22 | $30.286

72. ‖ 23 | $328.202 ‖ 24 | $248.011 ‖ 25 | $134.634

73. ‖ 1 | 875.0033 ‖ 2 | 368.5631 ‖ 3 | 7141.51354 ‖ 4 | 51.722

173. ‖ 5 | 2.7696 ‖ 6 | 1571.85 ‖ 7 | .6946 ‖ 8 | .89575

173. ‖ 9 | 603.925 ‖ 10 | 1379.25922 ‖ **174.** ‖ 11 | 99.706

174. ‖ 12 | 17.949 ‖ 13 | .699993 ‖ 14 \ 828.9992 ‖ 15 \ 999

174. ‖ 16 | 6314.9 ‖ 17 | 365.007495 ‖ 18 | 20.9942

174. ‖ 19 | 260.3608953 ‖ 20 | 10.030181 ‖ 21 | 2.0294

174. ‖ 22 | 999.999 ‖ 23 | 2499.75 ‖ 24 | 103.015 ‖ 25 | .4232

174. ‖ 26 | 171.925 ‖ 27 | $82.625 ‖ 28 | $26.60 ‖ 29 | 126.84194

174. ‖ 30 | $761.18 ‖ 175. ‖ 1 | .796875 ‖ 2 | 2 63872

175. ‖ 3 .0000500 ‖ 4 | 1.50050 ‖ 5 | 26.99178 ‖ 6 | 10376.283913

175. ‖ 7 165235.5195 ‖ 176. ‖ 8 | .0206211250 ‖ 9 | 28033.797-

176. ‖ 099 ‖ 10 | 175.26788356 ‖ 11 | .000432045770

176. ‖ 12 | 216.94165850 ‖ 13 | .000000000294 ‖ 14 | 18616.74

176. ‖ 15 | 933.8253150762 ‖ 16 | .00715248 ‖ 17 | .608785264

176. ‖ 18 | .02860992 ‖ 19 | 2.435141056 ‖ 20 | 1296

176. ‖ 21 | 312.5 ‖ 22 | .375 ‖ 23 | .0036 ‖ 24 | 148.28125

176. ‖ 25 | 12.13035 ‖ 26 | $24.0625 ‖ 27 | $3192.005625

176. ‖ 28 | $210.03125 ‖ 29 | $708.901875 ‖ 30 | $2.06525 gain.

177. ‖ 1 | 4796.4 ; 47964 ‖ 2 | 69472.9 ; 694.729 ‖ 3 | 415300. ;

177. ‖ 4153. ‖ 4 | 2704 ; 27040. ‖ 5 | 129072. ; 1290.72 ‖ 6 |

177. ‖ 871000. ; 8710. ‖ 7 | 140100. ; 1401. ‖ 179. ‖ 2 | 258.13007

179. ‖ 3 | 162.525 ‖ 4 | 2757.89785 ‖ 5 | 3566163 ‖ 180. ‖ 1 | 2.22

180. ‖ 2 | 8.522 ‖ 3 | 33.331 ‖ 4 | 1.0001 ‖ 5 | 12420.5 ‖ 6 | .005

180. ‖ 7 | 4.25 ‖ 8 | .007 ‖ 9 | .075 ‖ 10 | 1.27 ‖ 11 | .015

180. ‖ 12 | 17.008 ‖ 13 | 25.05068 ; 250.5068 ; 2505.068 ;

180. ‖ 25050.68 ; 250506.8 ‖ 14 | 48.65961 ; 4865.961 ;

180. ‖ 48659.61 ; 486596.1 ; 4865961. ‖ 15 | 41.622 ; 416.22 ;

180. ‖ 4162.2 ; 41622. ; 416220. ; 4162200. ‖ 16 | 254.7347748 ;

180. ‖ 25473.47748 ; 254734.7748 ; 2547347.748 ; 25473477.48 ;

180. ‖ 254734774.8 ‖ 17 | .1395646+ ‖ 18 | 1918.515+
181. ‖ 19 | .004735 ‖ 20 | 174.412 ‖ 21 | 69.7125 ‖ 22 | 1.36832+
181. ‖ 23 | 12976.816+ ‖ 24 | .004958+ ‖ 25 | 6.165
181. ‖ 26 | $9.875 ‖ 27 | $2.15 ‖ 28 | $.62 ‖ 29 | 18 ‖ 30 | 8
181. ‖ 31 | 14 ‖ 32 | 55.5 ‖ 33 | 269 acres ; $13573.204 cost;
181. ‖ $50.458 average price. ‖ 34 | $7631.8855 share of eldest;
181. ‖ $5723.914125 share of others. ‖ 182. ‖ 2 | 10970
182. ‖ 3 | 60200 ‖ 4 | 1000 ‖ 5 | 100 ‖ 6 | 10 ; 100 ; 1000 ;
182. ‖ 30 ; 20 ; 2000 ; 12 ; 1200 ; 500000 ‖ 183. ‖ 3 | 8.311+
183. ‖ 4 | 1.563+ ‖ 5 | 1.16049+ ‖ 6 | 16.11902+
184. ‖ 1 | 31.69274 ; 3.169274 ‖ 2 | 57.13562 ; 571.3562 ;
184. ‖ 5713.562 ‖ 3 | .675 ; .0675 ; .0000675 ‖ 4 | .049 ; .0049 ;
184. ‖ .00049 ‖ 5 | .030467 ; .0030467 ; .00030467 ‖ 6 | .004741 ;
184. ‖ .0004741 ; .00004741 ‖ 7 | .497 ; .0497 ; .00497
186. ‖ 1 | 79.1188 ‖ 2 | 35.2843 ‖ 3 | 11.5834036 ‖ 4 | 3202.8870
187. ‖ 1 | .25 ; .5 ; .75 ‖ 2 | .8 ; .875 ; .3125 ‖ 3 | .375 ; .04
187. ‖ 4 | .015625 ; .2666$\frac{2}{3}$ ‖ 5 | .125 ; .003 ‖ 6 | .25714+;
187. ‖ .44117+ ‖ 7 | .23903+ ‖ 8 | .07157+ ‖ 9 | .4375 ;
187 ‖ .078125 ‖ 10 | .00448 ‖ 11 | .536 ; .372 ‖ 12 | .9
187. ‖ 13 | .73333$\frac{1}{3}$ ‖ 14 | .48375 ‖ 15 | .51282+ ‖ 16 | .5375 ;
187. ‖ .005606+ ‖ 17 | .16666+ ‖ 18 | 1.5555$\frac{5}{9}$ ‖ 19 | .15909$\frac{1}{11}$
187. ‖ 20 | $100.80 ‖ 21 | $17.85 | 22 | 30.6111$\frac{1}{9}$ ‖ 23 | 2.9166$\frac{2}{3}$
187. ‖ 24 | 2.8412+ ‖ 188. ‖ 1 | $\frac{1}{4}$; $\frac{3}{4}$ ‖ 2 | $\frac{1}{8}$; $\frac{5}{8}$ ‖ 3 | $\frac{21}{200}$; $\frac{1}{400}$
188. ‖ 4 | $\frac{1603}{2000}$; $\frac{3021}{5000}$ ‖ 5 | $\frac{547}{800}$ ‖ 6 | $\frac{3}{160}$ ‖ 7 | $\frac{903}{4000}$ ‖ 8 | $\frac{17}{64}$
188. ‖ 9 | $\frac{1}{3}$ ‖ 10 | $\frac{4}{7}$ ‖ 189. ‖ 1 | .0546875 ‖ 2 | .325 ‖ 3 | 3.9875

189. ‖ 4 | .375 ‖ 5 | 71.15113+ ‖ 6 | .6625 ‖ 7 | .15375

189. ‖ 8 | .1225 ‖ 9 | .26175 ‖ 10 | 100511+ ‖ 11 | .64

189. ‖ 12 | .91111+ ‖ 13 | .875 ‖ 14 | .01587+ ‖ 15 | .71299+

189. ‖ 16 | .2325 ‖ 17 | .972916+ ‖ 18 | .48125 ‖ 19 | .55

189. ‖ 20 | .001617+ ‖ 21 | .25625 ‖ 22 | .063 ‖ 23 | .10416+

189. ‖ 24 | .00994318+ ‖ 25 | .791666+ ‖ 26 | .3375 ‖ 27 | .3125

189. ‖ 28 | .040909 ‖ 29 | .01875 ‖ 30 | .020265+

189. ‖ 31 | .19672+ ‖ 32 | .34895+ ‖ 33 | .01537+ ‖ 34 | .005

190. ‖ 1 | 2 qr. 17 lb. 4 oz. ‖ 2 | 1 hhd. 13 gal. 3.44 qts

190. ‖ 3 | 16s. 7d. 2.99 far. ‖ 4 | 2 gal. 1 qt. ‖ 5 |

190. ‖ 1 wk. 4 da. 23 hr. 59 min. 56.54+ sec. ‖ 6 | 8 P

190. ‖ 7 | 6 cwt. 3 qr. ‖ 8 | 1 hhd. 47 gal. 1 qt. ‖ 9 | 20 gal. 1 qt.

191. ‖ 10 | 10 oz. 18 pwt. 15.99+ gr. ‖ 11 | 3 qrs. 1.5 na.

191. ‖ 12 | 1 yd. 2 ft. 11.9+ in. ‖ 13 | 24 P. 23 sq. yd. 5 sq. ft.

191. ‖ 82.4832 sq. in. ‖ 14 | 32 mi. 1 fur. 14 rd. 4 yd. 2 ft. 9.408 in.

191. ‖ 15 | 2 ft. 7.5 in. ‖ 16 | 4 ℥ 1 ʒ 1 ℈ 9.6+ gr.

191. ‖ 17 | 3 R. 1 P. 13.31 sq. yd. ‖ 18 | 9 sheets. ‖ 19 | 11 lb.

191. ‖ 20 | 7d. 2 far. ‖ 21 | 1 R. 14 P. ‖ 22 | 286 da. 17 hr

191. ‖ 18 min. 36 sec. ‖ **193.** ‖ 1 | .06 ‖ 2 | .08125 ‖ 3 | .034375

193. ‖ 4 | .01328125 ‖ 5 | .0171875 ‖ 6 | .034 ‖ 7 | .028

193. ‖ 8 | .024219375 ‖ **194.** ‖ 1 | .71428571+ ‖ 2 | .2666+

194. ‖ 3 | .4545+ ‖ 4 | .3888+ ‖ **196.** ‖ 3 | $\frac{2}{3}$; $\frac{6}{37}$; $\frac{10}{13}$; $\frac{35}{37}$; $\frac{1}{11}$

196. ‖ 4 | $\frac{85}{143}$; $\frac{4}{11}$; $\frac{1}{7}$ ‖ **197.** ‖ 4 | $\frac{5}{36}$; $7\frac{269}{495}$; $\frac{29}{666}$; $37\frac{49}{90}$;

197. ‖ $\frac{223}{330}$; $\frac{75434}{99999}$ ‖ 5 | $\frac{34}{45}$; $\frac{217}{495}$; $\frac{7}{75}$; $4\frac{1256}{1665}$; $\frac{163}{16500}$; $\frac{41}{90}$

198. ‖ *2* | .1875'0' ‖ 3 | .0'0344827586206896551724137931

198. ‖ 4 | .‘09756′; .‘592′; .5‘3′ ‖ **200.** ‖ 2 | 2.4‘181818;

200. ‖ 5‘925925′; .008‘497133′ ‖ 3 | 165.16‘416416′; .04‘040404′

200. ‖ .03‘777777′ ‖ 4 | .5‘333333′; .4‘757575′; 1.7‘577577

201. ‖ 2 | 95.2‘829647′ ‖ 3 | 69.74‘203112′ ‖ 4 | 55.6‘209780437503′

201. ‖ 5 | 47.3‘763490′ ‖ 6 | 416.2‘542876′ ‖ **202.** ‖ 2 | 45.7‘755′

202. ‖ 3 | 2.9‘957′ ‖ 4 | 5.09‘0 ‖ 5 | .65‘370016280907′

202. ‖ 6 | 4.37‘4′ ‖ 7 | 4.619‘525′ ‖ 8 | 1.0923‘7′ ‖ 9 | 1.3462‘937′

202. ‖ 2 | 5.53780‘5′ ‖ 3 | 1.093‘086′ ‖ 4 | 1.6411‘7′ ‖ 5 | 1.7183‘39′

202. ‖ 6 | 1.4710‘037′ ‖ 7 | 6.1‘656′ ‖ 8 | 11.‘068735402′

202. ‖ 9 | .81654‘168350′ ‖ **203.** ‖ 2 | 13.570413‘961038′

203. ‖ 3 | 35.024‘0′ ‖ 4 | 7.719‘54′ ‖ 5 | 26.7837‘428571′

203. ‖ 6 | 3.1‘45′ ‖ 7 | 3.‘8235294117647058′ ‖ 8 | 1.2‘6′

203. ‖ 9 | 15.48‘423′

205. ‖ 2 | $\frac{21}{39}=\cfrac{1}{1+\cfrac{1}{1+\frac{1}{6}}}$ $=1$, $=\frac{1}{2}$ ‖ 4 | $\frac{17}{27}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\frac{1}{3}}}}}$ $=1$, $=\frac{1}{2}$, $=\frac{2}{3}$, $=\frac{5}{8}$

205. ‖ 3 | $\frac{47}{65}=\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{3}}}}}}}$ $=1$, $=\frac{2}{3}$, $=\frac{3}{4}$, $=\frac{5}{7}$, $=\frac{8}{11}$, $=\frac{13}{18}$ ‖ 5 | $\frac{67}{85}=\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}}}}$ $=1$, $=\frac{3}{4}$, $=\frac{4}{5}$, $=\frac{11}{14}$, $=\frac{15}{19}$, $=\frac{26}{33}$

205. ‖ 6 | $\frac{37}{87}=\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{5+\frac{1}{2}}}}}$ $=\frac{1}{2}$, $=\frac{2}{5}$, $=\frac{3}{7}$, $=\frac{17}{40}$ ‖ 7 | $\frac{109}{450}=\cfrac{1}{4+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\frac{1}{2}}}}}}$ $=\frac{1}{4}$, $=\frac{7}{29}$, $=\frac{8}{33}$, $=\frac{31}{128}$, $=\frac{39}{161}$

205. ‖ $\left(\frac{31}{128}+\frac{39}{161}\right)\div 2=\frac{9983}{41216}$ *Ans.*

208. ‖ 2 | $\frac{24}{6} = 4$ ‖ 3 | $\frac{35}{7} = 5$ ‖ 4 | $\frac{6}{15} = \frac{2}{5}$ ‖ 5 | 60 +

208. ‖ 20 = 3 ‖ 6 | $\frac{9 \times 4}{6 \times 3} = 2$ ‖ 7 | $\frac{5 \times 8 \times 5}{4 \times 9 \times 3} = \frac{200}{108} = 1\frac{23}{27}$

208. ‖ 8 | $\frac{4}{6} = \frac{2}{3}$ ‖ 9 | $\frac{5}{10} = \frac{1}{2}$ ‖ 10 | $\frac{17}{34} = \frac{1}{2}$ ‖ 11 | $\frac{300}{450} = \frac{2}{3}$

208. ‖ 12 | $\frac{16}{96} = \frac{1}{6}$ ‖ 13 | $\frac{2}{3}$ ‖ 14 | $\frac{1}{3}$ ‖ 15 | $\frac{1}{5}$ ‖ 16 | $\frac{1}{11}$ ‖ 17 | $\frac{9}{10}$

208. ‖ 1 | 112 cwt. ‖ 2 | 5 tons. ‖ **209.** ‖ 3 | 60 ‖ 4 | 5 ‖ 5 | $\frac{7}{10}$

209. ‖ 6 | 32 ‖ 7 | 28 ‖ 8 | $65 ‖ **211.** ‖ 1 | $x = 60$ ‖ 2 | $x = 27$

209. ‖ 3 | $x = 9$ ‖ 4 | $x = \frac{1}{8}$ ‖ 5 | 38 ‖ 6 | 56 ‖ 7 | 12 ‖ 8 | 40

213. ‖ 1 | $x = 21$ ‖ **215.** ‖ 1 | 330 ‖ 2 | 90 ‖ 3 | 504 ‖ 4 | 2.08

215. ‖ 5 | 875 ‖ **216.** ‖ 6 | 99 ‖ 7 | 2762½ ‖ 8 | 20 ‖ 9 | 122.85

216. ‖ 10 | 1400 ‖ 11 | 16485 ‖ 12 | 121.875 ‖ 13 | $27

216. ‖ 14 | 7⅕ oz. ‖ 15 | 3533.936 ‖ 16 | 86.62 ‖ 17 | £39679 10s

216. ‖ 18 | 9 ‖ 19 | 8⅖ rd. ‖ 20 | 160 yds. ‖ 21 | 7⅓ ‖ **217.** ‖ 22 | 10

217. ‖ 23 | 920 ‖ 24 | 54 ‖ 25 | 39.375 ‖ 26 | 382.85 ‖ 27 | 63

217. ‖ 28 | $.036 ‖ 29 | $7080.48 ‖ 30 | $1.925 ‖ 31 | 2.10

217. ‖ 32 | 52.50 ‖ 33 | $\$\frac{36}{49}$ ‖ 34 | 7200 ‖ 35 | $37.909+

217. ‖ 36 | 225 ‖ 37 | 20 ‖ **218.** ‖ 38 | 54 ‖ 39 | 12 ‖ 40 | 6

218. ‖ 41 | 160 ‖ 42 | 40.47 ‖ 43 | 10 yr. ‖ 44 | 5⅓ ‖ 45 | 132.589+

218. ‖ 46 | 112½ ‖ 47 | 18.66⅔ ‖ 48 | 56.355 ‖ 49 | 106⅔ ‖ 50 | 40

218. ‖ 51 | 112.86 ‖ 52 | 18090 ‖ **219.** ‖ 53 | 21 gal. ‖ 54 |

219. ‖ 2142 to A ; 1125 to B ‖ 55 | .625 ‖ 56 | $6\frac{2}{9}$ ‖ 57 | $15.86

219. ‖ 58 | 168 lbs. ‖ 59 | 93¾ ‖ 60 | 552 ‖ 61 | 1744

219. ‖ 62 | 6 hr. 32 min. $43\frac{7}{11}$ sec. ‖ 63 | 140 ‖ **220.** ‖ 64 | 1⅓ da

220. ‖ 65 | 22½ da. ‖ 66 | 45 ‖ 67 | $13\frac{7}{11}$ ‖ 68 | $20\frac{4}{7}$ ‖ 69 | 10

220. ‖ 70 | 13⅓ ‖ 71 | 810 ‖ 72 | 6 ‖ 73 | 10 hr. 40 min. $36\frac{7}{48}$ sec

220. ‖ 74 | 16½ times. ‖ **222.** ‖ 1 | 16⅕ ‖ 2 | 7200 ‖ 3 | 187

222. ‖ 4 | 72 ‖ 5 | 10 ‖ 6 | $92\frac{1}{4}$ ‖ **223.** 7 | 36 ‖ 8 | 292.5 ‖ 9 | 156

223. ‖ 10 | 9600 ‖ 11 | 50 ‖ 12 | $13\frac{3}{5}$ ‖ 13 | $8571\frac{3}{7}$ ‖ 14 | 3 hr.

223. ‖ 15 | $471.04 ‖ 16 | $3\frac{15}{16}$ ‖ **224.** ‖ 17 | 180 ‖ 18 | $13\frac{1}{8}$ in.

224. ‖ 19 | $14\frac{2}{5}$ ‖ 20 | $7\frac{1}{2}$ ‖ 21 | $97\frac{1}{5}$ ‖ 22 | 32 ‖ 23 | 32 ‖ 24 | 132

226. ‖ 2 | $1000, A's; $1200, B's ; $800, C's ‖ 3 | $1714.28\frac{4}{7}$, A's

226. ‖ $285.71\frac{3}{7}$, B's ‖ 4 | $4030, A's ; $3980, B's ; $3980, C's

226. ‖ $4010, D's ‖ 5 | 100, A's ; 140, B's ; 200, C's ‖ 6 |

226. ‖ $$3333\frac{1}{3}$, 1st ; $3000, 2d ; $3000, 3d ; $$2666\frac{2}{3}$, 4th

227. ‖ 7 | $3000, widow's ; $1500, son's ‖ 8 | $12961.50, A's ;

227. ‖ $15737.25, B's ; $10802.25, C's ; $1833, D's gain.

227. ‖ 9 | $450, A's ; $600, B's ; $750, C's ‖ 10 | 4242.50, A s

227. ‖ stock ; 1697, A's gain : 5939.50, B's stock ; 2375.80 B's

227. ‖ gain : 6788, C's stock ; 2715.20, C's gain. ‖ 11 | 237.75,

227. ‖ A's ; 181.0625, B's ; 125.4375, C's ; 70, D's. ‖ 12 |

227. ‖ 87.831+ A's ; 65.06+ B's ; 48.795, C's ; 68.313,

227. ‖ D's. ‖ 13 | 2553, A's ; 3401.70, B's ; 1405.30, C's.

228. ‖ 14 | $15063\frac{8}{9}$, B's ; $9586\frac{1}{9}$, A's. ‖ 15 | $1015.33\frac{1}{3}$, the first ;

228. ‖ 1523.00, the second ; $2030.66\frac{2}{3}$, the third. ‖ 2 | 16.38, A's ;

228. ‖ 35.10, B's ; 18.72, C's. ‖ **229.** ‖ 3 | $7 ‖ 4 | $6577.23\frac{543}{659}$,

229. ‖ A's ; $1822.76\frac{116}{659}$, B's. ‖ 5 | 288, A's ; 270, B's ; 240, C's.

229. ‖ 6 | 280, D's ; 168, C's. ‖ 7 | $2648.86\frac{4}{11}$, A's ; $2901.13\frac{7}{11}$,

229. ‖ B's ; 1850, C's. ‖ 8 | $800, B's stock ; 15 mo., C's time

231. ‖ 1 | 50.24 ‖ 2 | 114.78 ‖ 3 | 1.1875 ‖ 4 | 2.839375

231. ‖ 5 | 1.002 ‖ 6 | 12 ‖ 7 | 90 ‖ 8 | 16.74 ‖ 9 | 47.725

231. ‖ 10 | 27.54 ‖ 11 | 300.365 ‖ 12 | 15.75 ‖ 13 | 180

231. ‖ 14 | 478.125 ‖ 15 | 4344.35 ‖ 16 | 2625 ‖ 17 | 5144.625

231. ‖ 18 | 12500 ‖ 19 | 3867.01875 ‖ 20 | 15000 ‖ 21 | 22.95

231. ‖ 22 | 43.20 ‖ 23 | 65 ‖ 24 | 742.85 ‖ 25 | 205

232. ‖ 1 | 20 ‖ 2 | $12\frac{1}{2}$ ‖ 3 | $7\frac{1}{2}$ ‖ 4 | $13\frac{3}{5}$ ‖ 5 | 25 ‖ 6 | $87\frac{1}{2}$

232. ‖ 7 | $57\frac{1}{7}$ ‖ 8 | $\frac{3}{4}$% ‖ 9 | $47\frac{7}{69}$ ‖ 10 | $33\frac{1}{3}$ ‖ 11 | $37\frac{1}{2}$ ‖ 12 | $12\frac{1}{2}$

233. ‖ 13 | $87\frac{1}{2}$ ‖ 14 | 80 ‖ 15 | 70 ‖ 16 | $66\frac{2}{3}$ ‖ 17 | 160

233. ‖ 2 | 1900 ‖ 3 | 700 ‖ 4 | 400 ‖ 5 | 15000 ‖ 6 | $142\frac{6}{7}$

233. ‖ 7 | $1.9\frac{4}{9}$ ‖ 8 | 90 ‖ 9 | 7800 ‖ 10 | 4392 ‖ 11 | 20800

234. ‖ 1 | 388.1188 ‖ 2 | 9000 ‖ 3 | 550 ‖ 4 | 756 ‖ 5 | 30.123

235. ‖ 6 | 740 ‖ 7 | $37777.77\frac{7}{9}$ ‖ 8 | 5400 ‖ 9 | 4 ‖ 10 | 5425

235. ‖ 1 | 160 ‖ **236.** ‖ 2 | 750 ‖ 3 | 950 ‖ 4 | 30000 ‖ 5 | 13500

236. ‖ 6 | 50000 ‖ 7 | 5000 ‖ 8 | 2600 ‖ **237.** ‖ 1 | 33.75

237. ‖ 2 | $27.41\frac{1}{4}$ ‖ 3 | 236.25 ‖ 4 | $6.15\frac{5}{13}$ ‖ 5 | $.695 per bushel

238. ‖ 6 | 8766 ‖ 7 | $.56 ‖ 8 | 915.75 ‖ 9 | 733.20

238. ‖ 10 | $11765.31\frac{1}{4}$ ‖ 11 | 444.75 ‖ 12 | 1970.775 ‖ 13 | 2 cts.

238. ‖ 14 | 18860 ‖ 15 | $1\frac{1}{4}$% ‖ 16 | 90 cents. ‖ **239.** ‖ 17 | $3.20

239. ‖ 18 | $.96 ‖ 19 | $18.03 ‖ 20 | $.66 ‖ 21 | $1.80 ‖ 22 | 18%

239. ‖ 23 | 25% ‖ 24 | Neither. ‖ 25 | 80% ‖ 26 | 25%

239. ‖ 27 | 160.34375 gain; $4\frac{2}{3}$% ‖ **240.** ‖ 28 | $$1041.15908\frac{4}{7}$

240. ‖ 29 | 25% on gold; 20% on paper. ‖ 30 | $1612.90\frac{10}{31}$

240. ‖ 31 | 14980 ‖ 32 | 10562.50 ‖ 33 | 20000 ‖ 34 | 260000

240. ‖ 35 | $1\frac{1}{5}$% ‖ 36 | $426 ‖ 37 | $400 ‖ 38 | 170

241. ‖ 39 | 548.80 ‖ 40 | 350, 1st; 525, 2d; 70, gain. ‖ 41 | 6

241. ‖ 42 | 4375 ‖ 43 | 45% ‖ 44 | 25.65 lost. ‖ 45 | 3450, cost;

241. ‖ 5% loss. ‖ 46 | 40% ‖ 47 | $10\frac{2}{3}$% ‖ 48 | 339, cost; 508.50

242. ‖ 1 | 188.50, com.; 7351.50 paid over. ‖ 2 | 40.77, com.;

242. ‖ 1359 laid out. ‖ 3 | 34.8375 ‖ 4 | 164.53125

243. ‖ 5 | 96.33; 5831.67 ‖ 6 | 163.80, com.; 4340.70, cost.

243. ‖ 7 | 115.39$\frac{1}{8}$ ‖ 8 | 6835.283 ‖ 9 | 935 ‖ 10 | 420.922

243. ‖ 11 | $70 ‖ 12 | 2571.36 ‖ 13 | 39.1875, charges; 1267.0625

243. ‖ trans. ‖ **244.** ‖ 14 | 11764.705+; 235.295+ ‖ 15 | 15 tons

244. ‖ 16 | 63.625, com.; 4544.642+ bu. ‖ 17 | 158 bbls.,

244. ‖ $2412.66 ‖ 18 | 183.0607+; 3.6612+ ‖ 19 | 2$\frac{86}{557}$ %

244. ‖ 20 | 2$\frac{782}{1359}$ % ‖ 21 | 8$\frac{4}{7}$ % ‖ 22 | 11$\frac{1}{3}$ % ‖ **246.** ‖ 1 | 43.875

246. ‖ 2 | 60.9875 ‖ 3 | 224.91 ‖ 4 | 360.2832 ‖ 5 | 473.844

246. ‖ 6 | 1312.50 ‖ 7 | 283.8438 ‖ 8 | 422.8976 ‖ 9 | 1112.90

246. ‖ 10 | 265.2345 ‖ 11 | 1893.75 ‖ 12 | 373.2495 ‖ 13 | 735

246. ‖ 14 | 1016.075 ‖ 15 | 120.80 ‖ 16 | 5796 ‖ 17 | 20.909

246. ‖ 18 | 26.313 ‖ 19 | 458.88 ‖ 20 | 1979.5013$\frac{1}{3}$ ‖ 21 | 5618.75

246. ‖ 22 | 628.416$\frac{2}{3}$ ‖ 23 | 64.0625 ‖ 24 | 157.65625

249. ‖ 2 | 42.24325 ‖ 3 | 420.253125 ‖ 4 | 213 ‖ 5 | 181.25

249. ‖ 6 | 11.0415 ‖ 7 | 132.7707+ ‖ 8 | 26.9586+ ‖ 9 |

249. ‖ 416.1673+ ‖ 10 | 1334.2187+ ‖ 11 | 120.0693+

249. ‖ 12 | 40.0968 ‖ 13 | 81.6778+ ‖ 14 | 162 ‖ 15 | 221.266

249. ‖ 16 | 389.2466 ‖ 17 | 135.3714 ‖ 18 | 42.9404+

249. ‖ 19 | 84.6855 ‖ 20 | 55.6685+ ‖ 21 | 32.666

250. ‖ 22 | 8590.8333$\frac{1}{3}$ ‖ 23 | 36 ‖ 24 | 93.7843+ ‖ 25

250. ‖ 160.4408+ ‖ 26 | 12.963+ ‖ 27 | 82.036+ ‖ 28 | 70.964

250. ‖ 29 | 879.46703+ ‖ 30 | 801.769 ‖ 31 | 933.1573+

250. ‖ 32 | 499.339+ ‖ 33 | 140.6444+ ‖ 34 | 5085

251. ‖ 35 | 403.858 ‖ 36 | 9337.50 ‖ 1 | 394.325625 ‖ 2 | 697.986

251. ‖ 3 | 3339.613 ‖ 252. ‖ 4 | 823.902+ ‖ 5 | 4640.532+

252. ‖ 6 | 1976.6305+ ‖ 2 | £45 8s. 1¾d. ‖ 253. ‖ 3 | £45 12s. 4d.+

253. ‖ 4 | £154 7s. 0d. 2 far. ‖ 5 | £1133 10s. 9¼d. ‖ 6 |

253. ‖ £199 6s. 3¾d. ‖ 7 | £6 16s. 5d. ‖ 255. ‖ 2 | 5359.3664+

255. ‖ 3 | 8925.5443 ‖ 4 | 1127.041 ‖ 5 | 190.758 ‖ 256. ‖ 6 |

256. ‖ 156.20+ ‖ 257. ‖ 2 | 3976.782⅔ ‖ 3 | 439.80 ‖ 4 | 6234.76

257. ‖ 5 | 30000 ‖ 6 | 952.576+ ‖ 7 | 7% ‖ 8 | 10% ‖ 9 | 5½%

258. ‖ 2 | Amount, $200.3575; Prin., $175; Ans. 25.3575

258. ‖ 3 | Am't, $291.7215 ‖ 259 ‖ 4 | Am't, $664.66583281;

259. ‖ Prin., $540.05; A. 124.61583281 ‖ 5 | Am't, $151.5812

259. ‖ 6 | Am't, $926.744 ‖ 7 | $4773.567 ‖ 8 | $48165.9388+

259. ‖ 9 | $14523.55509+ ‖ 260. ‖ 1 | 562.50 ‖ 2 | 184.499+

261. ‖ 3 | 21 ‖ 4 | 5000 ‖ 5 | 1902.587+ ‖ 6 | 236.438 = dis.;

261. ‖ 2763.562 pres. value. ‖ 7 | 1379.6123+ ‖ 8 | 3538.0835+

261. ‖ cash value; 388.0835+ gain. ‖ 9 | 9890.23864+

261. ‖ 10 | .00414+ at 7½ cts. ‖ 11 | 13.33⅓ ‖ 12 | 2369.2617,

261. ‖ cash value; 61.9883, diff. ‖ 265. ‖ 1 | 6.15 ‖ 2 | 7.65

265. ‖ 3 | 23.2913 dis.; 476.708 pres. value. ‖ 4 | 1225.3555

265. ‖ 5 | 4.375 ‖ 6 | 82.5916 gain. ‖ 7 | 11.785 ‖ 8 | 15.4044

265. ‖ 9 | 981.21 ‖ 10 | 474.375 ‖ 266. ‖ 2 | 296.50 ‖ 3 | 697.20

266. ‖ 4 | 1041.666⅔ ‖ 5 | 3522.092 ‖ 268. ‖ 2 | 387. ‖ 3 | 90

268. ‖ 4 | 2559.06, A's; 3210.6375, B's. ‖ 5 | 153 ‖ 2 | 5320

268. ‖ 3 | 666 ‖ 4 | 17455.50 ‖ 269. ‖ 5 | 59110 ‖ 6 | 21375

269. ‖ 7 | 7999.6875 ‖ 8 | 213500 ‖ 9 | 307 ‖ 2 | 3529.411+

270. ‖ 3 | 56 ‖ 4 | 4000 ‖ 5 | 7235.142+ ‖ 6 | 800C

270. ‖ 7 | 10432.432+ ‖ 2 | 8 % ‖ **271.** ‖ 3 | 8 % ‖ 4 | 8 %

271. ‖ 5 | 5 % ‖ **272.** ‖ 2 | 20 % ‖ 3 | $41\frac{2}{3}$ % ‖ 4 | $12\frac{1}{2}$ % premium.

272. ‖ 2 | 7 % best. ‖ 3 | 8 % best. ‖ 4 | $166.66\frac{2}{3}$ ‖ **274.** ‖ 1 |

274. ‖ 5168.59 ‖ 2 | 158.40 ; 237.60 ‖ 3 | 252 ; 126 ‖ 4 | 300

274. ‖ 5 | 89.55 ‖ 6 | $47.81\frac{1}{4}$ ‖ 7 | $1252.12\frac{1}{2}$ ‖ 8 | 163.80

274. ‖ 9 | 16481.25 ‖ 10 | $5\frac{1}{2}$ % ‖ **275.** ‖ 11 | $1\frac{3}{5}$ % ‖ 12 | $4\frac{1}{5}$ %

275. ‖ 13 | 24000 ‖ 14 | 9020 ‖ 15 | 127.4625 ‖ 16 | 298.2546

277. ‖ 1 | 121.72 ‖ 2 | 232.50 ‖ 3 | 262.50 ‖ 4 | 20 ‖ 5 | 98.20

277. ‖ 6 | 120 ‖ 7 | 9101.635 ‖ **278.** ‖ 1 | 411.15 ‖ 2 | 757.908

279. ‖ 3 | 1227.395 ‖ **280.** ‖ 1 | 7051.63415 ‖ 2 | 9049.53795

280. ‖ 3 | 23058.6765 ‖ **281.** ‖ 4 | 2195.95 ‖ 5 | 2159.613+

282. ‖ 1 | $\frac{23}{50}$ % ‖ 2 | 37901125 ‖ **283.** ‖ 3 | $1\frac{1}{2}$ % ; 82.25, A's tax ;

283. ‖ 56.9075, B's tax. ‖ 4 | $\frac{3}{4}$ % ; 15.50 ‖ 5 | 5820 ‖ 6 |

283. ‖ 22236.197 ‖ 7 | 4656.05, whole tax ; 5 mills on $1 ; $27 ;

283. ‖ 6.8775, G's tax ; 12.78, H's tax. ‖ **284.** ‖ 8 | $1\frac{1}{2}$ cts. on $1 ;

284. ‖ 112.50 ; 18. ‖ 9 | 7.40 ; 9.225 ‖ **285.** ‖ 1 | 12 mo. ‖ 2 | 9

285. ‖ 3 | $8\frac{2}{3}$ mo. ‖ 4 | 7 mo. 3 da. ‖ 5 | $6\frac{1}{4}$ mo. ‖ 6 | 6 mo. 6 da.

286. ‖ 7 | $26\frac{1}{2}$ da., or July 28th. ‖ **287.** ‖ 2 | $28\frac{51}{213}$, or April

287. ‖ 29th, Eq. time of purchase ; Dec. 29th, Eq. time of paym't.

287. ‖ 3 | $78\frac{5}{17}$ da., or Oct. 18th. ‖ **288.** ‖ 4 | $76\frac{55415}{167338}$ da., or

288. ‖ July 13th. ‖ 5 | $21\frac{3}{562}$, or 21 days. ‖ 2 | $9\frac{3}{8}$mo

289 ‖ 3 | $5\frac{5}{8}$ mo. ‖ 2 | $2\frac{7}{9}$ mo. ‖ 3 | $5\frac{3}{5}$ mo. ‖ 4 | 421 da., from

289. ‖ Jan. 1st ; or on Feb. 26d, next year. ‖ 5 | July 8th, 1857.

291. ‖ 2 | $.58 int. balance ; $700.58 cash balance. ‖ **292.** ‖ 3 |

292. ‖ $46.20 int. balance ; $403.80 cash balance. ‖ 4 | $.70 int.

292. ‖ balance ; $620.70 cash balance. ‖ 293. ‖ 1 | 109 da. from

293. ‖ April 2d, or Dec. 14, 1860. ‖ 294. ‖ 2 | 2449.75 balance ;

294. ‖ April 9th. ‖ 3 | July 13th. ‖ 295. ‖ 1 | $.50 ‖ 2 | $.66

296. ‖ 3 | $.49 ‖ 4 | $1.00 ‖ 5 | 75° ‖ 6 | 19 ‖ 7 | $.13½

296. ‖ 8 | $.30 loss. ‖ 298. ‖ 1 | 1 lb. at 8 cts. ; 1 lb. at 10 cts.

298. ‖ 3 lb. at 14 cts. ‖ 2 | 1 lb. of each. ‖ 299. ‖ 3 | 1 calf,

299. ‖ 2 cows, 1 ox, 1 colt. ‖ 4 | 3 gallons. ‖ 300. ‖ 1 | 20 lb. o

300. ‖ each ‖ 2 | 75 lb. of each. ‖ 3 | 36 gal. at 7s. ; 24 gal. at

300. ‖ 7s. 6d. and 9s. 6d. ; 12 gal. at 9s. ‖ 4 | 10 at $2, and 15 at

300. ‖ $¾ ‖ 5 | 25 lb. at 5 and 7 ; 100 lb. at 7½ ; 37½ at 9⅓ ; and

300. ‖ 50 at 10 ‖ 301. ‖ 1 | 22 lb. of each. ‖ 2 | 9 gal. water,

301. ‖ 40½ gal. at $2.50, and 13½ gal. at $3.00 ‖ 3 | 12 sheep, 16

301. ‖ lambs, 12 calves. ‖ 4 | 8 at $6, 8 at $7, 4 at $19

301. ‖ 5 | 90 gal. at 4s., and 10 gal. each at 6s., 8s., and 10s.

301. ‖ 6 | 6 vests, 12 pants, 6 coats. ‖ 7 | 30 at 15, 4 each of 20,

301. ‖ 22, and 24 ‖ 8 | 10 at $½, 15 at $1, 10 at $5

304. ‖ 3 | $260.9932 ‖ 4 | 713.37 ‖ 5 | 6 T. 14 cwt. 1 qr. 16.68 lb.

304. ‖ 6 | 6 T. 13 cwt. 2 qr. 4 lb ; $308.4774 ‖ 7 | 792.612

305. ‖ 8 | 1196.343+ ‖ 9 | 255.835+ ‖ 10 | 4.09+ ‖ 11 |

305. ‖ 398.1199 ‖ 12 | 466.27875 ‖ 13 | 1101.24 gain ; 14 cts., price

305. ‖ 14 | 7936.50 ‖ 15 | 820.4625 ‖ 16 | 423.36 ‖ 17 | 251.453+

306. ‖ 18 | 1457.75 ‖ 19 | 22.605 cwt., tare ; $68.5856 duty

307. ‖ 1 | $225\frac{9}{19}$ tons. ‖ 2 | $438\frac{45}{76}$ tons. ‖ 3 | $729\frac{97}{855}$ tons.

307. ‖ 4 | 1006.57+ tons. ‖ 316. ‖ 1 | 8591.975 ‖ 2 | 8637.16875

316. ‖ 3 | 9777.636 ‖ 4 | 9970 ‖ 5 | $15006.305 ‖ 6 | 9801.9299+

317. ‖ 2 | 176204.4729+ ‖ **318.** ‖ 3 | £14014 17s 7½d.

318. ‖ 4 | 6005.368+ ‖ 5 | 807.874+ ‖ 6 | 9096.806+

319. ‖ 2 | 7 % premium. ‖ 3 | 12286.06 ‖ 4 | 84597 francs 66

319. ‖ centimes. ‖ 1 | 6657.693 ‖ 2 | 1250.52 ; 3 % nearly below

319. ‖ par. ‖ **321.** ‖ 2 | 5761.31+ florins. ‖ 3 | 9962.219+

322. ‖ 4 | 3495.839+ Sp. doll. ‖ **323.** ‖ 1 | 16 ‖ 2 | 225

323. ‖ 3 | 20164 ‖ 4 | 214369 ‖ 5 | 1795600 ‖ 6 | 605.16

323. ‖ 7 | .276676 ‖ 8 | 9.765625 ‖ 9 | .00274576 ‖ 10 | $\frac{9}{16}$

323. ‖ 11 | $\frac{36}{49}$ ‖ 12 | $\frac{49}{81}$ ‖ 13 | $\frac{1225}{7056}$ ‖ 14 | $\frac{15625}{61009}$ ‖ 15 | 58.140625

323. ‖ 16 | $250\frac{26}{121}$ ‖ 17 | 51030.81 ‖ 18 | 216 ‖ 19 | 13824

323. ‖ 20 | 1953125 ‖ 21 | 2515456 ‖ 22 | 20736 ‖ 23 | 59049

323. ‖ 24 | 76.765625 ‖ 25 | 10.4976 ‖ 26 | .0184528125

323. ‖ 27 | $\frac{3375}{4096}$ ‖ 28 | $\frac{125}{216}$ ‖ 29 | $\frac{81}{4096}$ ‖ 30 | $57\frac{681}{1024}$

323. ‖ 31 | $\frac{390625}{531441}$ ‖ 32 | 14886.936 ‖ 33 | .000244140625

323. ‖ 34 | 2893640.625 ‖ **327.** ‖ 3 | 7 ‖ 4 | 12 ‖ 5 | 15 ‖ 6 | 48

327. ‖ 7 | 89.409+ ‖ 8 | 2505 ‖ 9 | 137.84+ ‖ 10 | 1663.8677+

327. ‖ 11 | 191.713+ ‖ 12 | 1000 ‖ 13 | 311.011+ ‖ 14 | 173.853+

329. ‖ 4 | $\frac{6}{9}$ ‖ 5 | $\frac{15}{48}$ ‖ 6 | .14 ‖ 7 | 2.5 ‖ 8 | 16.7 ‖ 9 | .453 ‖ 10 |

329. ‖ 93+ ‖ 11 | .9682+ ‖ 12 | .1581+ ‖ 13 | $\sqrt{5\frac{4}{9}}=2\frac{1}{3}$. *Ans.*

329. ‖ 14 | $\sqrt{.7994}=.89409+$ *Ans.* ‖ 15 | $\sqrt{.222\frac{2}{9}}=.4714+$ *Ans*

329. ‖ 16 | .779+ ‖ 17 | .149 ‖ 18 | 5.01 ‖ 19 | 14.015

329. ‖ 20 | 1.2247+ ‖ 21 | $\frac{53}{79}$ ‖ 22 | $\frac{3}{7}$ ‖ 23 | .2828+ ‖ 24

329. ‖ 11.618+ ‖ 25 | .885+ ‖ 26 | 75.15 ‖ 27 | 400.06

331. ‖ 1 | 343 ‖ 2 | 221 ‖ 3 | $21\frac{2}{11}$ ‖ 4 | 60 rd. wide ; 180 rd. long.

331. ‖ 5 | 40 rows ; 80 trees in a row ; 10 A. 0 R. 29 P. $168\frac{3}{4}$ sq.

√331. ‖ ft. area. ‖ 6 | 75 ft. ‖ 332. ‖ 7 | 135 ‖ 8 | 94.708+

332. ‖ 9 | $53.33\frac{1}{3}$ ft. ‖ 10 | 8.66+ ft. ‖ 11 | 825.8 mi. ‖ 12 | $100

332. ‖ 13 | 75 ‖ 14 | 28.28+ ft. ‖ 333. ‖ 15 | 6 in. ‖ 16 |

333. ‖ 11.041+ rd. ‖ 17 | 4.405+ in., 1st man's share

333. ‖ 5.739+ in., 2d man's share ; 13.856+ in., 3d man's share

336. ‖ 1 | 12 ‖ 2 | 49 ‖ 3 | 36 ‖ 4 | 247 ‖ 5 | 179 ‖ 6 | 364

336. ‖ 7 | 439 ‖ 8 | 3072 ‖ 337. ‖ 1 | 2.028+ ‖ 2 | 12.0016+

337. ‖ 3 | .232+ ‖ 4 | 27.0002+ ‖ 5 | .729+ ‖ 6 | .015

337. ‖ 7 | .188+ ‖ 8 | 4.339+ ‖ 1 | $\frac{4}{5}$ ‖ 2 | $\frac{7}{9}$ ‖ 3 | $3\frac{1}{7}$ ‖ 4 | $4\frac{1}{2}$

337. ‖ 5 | $\frac{7}{8}$ ‖ 6 | $\frac{9}{25}$ ‖ 7 | $\frac{27}{64}$ ‖ 8 | $\frac{24}{35}$ ‖ 9 | 1.987+ ‖ 10 | 3.83+

338. ‖ 1 | 27 ft. ‖ 2 | 19 ft. long ; 2166 sq. ft. area. ‖ 3 | 36 ft.,

338. ‖ length of side. ‖ 4 | 8.57+ ft. ‖ 5 | 9.77+ ft., length ;

338. ‖ 19.54+ ft., height. ‖ 6 | 10.125 cu. ft. ‖ 7 | 45 cts. per yd. ;

338. ‖ 2025 yd. ‖ 9 | 64 lb. ‖ 10 | 8 ft., length of side. ‖ 11 | 8

338. ‖ 12 | $1331 ‖ 13 | 12 in. long ; 6 in. wide ; 1 in. thick.

339. ‖ 14 | 24 ft. long ; 20 ft. wide ; 9 ft. deep. ‖ 15 | 20 ft.

339. ‖ 16 | .54+ in., 1st woman's share ; .69+ in., 2d woman's

339. ‖ share ; .99+ in., 3d woman's share ; 3.77+ in., 4th

339. ‖ woman's share. ‖ 340. ‖ 1 | 89 ‖ 2 | $80 ‖ 341. ‖ 3 | $396

341. ‖ 4 | $1550 ‖ 5 | $17\frac{1}{2}$ rd. ‖ 6 | 201 ft. ‖ 342. ‖ 1 | 5 miles

342. ‖ 2 | $2 ‖ 3 | $\frac{3}{4}$ inches. ‖ 4 | 15, 18, 21, 24, 27, 30, 33

343. ‖ 1 | 2730 ‖ 2 | 226 last term ; $64.96 whole. ‖ 3 | $791\frac{1}{4}$ mi

343. ‖ 4 | 10 mi. 7 fur. 27 rd. $1\frac{1}{2}$ yd. ‖ 344. ‖ 1 | 5551

344. ‖ *2* | *13 da.* ; 312 m. ‖ 3 | 6 ‖ 346. ‖ 1 | 1 ‖ 2 | 3125000

346. ‖ 3 | $\frac{1}{27}$ ‖ 4 | $100000000000 ‖ 5 | $3200 ‖ 6 | $54000

346. ‖ 7 | $327.68 ‖ 8 | $595.508 ‖ 347. ‖ 1 | 118081 ‖ 2 | 2044

347. ‖ 3 | 11184810 ‖ 4 | $42949672.95 ‖ 348. ‖ 5 | 938249

348. ‖ 922+ ships. ‖ 355. ‖ 16 | $4.4954 ‖ 359. ‖ 22 | 4166.40

359. ‖ 23 | $9\frac{5}{8}$ ‖ 24 | 36 ft. ‖ 25 | 1770 ‖ 360. ‖ 26 | $2\frac{19}{28}$

360. ‖ 27 | $4.33\frac{1}{3}$ ‖ 28 | 100 ‖ 29 | 5500 yd. ‖ 30 | 117 ‖ 31 | 693

360. ‖ 32 | 11 cts. ‖ 33 | $56 ‖ 34 | $5600 ‖ 35 | $7\frac{1}{5}$ cts. ‖ 36 |

360. ‖ $43 ‖ 37 | $246.75 gain. ‖ 361. ‖ 38 | 5 hr. 27 min. $16\frac{4}{11}$ sec

361. ‖ 39 | 40 ‖ 40 | 10 ‖ 41 | 36 days. ‖ 42 | $1\frac{13}{47}$

361. ‖ 43 | 60 = 1st part; 100 = 2d part; 140 = 3d part;

361. ‖ 180 = 4th part. ‖ 44 | $16\frac{4}{5}$ in. ‖ 45 | $2\frac{5}{14}$ mo. ‖ 46 | $2.20

361. ‖ 47 | $12 ‖ 48 | $1.20 ‖ 362. ‖ 49 | 78.652+ discount.

362. ‖ 50 | $129.60 ‖ 51 | $19.375 most advantageous on time.

362. ‖ 52 | 292.823 gain ‖ 53 | 122.70 = A's share; 163.60 =

362. ‖ B's share; 196.32 = C's share. ‖ 54 | $2317.15 = A's;

362. ‖ $1853.72 = B's; $2317.15 = C's; $2780.58 = D's.

362. ‖ 55 | $95.10 = A's; $95.10 = B's; $133.14 = C's;

362. ‖ 152.16 = D's. ‖ 56 | $7\frac{1}{7}$ oz. ‖ 57 | $8\frac{8}{9}$ da. ‖ 58 | 17 times.

363. ‖ 59 | $4\frac{12}{17}$ da. ‖ 60 | 5 mo. 24 da. ‖ 61 | 68 da. ‖ 62 | 126 gal.

363. ‖ 63 | $172.78 loss on stocks. ‖ 64 | $3312.417+ ‖ 65 | $42.60

364. ‖ 66 | 4 yd. ‖ 67 | 140 miles. ‖ 68 | $2, the 1st; $6, the

364. ‖ second. ‖ 69 | 100 ‖ 70 | $3825 ‖ 71 | $144 gain by borrowing. ‖ 72 | $36000 ‖ 73 | 41.183+ ‖ 365. ‖ 74 | The

365. ‖ second, 10 days after the 3d; the 1st, 8 days after the

365. ‖ 2d, or 18 days after the 3d. ‖ 75 | $6890 ‖ 76 | 100 A.

365. ‖ 1st Co.; 88 A., 2d Co.; \$7 per acre. ‖ 77 | 74 da., or

365. ‖ March 16th. ‖ 78 | 512 slabs; $\$302.22\frac{2}{9}$ ‖ 79 | 350, A's;

365. ‖ 210, C's; 297.50, B's; 175, D's; 122.50, E's. ‖ 80 | 72

365. ‖ 81 | 5 hr 20 min. P. M. ‖ 82 | $\$0.66\frac{2}{3}$ ‖ 83 | 24 chickens

365. ‖ and 36 turkeys. ‖ 366. ‖ 84 | 8 days. ‖ 85 | 1797.50, 1st

366. ‖ 2157, 2d; 2516.50, 3d. ‖ 86 | \$640 stock, and \$120 gain

366. ‖ 2d.; \$960 stock, and \$180 gain, 1st. ‖ 87 | 49.945+ ft

366. ‖ 88 | $\frac{8}{9}$ wk. ‖ 89 | 11$\frac{2}{3}$ hr.; 134$\frac{1}{6}$ miles. ‖ 90 | 533$\frac{1}{3}$, A's;

366. ‖ 888$\frac{8}{9}$, B's; 177$\frac{7}{9}$, C's. ‖ 91 | 36$\frac{1}{2}$ days. ‖ 367. ‖ 92

367. ‖ 84485.006+ ft. ‖ 93 | \$206.06+ in favor of 1st invest.

367. ‖ 94 | 23599680 cu. yd. ‖ 95 | 4646.363+ ‖ 96

367. ‖ \$1555.017+, 1st; 4354.717+, 2d; 4304.663+, 3d,

367. ‖ 5781.263+, 4th; 4004.338+, 5th. ‖ 97 | 2160

367. ‖ 98 | \$564, A's; \$423, B's. ‖ 368. ‖ 99 | \$31 ‖ 100 | 8 hr.

368. ‖ 101 | \$30 com. diff.; \$2460 cost. ‖ 102 | \$14467.505

368. ‖ 103 | 97$\frac{1}{5}$ lb. ‖ 104 | $\frac{3}{4}$ ct., cost; $\frac{4}{5}$ ct. sold for; $\frac{1}{20}$ ct.,

368. ‖ gain on each; 80 eggs sold. ‖ 105 | 84 years old.

369. ‖ 106 | 942.48+ cubic feet. ‖ 107 | 155 A. 3 R. 38.72 P

369. ‖ 108 | \$365.837+ ‖ 109 | 6$\frac{6}{19}$ hours. ‖ 110 | 5 inches

369. ‖ 111 | \$4006.54+ ‖ 371. ‖ 2 | 36 A. ‖ 3 | 5 A. 1 R. 15 P. ‖ 4

371. ‖ 135 A. ‖ 372. ‖ 1 | 437 A. 2 R. 34.32+ P. ‖ 2 | 291 A. 2 R. 16 P

372. ‖ 3 | 35 A. 0 R. 25 P. ‖ 4 | 20 A. ‖ 5 | 40 A. ‖ 6 | 15 A

372. ‖ 7 | 24 A. 1 R. 8 P. ‖ 8 | 26 A. 3 R. 20 P. 5 sq. yd.

372. ‖ 9 | 120 feet. ‖ **373.** ‖ 2 | 21 A. 0 R. 39.824 P. ‖ 3 |

373. ‖ 221.875 sq. ft. ‖ 4 | 704.125 sq. yd. ‖ 5 | 60 A. 3 R. 12.8 P.

373. ‖ 6 | 270 A. 1 R. 24 P. ‖ **374.** ‖ 2 | 584.3376 ‖ 3 | 125.664

374. ‖ 4 | 179.0712 ‖ 1 | 50 ‖ 2 | 7418 ‖ 3 | 4360.835+

374. ‖ 1 | 113.0976 ‖ 2 | 19.635 ‖ 3 | 153.9384 ‖ **375.** ‖ 4 |

375. ‖ 1.069+ ‖ 5 | 20 A. 0 R. 16.9984 P. ‖ 1 | 113.0976

375. ‖ 2 | 615.7536 ‖ 3 | 4071.5136 ‖ 4 | 196996571.722104 sq. mi.

376. ‖ 2 | 268.0832 ‖ 3 | 2144.6656 cu. in. ‖ 4 | 259992792-

376. ‖ 082.6374908 ‖ 5 | .9047808 cu. ft. ‖ **377.** ‖ 1 | 9100 sq. ft

377. ‖ 2 | 1440 sq. ft. ‖ 2 | 110592 cu. in. ‖ 3 | 42$\frac{2}{3}$ cu. ft.

377. ‖ 4 | 315$\frac{65}{77}$ gal. ‖ 5 | 13820 cu. ft. ‖ **378.** ‖ 1 | 2513.28

378. ‖ 2 | 233.33$\frac{1}{3}$ sq. ft. ‖ 3 | 2827.44 sq. in. ‖ 4 | 6283.2 sq. ft.

379. ‖ 2 | 36442.56 ‖ 3 | 13571.712 ‖ 4 | 9650.9952

379. ‖ 5 | 7363.125 ‖ 2 | 4380 ‖ 3 | 2484 ‖ 4 | 5620

380. ‖ 5 | 5760 ‖ 6 | 14400 ‖ 7 | 1800 ‖ 2 | 9160.9056

380. ‖ 3 | 8659.035 ‖ 4 | 2827.44 ‖ **382.** ‖ 2 | 32.4938 inches.

382. ‖ 3 | 28.2574 in. ‖ **383.** ‖ 1 | 197.459+ gal. wine. ‖ 2 |

383. ‖ 136.9209+ gallons wine; 112.7583+ gallons beer.

383. ‖ 3 | 148.3772+ gal. wine. ‖ **385.** ‖ 1 | 40 lb. ‖ 2 | 25 lb

385. ‖ 3 | 50 lb. ‖ 4 | 20 lb. ‖ 5 | 40 lb. ‖ 6 | 1 in.; 1$\frac{1}{2}$ in.; 2 in.; 4 in.

386. ‖ 7 | 64 lb. ‖ 8 | 150 lb. ‖ **388.** ‖ 1 | 60 lb ‖ 2 | 40 lb.

388. ‖ 3 | 25 lb. ‖ 1 | 7$\frac{1}{2}$ ft. ‖ **389.** ‖ 2 | 1$\frac{1}{2}$ ft. ‖ 1 | 40 lb.

389. ‖ 2 | 100 lb. ‖ 3 | 60 lb. ‖ **390.** ‖ 1 | 576 ‖ 2 | 2250

390. ‖ 3 | 1066$\frac{2}{3}$ ‖ 4 | 3000 ‖ **391.** ‖ 1 | 259200 lb. ‖ 2 | 1.47+ lb.

392. ‖ 3 | 1.15+ lb. ‖ 4 | 1.2 in. ‖ **393.** ‖ 1 | 369$\frac{11}{12}$ ft.; 2316 ft.

394. ‖ 2 | $3618\frac{3}{4}$ ft.; $482\frac{1}{2}$ ft. ‖ 3 | $223\frac{161}{193}$ ft. ‖ 4 | $2\frac{1}{2}$ sec. nearly.

394. ‖ 5 | $1608\frac{1}{3}$ ft. space; $321\frac{2}{3}$ velocity. ‖ 6 | 2 mi. $4984\frac{8}{193}$ ft.

394. ‖ 7 | $164.69\frac{1}{3}$ ‖ 8 | $100.52\frac{1}{12}$ ft. ‖ 9 | $397\frac{179}{193}$ ft.; $4\frac{188}{193}$ sec.

394. ‖ 10 | $160\frac{5}{6}$ ft. = velocity; $402\frac{193}{2316}$ ft. ‖ 11 | $197\frac{1}{48}$ ft

394. ‖ 12 | $77\frac{139}{193}$ sec. ‖ 13 | $1904\frac{28}{193}$ ft. = height; $10\frac{179}{193}$ =

394. ‖ time of ascent. ‖ 14 | $94\frac{98}{193}$ sec. ‖ 15 | 1447.5

395. ‖ 16 | 61.24 sec. ‖ 17 | 14.28+ sec. ‖ 18 | $15050\frac{118}{193}$ ft

395. ‖ 19 | 12 sec.; 2316 ft. ‖ **396.** ‖ 1 | 8.857 ‖ 2 | $38\frac{138}{185}$ cu. ft

397. ‖ 3 | .980 ‖ 4 | 2 ft. 11.388 in. ‖ 5 | 190 T. 709 lb. ‖ 6 | 2.75

397. ‖ 7 | 7.234+ ‖ 8 | .786 ‖ 9 | .875 ‖ 10 | 177 lb. 5 oz

397. ‖ 11 | 1.103 ‖ 398. ‖ 1 | 3.49 qt. ‖ 2 | 37.5 ‖ 3 | 2.46 gr

398. ‖ 4 | $\frac{25}{47}$ = .5319.

MJ